Astropolarimetry: reduced form of statistical equilibrium equations

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In a typical astrophysical spectropolarimetry study, it is assumed that the gas surrounding an exoplanet is irradiatiated by the nearest star, causing polarization of the atoms in the gas. Then the polarized gas emits or scatters polarized light, which is observed from Earth (transit spectroscopy). The interaction between polarized atoms and polarized radiation is described by statistical equilibrium equations (SEE), which are usually given in a rather complicated form [1](=DL). In this work we reformulated them in a new form, which is shorter and its physical meaning is easily verifiable, but it is equivalent to initial DL's expressions. The SEEs for the density-matrix elements $\rho_{mm'} = \langle m | \rho | m' \rangle$ describe polarization of gas species by radiation: $\frac{d}{dt}\rho_{mm'} = -i2\pi\nu_{mm'}\rho_{mm'} + \sum_{nn'} \rho_{nn'} T_A(m,m',n,n') + \sum_{pp'} \rho_{pp'} [T_E(m,m',p,p') + \ldots$ where m, m', m'', n, n', … are eigenvectors of atomic Hamiltonian. In our case $|m\rangle = |\tau, j, m\rangle$ and $|m'\rangle = |\tau, j, m'\rangle, \ldots$ etc. Usually there are seven terms here, only three are shown. Let us consider only the second term, that describes absorption of light via transition $\tau_l j_l m_l \to \tau, j, m$. Using the algebra of irreducible spherical tensor operators (ITOs) DL rewrote the SEEs in the ITO-form like $\frac{d}{dt}\rho^K_Q(\tau j) = -i2\pi\nu_L g_{\tau j} Q \rho^K_Q(\tau j) + \mathrm{T}_A + \ldots$ where $\mathrm{T}_A \equiv \sum_{\tau_l j_l K_l Q_l} \rho^{K_l}_{Q_l}(\tau_l j_l) \sum_{K_r Q_r} 3(2j_l+1) \times$

$$\times(-1)^{K_l+Q_l}\sqrt{(2K+1)(2K_l+1)(2K_r+1)}\begin{pmatrix} K & K_l & K_r \\ -Q & Q_l & -Q_r \end{pmatrix}\begin{pmatrix} j & jl & k \\ j' & j'_l & k'_l \\ K & K_l & K_r \end{pmatrix} B(\tau_l j_l \to \tau j) \mathrm{J}^{K_r}_{Q_r}(\nu_{\tau_l j_l \to \tau j})$$

here Einstein coefficient is $B(\tau_l j_l \to \tau j) = = C_A |\langle \tau j \| d \| \tau_l j_l \rangle|^2 / (2j_l+1)$, the polarization tensor is $\mathrm{J}_{q'q}(\nu) \equiv \oint \mathrm{I}_{q'q}(\nu,\Omega) d\Omega / 4\pi$, where $\mathrm{I}_{q'q}(\nu,\Omega) = \sum_{\alpha,\beta=\pm 1} I_{\beta,\alpha}(\nu,\Omega) D^1_{\alpha q}(\Omega) D^1_{\beta q'}(\Omega)$, here $I_{\beta,\alpha}(\nu,\Omega)$ is the photon flux tensor, and $C_A \equiv 32\pi^4 / 3h^2 c$. In this work, an extension of the ITO vector algebra has been proposed, which involves coupling two ITOs into a third one , where one of the terms is a complex conjugate and therefore is not an ITO. This approach has been applied to rewrite the SEEs. Thus, the rate coefficient T_A in our form is $\mathrm{T}_A = \frac{3C_A}{\sqrt{2K+1}} \sum_{\tau_l j_l K_l K_r} (\rho^{(K_l)}(\tau_l j_l) \otimes \mathrm{J}^{(K_r)*})^{(K)}_Q \langle \tau (j \otimes j'^*)^{(K)} \| (d^{(k)} \otimes d'^{(k')*})^{(K_r)} \| \tau_l (j_l \otimes j_l'^*)^{(K_l)} \rangle$. It has two terms, the reduced matrix element of the dipole moments and the vector product of the tensors $\rho^{(K_l)}(\tau_l j_l)$ and $\mathrm{J}^{(K_r)*}$: in all our equations polarimetry and geometry are clearly separated.

References

1. (DL) E.L. Degl'innocenti, M.Landolfi *Polarization in Spectral Lines*, Springer, 2004