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LIFT OF THIN AIRFOIL WITH SLAT

Geometric Modeling of the Airfoil with Slat

Fig. 1 describes the geometric structure of the airfoil with slat. The model has two parts. The analytical part is an ellipse whose center is located in the coordinate system center O, and the numerical part is the model of slat which located on the left side of the ellipse. C_1D_1, C_2D_2 are different positions when the slat rotates around point A_1. C_2D_2 coincides with X-axis. The deflection angle δ is the angle between chord line C_2D_2 and chord line in other position, like C_1D_1.

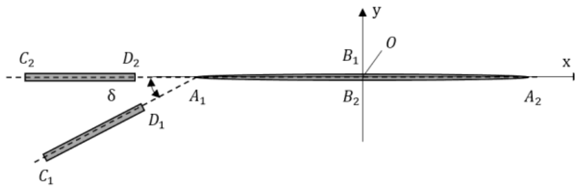

Fig. 1. Diagram of Geometric Structure

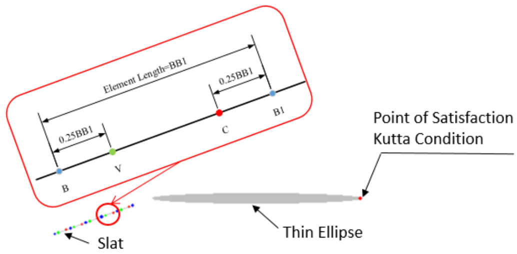

Fig. 2. Boundary Elements of the Model

The numerical part of the model is divided into certain number of elements. In fig. 2 blue points are boundary points of each element. Green points are vortex points that generate circulation. Red points are control points in which the velocity perpendicular to the slat equal to zero, preventing the streamline from going through the slat. The detail of each element is also shown in fig. 2. Point B and point B1 are boundary points. Points V and C respectively represent vortex point and control point. As a number of elements equal to *N*, the total number of boundary points is *N+1*. In numerical part of the model, there are *N* vortex points and *N* control points. To satisfy Kutta condition [1], a control point needs to be added on the trailing edge of the plate. The corresponding vortex point is in the center of the ellipse. Thus the total number of vortex points and control points is *N+1*.

Using NAM to Predict Flow Field and Calculate Lift Coefficient

After getting the discrete numerical model, conform mapping method [2] is used. By using Zhukovsky formula [1], the ellipse in the physical plane is transformed into a cylinder, and the discrete points are also mapped into new points. The mapping result is shown in fig. 3. The green points inside the cylinder have existed in order to counter the influence of external vortex points.

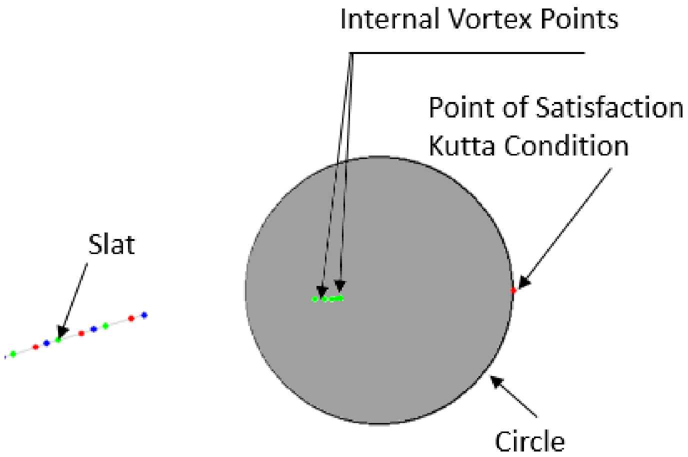

Fig. 3. Conform Mapping Result

Zhukovsky Formula is

$$\zeta = z + \sqrt{z^2 - \varepsilon^2},$$

where $\varepsilon^2 = a^2 - b^2$, and *a*, *b* are the lengths of the semi-axes of the ellipse.

