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COMPUTER MODELING OF SURFACE RAYLEIGH WAVES IN THE ELASTIC MEDIA

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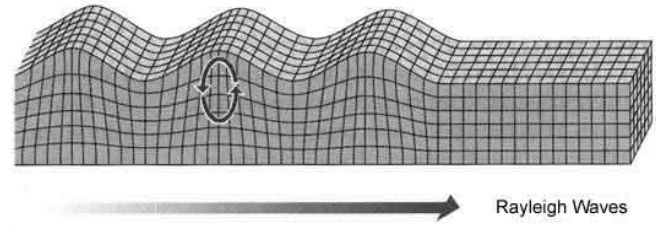

Fig. 1. Rayleigh waves

Perturbations on a surface bounding an elastic half-space can cause the propagation of waves due to the fact that their amplitude reaches a maximum on the surface and that they decay rapidly with depth. These waves are of great importance in seismology. Consider the elastic half-space $x_1 \geq 0$ and assume that the surface wave arrives in the direction of the x_2 axis. This kind of wave can arise if the perturbation causing it does not depend on the variable x_3. The independence of the surface wave from the variable x_3 is the reason that $u_3 = 0$ and $\varepsilon_{33} = \varepsilon_{13} = \varepsilon_{23} =$Here we are dealing with a skew deformed state.

The motion recorded in three-dimensional problems with the help of the potentials φ and ψ in the form

$$u_i = \varphi_i + \epsilon_{ijk}\psi_{kj}, \qquad i, j, k = 1,2,3, \tag{1}$$

will take the following form in the two-dimensional problem:

$$u_1 = \varphi_1 + \psi_2, \; u_2 = \varphi_2 - \psi_1, \tag{2}$$

where φ and ψ are scalar potentials.

The Lame equation is simplified:

$$ {}^2_1\varphi = 0, \qquad {}^2_2 = 0; \tag{3}$$

here

$$ {}^2_\alpha = \nabla_1^2 - \frac{1}{c_\alpha^2}\partial_t^2, \qquad \nabla_1^2 = \partial_1^2 + \partial_2^2, \; \alpha = 1,2.$$

The solution of the wave equations will be sought in the form

$$\varphi = \quad (x_1)\exp\left[-i(\omega t - kx_2)\right], \; \psi = \quad (x_1)\exp\left[-i(\omega t - kx_2)\right]; \tag{4}$$

hence it is seen that the harmonic wave is in time and propagate in the x_2 axis. The phase velocity, as yet unknown, is. Substituting (4) into (3), we obtain from the latter the ordinary differential equations $c = \omega/k$

$$\frac{d^2\Phi}{dx_1^2} - \nu_1^2\Phi = 0, \qquad \frac{d^2\Psi}{dx_1^2} - \nu_2^2\Psi = 0, \qquad \text{where} \qquad \nu_\alpha = \left(k^2 - \frac{\omega^2}{c_\alpha^2}\right)^{1/2}. \tag{5}$$

From the general solutions of equations (5) we choose only those that correspond to a decrease in the amplitude of the wave with depth:

$$ = Ae^{-\nu_1 x_1}, \quad = Be^{-\nu_2 x_1}, \qquad \nu_\alpha > 0\alpha = 1,2. \tag{6}$$

Adopted here the postulate of extinction waves with depth leads to the assertion $\nu_\alpha \; (\alpha = 1,2)$ that the values must be real and positive.

$$\sigma_{11}(0, x_2, t) = 0, \qquad \sigma_{12}(0, x_2, t) = 0, \qquad \sigma_{13}(0, x_2, t) = 0. \tag{7}$$

The latter condition is identically satisfied by the assumption of independence of the deformations of the variable x_3 as

$$\sigma_{11} = 2\mu\varepsilon_{11} + \lambda\varepsilon_{kk},\ \sigma_{12} = 2\mu\varepsilon_{12},\ \varepsilon_{kk} = \partial_1 u_1 + \partial_2 u_2, \qquad (8)$$

Then substituting the relations (2) into (8), we obtain

$$\sigma_{11} = 2\mu\partial_1^2\varphi + \lambda\nabla_1^2\varphi + 2\mu\partial_1\partial_2\psi,\ \sigma_{12} = \mu[2\partial_1\partial_2\varphi + \partial_2^2\psi - \partial_1^2\psi]. \qquad (9)$$

Substituting in (9) the expressions

$$\varphi = Aexp[-\nu_1 x_1 - i(\omega t - kx_2)],\ \ \psi = Bexp[-\nu_2 x_1 - i(\omega t - kx_2)]$$

From the compatibility condition for this system of homogeneous linear equations,

$$[\lambda(\nu_1^2 - k^2) + 2\mu\nu_1^2](k^2 + \nu_2^2) - 4\mu\nu_1\nu_2 k^2 = 0. \qquad (10)$$

This solution,

$$\left(2k^2 - \frac{\omega^2}{c_2^2}\right)^2 - 4v_1 v_2 k^2 = 0, \quad \text{or} \quad (2-\eta)^2 = 4(1-\vartheta\eta)^{1/2}(1-\eta)^{1/2} \qquad (11)$$

We need to add a few words about the velocity of the Rayleigh waves. We know that $c_R < c_2 < c_1$ and that in an unbounded space, longitudinal waves call changes in volume, and transverse waves - change in shape. The resistance change of the medium volume much larger than change in shape. Therefore, the phase velocity of longitudinal waves is greater than the phase velocity of transverse waves. Surface waves propagate near the boundary of the medium in the region of discontinuity of material constants between the elastic medium and the atmosphere. Near the boundary, the resistance of the medium to the propagation of waves is the least, the medium is more pliable. Therefore, the velocity of surface waves is less than the velocity of spatial (longitudinal and transverse) waves.

Surface waves are also of great importance in ultrasonic studies and in flaw detection in the study of surface defects of a structure.

The discovery of surface waves by Rayleigh theoretically and later their finding experimentally is an excellent example of the effectiveness and fruitfulness of theoretical studies.

References

1. Alexeeva L. A., Kupesova (Alipova) B. N, Method of generalized functions for boundary problems of coupled thermoelastodynamics // Journal of Applied Mathematics and Mechanics (of Russian Academy of Sciences), V.65, N 2, 2001, pp. 334-345

2. Alipova B. N. Method of Boundary integral equations (BIEM) and generalized solutions of transient problems of thermoelastodynamics. ICNPAA 2012, 9-15 July, AIP Conference Proceedings, 1493, 39(2012); doi: 10.1063/1.4765466, American Institute of Physics, Vien, Austria, http://dx.doi.org/10.1063/1.4765466, pp.39-46 http://www.scimagojr.com/journalsearch.php?q=26916&tip=sid&clean=0

3. Aytaliev Sh. M., L. A. Alexeyeva, Sh. A. Dildabayev, N. B. Zhanbyrbaev. Method of boundary integral equations.

4. Dortman N. B. (1984). Physical properties of rocks and minerals

5. Kupradze V. D, Gegelia T. G., Bashelishvili M. O., Burchuladze T. V. (1976). Three-dimensional problems Mathematical theory of elasticity and thermoelasticity. - M.: "Science".

6. Novacki V. (1975). Theory of elasticity. "MIR". Moscow.