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About geometry of completely integrable Hamiltonian systems

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We are interested in geometry of Liouville foliation generated by completely integrable Hamiltonian systems.

Definition 1 [2] Let M^{2n} be a symplectic manifold and $sgradH$ Hamiltonian vector field with a smooth Hamiltonian function H.

Hamiltonian system $sgradH$ is called *completely integrable in the sense of Liouville or completely integrable*, if exists set of smooth functions $f_1, ..., f_n$ as:

1) $f_1, ..., f_n$ are first integrals of $sgradH$ Hamiltonian vector field,

2) they are functionally independent on M, that is, almost everywhere on M their gradients are linearly independent,

3) $\{f_i, f_j\} = 0$ for any i and j,

4) the vector fields $sgradf_i$ are complete, that is natural parameter on their integral trajectories is defined on the whole number line.

Definition 2 [1] Partition of the manifold M^m into connected components of joint level surfaces of the integrals $f_1, ..., f_n$ is called *The Liouville foliation* corresponding to the completely integrated system.

Level surfaces of these first integrals generates Liouville foliation. If the dimension of the leaf L is maximal, it is called regular, otherwise L is called singular.

Definition 3 [3] A partition F of the manifold M by path-connected immersed submanifolds L_α is called *a singular foliation of* M if it verifies condition:

for each leaf L_α and each vector $v \in T_pL_\alpha$ at the point p there is $X \in XF$ such that $X(p) = v$, where T_pL_α is the tangent space of the leaf L_α at the point p, XF is the module of smooth vector fields on M tangent to leaves (XF acts transitively on each leaf).

If the dimension of L is maximal, it is called regular, otherwise L is called singular. It is known that orbits of vector fields generate singular foliation.

Definition 4 [4]. *The orbit* $L(x)$ of a system D of vector fields through a point x is the set of points y in M such that there exist $t_1, t_2, \ldots, t_k \in R$ and vector fields $X_1, X_2, \ldots, X_k \in D$ such that

$$y = X_k^{t_k}(X_{k-1}^{t_{k-1}}(\ldots(X_1^{t_1}))),$$

where k is an arbitrary positive integer.

The fundamental result in study of orbits is Sussman theorem.

Definition 5 [3] A *distribution* P on M is a map which assigns to every point $x \in M$ an vector subspace $P(x)$ of T_xM.

Every set of smooth vector fields D generates distribution, where for every point $x \in M$ matches subspace $P(x) \subset T_xM$, that generated by set of vectors $D(x) = \{X(x) \colon X \in D\}$.

The distribution P is called *completely integrable*, if for every $x \in M$ there is a submanifold L_x of the manifold M such, that $T_yL_x = P(y)$ for all $y \in L_x$. The submanifold L_x of M is called an *integral submanifold (or integral manifold)* of the distribution P. A *maximal integral manifold of* P is a connected submanifold L of M such that

(a) L is an integral manifold of P,

(b) every connected integral manifold of P which intersects L is an open submanifold of P.

We say that P is completely integrable if through every point $x \in M$ there passes a maximal integral manifold of P.

Sussman Theorem [4]. Let M be a smooth manifold, and let D be a set of vector fields. Then

(a) L is an orbit of D, then L admits a unique differentiable structure such that L is a submanifold of M. The dimension of L is equal to its rank.

(b) With the topology and differentiable structure of (a), every orbit of D is a maximal integral submanifold of distribution P.

(c) P has the maximal integral manifolds property,

(d) P is involutive.

Let $sgradH$ completely integrable Hamiltonian vector field and with Hamiltonian function $H\colon \mathbb{R}^4 \to \mathbb{R}$ on the four dimensional Euclidean space with the Cartesian coordinates (p_1, p_2, q_1, q_2) with equation:

$$H = H(p_1, p_2, q_1, q_2).$$

We assume that Hamiltonian system is completely integrable and following functions

$$F^1 = F^1(p_1, q_1), \quad F^2 = F^2(p_2, q_2)$$

are first integrals of Hamiltonian system (1).

Let us denote by P the distribution generated by vector fields

$$\begin{array}{l} gradf^1 = \{p_1'(u); 0; q_1'(u); 0\} \\ gradf^2 = \{0, p_2'(v), 0, q_2'(v)\}. \end{array} \qquad (1)$$

Theorem. The distribution P generates foliation $\mathcal{F}^{\perp}$, which is orthogonal to Liouville foliation $\mathcal{F}$.

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