КОРОТКИЕ СООБЩЕНИЯ

Orbital angular momentum of the spiral beams

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Abstract

At first sight, any rotation generates some angular momentum (it is true for a solid body). But these characteristics (rotation and orbital angular momentum) are rather different for optics and mechanics. In optics there are the situation when the rotation is important. On the other hand, there are the cases where the nonzero orbital angular momentum is necessary. The main goal of this article is to investigate a relationship between a rotation under propagation of spiral beam and its angular momentum.

It can be done the following conclusion: there is no any relation between rotation under propagation of spiral beam and its OAM.

Keywords: spiral beams, singular optics, orbital angular momentum.

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Introduction

Spiral beams are a family of the lightfields whose intensity distribution remains invariant, up to scale and rotation, during propagation [1, 2]. It is seen that history of the spiral beams is sufficiently extensive. At first sight any rotation generates some angular momentum (it is true for a solid body) but it is not so for the spiral beams. These characteristics (rotation and orbital angular momentum) are rather different. There are the situation when the rotation is important [3, 4]. On the other hand, there are the cases where the nonzero orbital angular momentum is necessary [5]. The relationship between the rotation and orbital angular momentum (OAM) in some cases have considered partly. Therefore, in [1] the examples of the light fields with opposite rotation and equal OAM takes place. In [5] the example of light field with nonzero rotation but zero OAM has been presented. But there is no the general consideration of this question. In this article, there is the investigation of relationship between a rotation under propagation of spiral beam and its angular momentum.

1. Orbital angular momentum

Spiral beams are a family of the lightfields whose intensity distribution remains invariant, up to scale and rotation, during propagation, namely:

$$I(x,y,l) = D(l)I_0 \times \times\left(\frac{x\cos\theta(l)-y\sin\theta(l)}{d(l)},\frac{x\sin\theta(l)+y\cos\theta(l)}{d(l)}\right), \quad (1)$$

where $\theta(l)$ – rotation parameter (see below), $d(l)$ – scale parameter.

In accordance with [1] the density of OAM, M of light field is the following:

$$M = \mathrm{Im}(\overline{F}\times F_\alpha), \quad (2)$$

where F is complex amplitude and α – polar angle.

The mode index condition for the spiral beams with Gaussian parameter ρ is [1, 2]:

$$2n+|m|+\theta_0 m = \mathrm{const}, \quad (3)$$

where θ_0 – is so called constant of rotation,

$$\theta(l) = \theta_0 \mathrm{arctg}\left(\frac{2l}{k\rho^2}\right), \quad (4)$$

l – variable of propagation, k – wavenumber; n, m is number of Laguerre–Gauss modes.

Imaginary part of real quantity is equal zero:

$$\mathrm{Im}\left(\frac{\partial}{\partial\alpha}F\overline{F}\right) = \mathrm{Im}(F\overline{F}_\alpha + F_\alpha\overline{F}) \quad (5)$$

hence two complex conjugate light fields have opposite OAM:

$$\mathrm{Im}\left(\frac{\partial}{\partial\alpha}F\overline{F}\right) = \mathrm{Im}(F\overline{F_\alpha} + F_\alpha\overline{F}) = 0. \quad (6)$$

Here α is polar angle.

Analytic formulae for the orbital angular momentum is by definition:

$$L = \iint_{R^2} M\,\mathrm{d}x\mathrm{d}y / E, \quad (7)$$

where

$$E = \iint_{R^2} F\overline{F}\,\mathrm{d}x\mathrm{d}y \;-$$

the energy. Any light field F with finite energy can be represented as decomposition of normalized Laguerre–Gauss modes LG_{nm}, because it is hole system:

$$F = \sum_{n,m} c_{n,m} LG_{nm}\,\mathrm{d}x\mathrm{d}y \quad (8)$$

and OAM is the following:

$$L = \sum_{n,m} \left| c_{n,m} \right|^2 m \Big/ \sum_{n,m} \left| c_{n,m} \right|^2 . \tag{9}$$

2. OAM and rotation of the spiral beams

To investigate the question let us consider the following compositions:

$$\begin{aligned} &1.\ F_1 = c_1 LG_{0,0} + c_2 LG_{0,n};\\ &2.\ F_2 = c_1 LG_{n,0} + c_2 LG_{0,n};\\ &3.\ F_3 = c_1 LG_{2n+1,-n} + c_2 LG_{n,n}, n \neq 0;\\ &4.\ F_4 = c_1 LG_{n,0} + c_2 LG_{0,2n}. \end{aligned} \tag{10}$$

In Fig. 1–4 the distributions of intensities and phase of these compositions are shown. It is convenient because any sum of two Laguerre–Gauss modes is some spiral beam with constant of rotation:

$$\theta_0 = \frac{2n_1 - 2n_2 + |m_1| - |m_2|}{m_2 - m_1}. \tag{11}$$

Two first compositions have opposite rotation parameters but equal OAM:

$$\theta_0^1 = -\theta_0^2 = -1, L_1 = L_2 = \frac{n}{2}, |n_1| = |n_2|, c_1 = c_2 . \tag{12}$$

The third composition is rotated without OAM:

$$\theta_0 = \frac{n+1}{|n|}, L = 0, |n_1| = |n_2|, n \neq 0 . \tag{13}$$

In [6] the example of lightfield with nonzero rotation but zero OAM has been presented also.

