

CONVECTIVE INSTABILITY AND NONLINEAR TRANSITION IN ROTATING-DISK BOUNDARY LAYERS OF AN INFINITE ROTOR-STATOR CAVITY

Xie Y., Du Q.

Advanced Gas Turbine Laboratory, Institute of Engineering Thermophysics,
Chinese Academy of Sciences, Beijing
xieyaguang@iet.cn

Key words: convective instability, rotating-disk boundary layer, stationary mode

The accurate prediction of flow state evolution and laminar-to-turbulent transition within rotor-stator cavities is fundamentally critical for the predictive analytics and advanced design of modern aero-engines [1]. Given that the flow in the rotating-disk boundary layer of an infinitely rotor-stator cavity closely resembles the von Kármán boundary layer, it theoretically exhibits both convective and absolute instabilities [2]. This study systematically investigates the convective instability characteristics induced by the stationary mode—defined precisely by a temporal frequency of $\omega = 0$ in a co-rotating frame—and its subsequent transition process to turbulence [3].

To isolate the specific velocity distribution of the rotating disk boundary layer, the laminar self-similarity solution for the infinite rotor-stator cavity model is transformed into the framework of the Bödewadt-Ekman-von Kármán (BEK) model [4], corresponding to a Rossby number of $Ro = -0.751$. A stepwise methodology involving local linear stability analysis (LLSA), global linear simulations, and direct numerical simulations (DNS) is employed using the spectral element solver Nek5000 [5, 6].

Local linear stability analysis reveals three primary families of instability modes: inviscid crossflow (Type I), viscous (Type II), and upstream-propagating (Type III) modes. The critical Reynolds number for the onset of convective instability is determined by $Re = r^*|Ro|/\delta^*$. Where r^* is the radius position, and δ^* represents the viscous length scale. The critical onset is identified at $Re = 204$, corresponding to an azimuthal wavenumber of $\beta = 27$, as illustrated by the neutral curves.

Global linear simulations demonstrate that roughness elements can trigger convective instability spiral waves. For instability spiral waves with different azimuthal wavenumbers, their spatial evolution characteristics align strictly with predictions derived from the spatial mode analysis of the local linear stability theory. As illustrated in Fig. 1, the contours representing the growth rates from the global linear simulations are superimposed with the neutral curves (solid and dashed lines) predicted by the local linear theory, demonstrating excellent agreement.

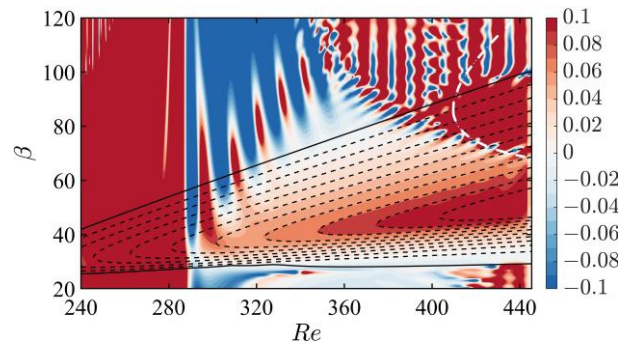


Fig. 1 – Comparison of the growth rates in the $Re - \beta$ plane: contours represent global linear simulation results, and lines denote local linear stability theory predictions

In direct numerical simulations, radially developing convective instability spiral waves enter a nonlinear regime following linear saturation, characteristic of supercritical transition

behavior. At lower Reynolds numbers, the first nonlinear growth phase does not lead to turbulence. However, a second nonlinear growth phase culminates in full-spectrum energy filling at $Re = 485$, which subsequently triggers localized turbulence. Fig. 2 shows the vortex structures highlighting the secondary spiral wave structures appearing above the larger primary spiral wave structures.

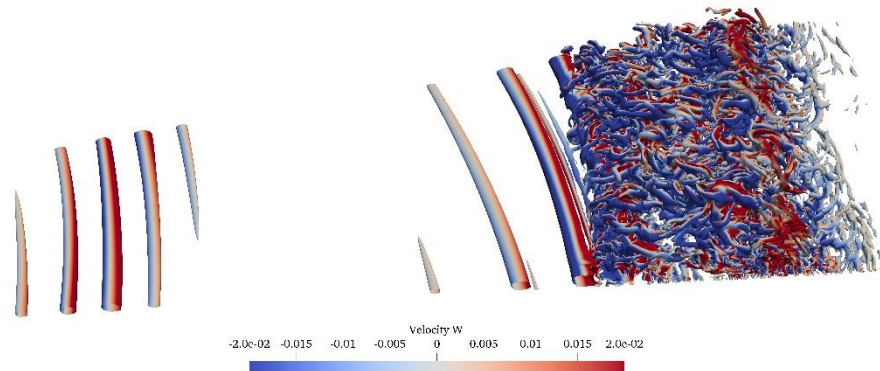


Fig. 2 – Q -criterion vortex structure visualization at $Q = 20$

In conclusion, all three computational methodologies reveal the same underlying physical mechanism of convective instability in the rotating disk boundary layer. The precise description of this instability behavior and the subsequent non-linear saturation provides a rigorous theoretical foundation for the aerodynamic optimization of rotating cavity flows in wide variable Rossby number environments.

References

1. Launder B., Poncet S., Serre E. Laminar, transitional, and turbulent flows in rotor-stator cavities // Annual Review of Fluid Mechanics. – 2010. – Vol. 42. – P. 229-248.
2. Lingwood R.J. Absolute instability of the boundary layer on a rotating disk // Journal of Fluid Mechanics. – 1995. – Vol. 299. – P. 17-33.
3. Gregory N., Stuart J.T., Walker W. On the stability of three-dimensional boundary layers with application to the flow due to a rotating disk // Philosophical Transactions of the Royal Society of London. Series A. – 1955. – Vol. 248. – P. 155-199.
4. Faller A.J. Instability and transition of disturbed flow over a rotating disk // Journal of Fluid Mechanics. – 1991. – Vol. 230. – P. 245-269.
5. Appelquist E., Schlatter P., Alfredsson P.H., Lingwood R.J. Global linear instability of the rotating-disk flow investigated through simulations // Journal of Fluid Mechanics. – 2015. – Vol. 765. – P. 612-631.
6. Appelquist E., Schlatter P., Alfredsson P.H., Lingwood R.J. Transition to turbulence in the rotating-disk boundary-layer flow with stationary vortices // Journal of Fluid Mechanics. – 2018. – Vol. 836. – P. 43-71.

Information about the authors

Xie Yaguang – Postdoctoral Researcher, Institute of Engineering Thermophysics, Chinese Academy of Sciences. Field of scientific interests: fluid mechanics, rotating boundary layer stability.

Du Qiang – Professor, Institute of Engineering Thermophysics, Chinese Academy of Sciences. Field of scientific interests: fluid mechanics, rotating boundary layer stability.