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COMPUTER MODELING OF DYNAMICS OF ELASTIC HALF-SPACE BY REFLECTION OF PLANE WAVE FROM THE UNLOADED SURFACE

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Characteristic of earthquakes is the occurrence in the Earth's crust under the influence of tectonic stresses of deep cracks. In this case, a sudden drop in stresses on the crack occurs, generating nonstationary elastic waves, which, diffracting on the earth's surface, generate surface waves, destructive for land installations. Here is developed a mathematical model for studying such phenomena to solve non-stationary problems in elastic media is one of the most convenient in applications of methods is the method of characteristics using the ideas of the method splitting. In the present work this method is developed for the solution of contact problems of the interaction of elastic bodies with angular points in conditions of plane deformation. An explicit difference scheme based on the method of characteristics with the idea of splitting into spatial coordinates. We obtain resolving difference equations for internal, boundary, angular, singular, and contact points of conjugation of a strip and a half-plane. For modeling of the process of stress relief on a crack uses singular generalized functions. Numerical experiments were carried out to determine the stress-strain.

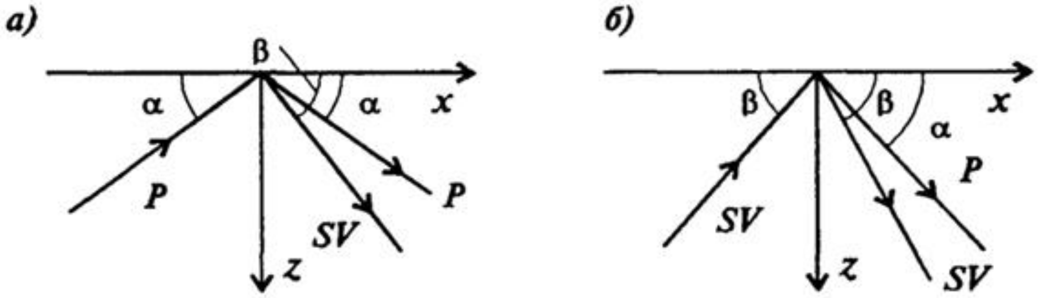

Fig. 1. Incidence and reflection of longitudinal P and transverse SV waves

The main problem in the study of waves is the clarification of the law of change in time and space of physical quantities. In the case of elastic waves, such a quantity may, for example, is an offset of small sections of the environment relative to their equilibrium positions. The dependence on spatial coordinates and time is called the wave equation.

Mathematical model of this thesis consists of several wave equations. As thesis about a plane wave, we used the equations of movement.

$$(d_1^2 + d_3^2 - \frac{d_t^2}{c_1^2})\Phi = 0 \qquad (1.1)$$

$$(d_1^2 + d_3^2 - \frac{d_t^2}{c_2^2}) \quad = 0 \qquad (1.2)$$

d_1- is a derivative from x_1, d_3- is a derivative from x_3, d_t- is a derivative from time of the wave propagation, c_1- is a velocity of longitudinal wave, c_2- is a velocity of transverse wave.

The solution of the equations we can take that form:

$$\Phi = G_1(x_3)e^{lk(x_1-ct)} \qquad (1.3)$$

and

$$= G_2(x_3)e^{lk(x_1-ct)} \qquad (1.4)$$

c – is a velocity of the wave propagation, t – is a time of the wave propagation, Φ and are potentials which describe the propagation of waves. They can be represented as:

$$\Phi = A_1 \exp[ik(x_1 + v_1 x_3 - ct)] + B_1 \exp[ik(x_1 - v_1 x_3 - ct)] \quad (1.5)$$

$$\psi = A_2 \exp[ik(x_1 + v_2 x_3 - ct)] + B_2 \exp[ik(x_1 - v_2 x_3 - ct)] \quad (1.6)$$

Here A_1, A_2, B_1, B_2 are constants which are included in the boundary equation, v_1- is a velocity of a falling wave, $v_2 -$ is a velocity of a wave reflection.

For the solution of the mathematical model, we can use the Tensor of stress:

$$\sigma_{33} = 2\mu d_3 u_3 + \lambda(d_1 u_1 + d_3 u_3) \quad (1.7)$$

$$\sigma_{13} = \mu(d_1 u_3 + d_3 u_1) \quad (1.8)$$

σ_{13}, $\sigma_{33} -$ Stress Tensor, $u_1, u_3 -$ is a displacement to the coordinates x_1 andx_3, μ −is the shear modulus, $\lambda -$ Lame's first parameter,

Substituting in u_1, u_3

$$u_1 = d_1\Phi - d_3\psi \quad (1.9)$$

$$u_3 = d_3\Phi + d_1\psi \quad (1.10)$$

We can write the tensor stress in this form:

$$\sigma_{33} = 2\mu(d_3^2\Phi + d_1 d_3\psi) + \lambda\nabla_1^2\Phi \quad (1.11)$$

$$\sigma_{13} = \mu(2d_1 d_3\Phi + d_1^2\psi - d_3^2\psi) \quad (1.12)$$

$\nabla_1^2 -$ is gradient.

There are also boundary conditions:

$$\sigma_{33}(x_1, 0, t) = 0 \quad (1.13)$$

$$\sigma_{13}(x_1, 0, t) = 0 \quad (1.14)$$

Let us consider the case of propagation of P-waves. In this case, we should put in $A_2 = 0$; it means that the incident wave is a P wave.

Now we have the solution of all 4 components in order to plot the propagation of a plane wave reflecting from the free surface.

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