The flow around a cylinder with circulation has been studied extensively, the complex potential[Ошибка! Источник ссылки не найден.] of which is given by the following equation:

$$W(\zeta) = \frac{1}{2}\left(\overline{V_\infty}\zeta + V_\infty \frac{r^2}{\zeta}\right) + \sum_{j=1}^{N}\left\{\frac{\Gamma_j}{2\pi i}\left[\ln(\zeta - \zeta_{vj}) - \ln\left(\zeta - \frac{r^2}{\overline{\zeta_{vj}}}\right) + \ln\zeta\right]\right\} + \frac{\Gamma_{n+1}}{2\pi i}\ln\zeta, \qquad (1)$$

where V_∞ and $\overline{V_\infty}$ are the complex velocity and conjugate complex velocity of infinity flow; ζ is complex coordinate on the mapped plane; *r* is the radius of the circle on the mapped plane; *N* is number of boundary elements on the slat; $i = \sqrt{-1}$ is the imaginary unit; Γ_j is the circulation of the *j*-vortex; ζ_{vj} and $\overline{\zeta_{vj}}$ are the complex coordinate and conjugate complex coordinate of *j*-vortex points; Γ_{n+1} represents the value of the circulation of the vortex point that is used to satisfy the Kutta condition.

In order to use Eq. (1) to predict the flow, it is necessary to calculate circulations generated by point vortices. The impermeability boundary conditions are used in each control point on the model of the slat, and the Kutta condition is used in the control point at the trailing edge of the plate. Thus circulations are calculated by an (*N*+1)-variable linear system of equations

$$\begin{bmatrix} A_{1,1} & \cdots & A_{1,n+1} \\ \vdots & \ddots & \vdots \\ A_{n+1,1} & \cdots & A_{n+1,n+1} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_{n+1} \end{bmatrix} = \begin{bmatrix} R_1 \\ \vdots \\ R_{n+1} \end{bmatrix}, \qquad (2)$$

where $A_{i,j} \cdot \Gamma_j$ is the influence of the *j*-th vortex point on the normal velocity of the *i*-th control point; R_i is the influence of the analytical part (the first term of the complex potential, see eq. (1)) on the normal velocity of the *i*-th control point.

The circulation of each vortex point is obtained by solving eq. (2). The velocity of an arbitrary point on the physical plane is obtained by the following equation

$$u = \mathrm{Re}\left[\frac{dW(\zeta)}{d\zeta} \cdot \frac{d\zeta}{dz}\right], \qquad v = -\mathrm{Im}\left[\frac{dW(\zeta)}{d\zeta} \cdot \frac{d\zeta}{dz}\right],$$

where *u* is the velocity along *X*-axis; *v* is the velocity along *Y*-axis.

The flow field can be predicted. First, the streamline on mapping plane is gotten as is shown in fig. 4. Then, the streamline cluster around the thin airfoil with a slat in an arbitrary case is predicted in fig. 5. In this figure, the red point indicates the stagnation point, on which the velocity of fluid equal to zero. After flowing through the stagnation point, the fluid is divided into three parts, the streamline of them are called zero streamlines (blue). The two streamlines advance along the upper and lower surface of the plate.

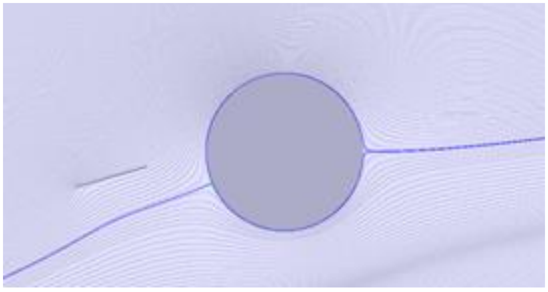

Fig. 4. Streamlines on Mapping Plane

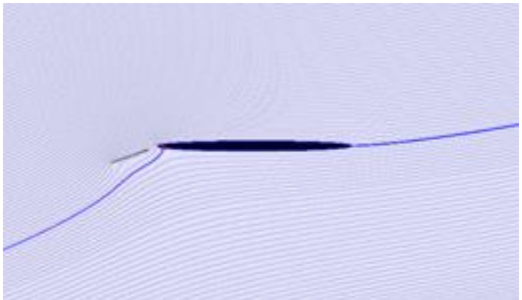

Fig. 5. Streamlines on Physical Plane

According to Kutta-Zhukovsky theorem, the lift coefficient can be expressed as by the following formula