And fourth composition is not rotated of all but has some OAM:

$$\theta_0 = 0, L = n, |n_1| = |n_2| . \tag{14}$$

It is seen from these examples the following: in the first place, there is no any relation between rotation of spiral beam and its OAM, secondly, it is difficult to reveal OAM directly from these distributions, because OAM is integral characteristic of lightfields. There is no any optical method of detecting OAM at present.

From formal point of view, it is clear: rotation parameter is defined only indexes of mode whereas OAM depends on weight coefficients too.

To do this more physical clearly let us consider two modes: real Laguerre-Gauss mode and complex one with timedependent components:

$$\begin{aligned} &F_1 = \operatorname{Re} LG_{0,1} \times \exp(-i\omega t),\\ &F_2 = \operatorname{Re}(LG_{0,1} \times \exp(-i\omega t)).\\ &F_1 = \operatorname{Re}(r\cos(\alpha)\exp(-r^2) \times\\ &\quad \times \exp(-i\omega t)) =\\ &\quad = r\cos\alpha\cos(-\omega t)\exp(-r^2),\\ &F_2 = \operatorname{Re}(r\exp i\alpha \times \exp(-i\omega t)\exp(-r^2)) =\\ &\quad = r\cos(\alpha - \omega t)\exp(-r^2). \end{aligned} \tag{15}$$

It is seen that second distribution is rotated. Namely, this rotation defines OAM.

Of course, this rotation is not the rotation under propagation.

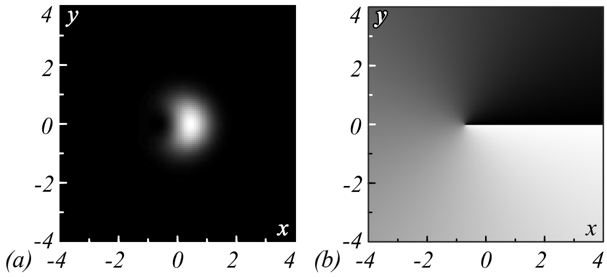

Fig. 1. The distributions of intensity (a) and phase (b) for the first composition (at n=1)

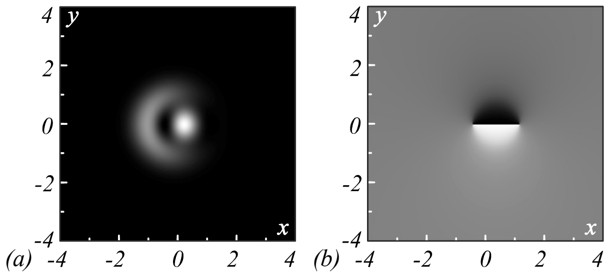

Fig.2. The distributions of intensity (a) and phase (b) for the second composition (at n=1)

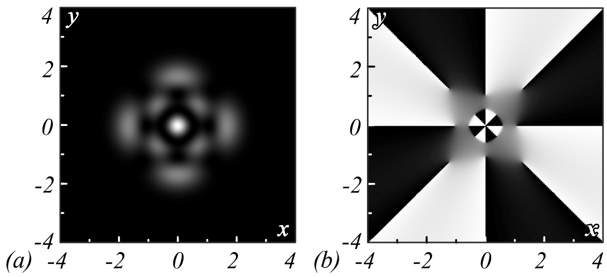

Fig.3. The distributions of intensity (a) and phase (b) for the third composition (at n=2)

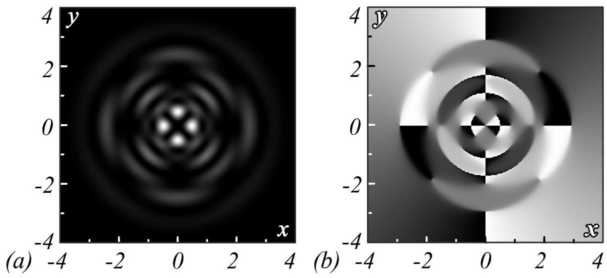

Fig.4. The distributions of intensity (a) and phase (b) for the fourth composition (at n=2)

Conclusion

The main purpose of the article is to consider the question: is it some relation between the rotation under propagation of spiral beam and its OAM. We consider four type of spiral beams. The two types of beams have opposite rotation parameters but equal OAM. The third typerotated without OAM. The fourth type of spiral beam is not rotated of all but has some OAM. From these examples it can be done the following conclusion. There is no any relation between rotation under propagation of spiral beam and its OAM. It should be noted [7], where the possibility of OAM measurement has been considered but only for the special case of lightfields in the form of superposition of Laguerre-Gauss modes with zero lower index.

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