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On the description of Leibniz dialgebras

M.E. Aziziv
Institute of mathematics, Tashkent, Uzbekistan
azizovmajidkhan@gmail.com

In this work we investigate Leibniz dialgebras and give the complete classification of two dimensional Leibniz dialgebras.

Leibniz algebras are non-commutative analogues of Lie algebras. The identity of the variety of right Leibniz algebras is the condition stating that each operator of right multiplication is a derivation, i.e. [1]:

$$(xy)z = x(yz) + (xz)y.$$

Firstly, J.–L. Loday gives definition of associative dialgebras in [2], which is the vector space A with two bilinear operations "$\dashv$","$\vdash$", which satisfies following identities:

$$(x \dashv y) \vdash z = (x \vdash y) \vdash z, x \dashv (y \vdash z) = x \dashv (y \dashv z), \quad (1)$$

$$(x \vdash y) \vdash z = x \vdash (y \vdash z), (x \dashv y) \dashv z = x \dashv (y \dashv z), \quad (2)$$

$$(x \vdash y) \dashv z = x \vdash (y \dashv z). \quad (3)$$

The notion of dialgebras can be defined for any class of algebras. In [3] Kolesnikov gave the method of construction of dialgebra for any variety of algebras. In [3] assumed that (1) identity must hold for any kind of dialgebras.

Definition 1 A vector spaces A with two "$\dashv$", "$\vdash$" bilinear operations is said to be a "0−dialgebra" if A satisfies the identities (1). By using the method of Kolesnikov, we find identities of Leibniz dialgebras and it given by following definition.

Definition 2 A 0-dialgebra A is said to be a Leibniz dialgebra if A satisfies the identities

$$(x \dashv y) \dashv z - (x \dashv z) \dashv y - x \dashv (y \dashv z) = 0, \quad (4)$$

$$(x \vdash y) \dashv z - (x \vdash z) \vdash y - x \vdash (y \dashv z) = 0, \quad (5)$$

$$(x \vdash y) \vdash z - (x \vdash z) \dashv y - x \vdash (y \vdash z) = 0. \quad (6)$$

From (4) it implies that $(A, \dashv)$ is the (right) Leibniz algebra. From this assertion it can be concluded that any Leibniz dialgebra corresponds to certain Leibniz algebra. In this work we consider the description of 2-dimensional Leibniz dialgebras by using the classification of 2-dimensional Leibniz algebras [4].

Theorem 1 [4]. Up to isomorphism, there exist four families of two dimensional Leibniz algebras over any field:

$L(1)$: abelian algebra

$L(2)$: $e_1 \dashv e_2 = -e_2 \dashv e_2 = e_1$;

$L(3)$: $e_2 \dashv e_2 = e_1$;

$L(4)$: $e_1 \dashv e_2 = e_1$.

(Omitted products are zero.)

Theorem 2 Up to isomorphism, there exist one parametric and five non parametric families of two dimensional Leibniz dialgebras over any field F:

$(L1(1), \dashv, \vdash) : e_1 \vdash e_1 = e_2$;

$(L1(2), \dashv, \vdash) : e_1 \dashv e_2 = e_1, e_1 \dashv e_2 = -e_1, e_1 \vdash e_2 = e_1$;

$(L2(2), \dashv, \vdash) : e_1 \dashv e_2 = e_1, e_1 \dashv e_2 = -e_1, e_1 \vdash e_2 = e_1, e_2 \vdash e_1 = -e_1$;

$(L1(3), \dashv, \vdash) : e_2 \dashv e_2 = e_1, e_2 \vdash e_2 = \delta e_1, \forall \delta \in F$;

$(L1(4), \dashv, \vdash) : e_1 \dashv e_2 = e_1, e_1 \vdash e_2 = e_1$;

$(L2(4), \dashv, \vdash) : e_1 \dashv e_2 = e_1, e_2 \vdash e_1 = -e_1$

(Omitted products are zero).

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