$$c_l = -\frac{\rho v_\infty \Gamma}{\frac{\rho v_\infty^2}{2} \cdot c} = -\frac{2\Gamma}{v_\infty c}.$$

where ρ is the density of approaching flow; *c* is the sum of the chord of plate and chord of the slat; Γ is the total circulation around the airfoil.

The total circulation around the airfoil could be calculated by integrating every point's velocity along a curve enclosing airfoil in anticlockwise direction or by summing values of all the circulations of the vortices using Stokes's theorem

$$\Gamma = \oint (udx + vdy) = \sum_{j=1}^{n+1} \Gamma_j \quad .$$

A quadratic Richardson extrapolation is used to improve the calculation accuracy. In quadratic extrapolation, calculate the lift coefficient for three times. The number of elements is N_{start}, $2N_{start}$, and $4N_{start}$. Assuming the quadratic relationship between the lift coefficient and the reciprocal of the number of elements ($1/N$) $C_{l|1/N}=a(1/\mathrm{N})^2+k(1/\mathrm{N})+b$ and by solving three equations, the values b, k and a can be easily obtained. When $N=\infty$, $1/N$ is approximating to zero. The value b is a quasi-exact result of lift coefficient for the case $N=\infty$.

Verification of Calculation Model

Use two methods to verify the correctness of the results. Firstly, make a comparison of numerical results with analytical results in special condition that the model is a plate without deflection angle and slot. The analytical formula [1] for calculating the lift coefficient is as follow

$$c_{l_{plate}} = 2\pi sin\alpha, \quad (3)$$

where α is angle of attack.

Fig. 6 shows that the numerical results of the lift coefficient match the analytical results very well when the angle of attack range from 0 to 30°.

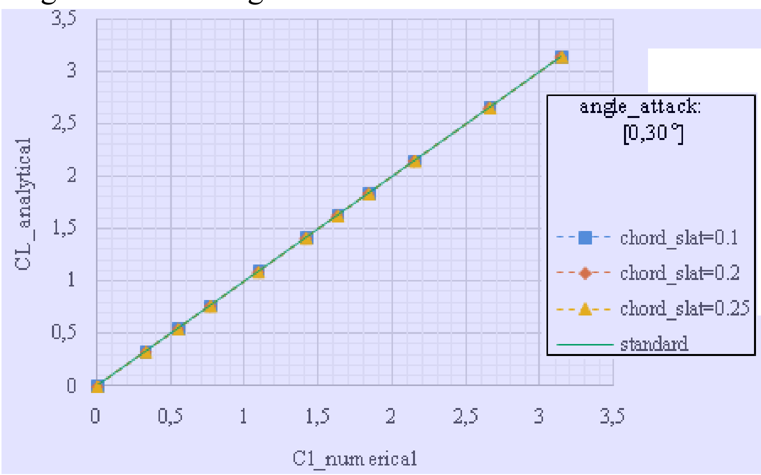

Fig. 6. Comparison of Analytical and Numerical Lift Coefficients

The results show that the accuracy is always higher than 10^{-3}. From this figure, we can conclude that the mathematical model and computing program are correct.

Secondly, compare the numerical result and the results from engineer formula when the angle of attack equal to zero. The leading-edge droop causes a lift loss at zero angles of attack which can be derived from Glauert's linear thin airfoil theory [4]

$$_{s}c_{l_0} = -\frac{\theta_s - sin\theta_s}{\pi}\delta_s C_{l_\alpha}.$$

In this formula, δ_s is the deflection angle of the slat, $C_{l_\alpha}$is the derivative of the angle of attack with lift coefficient when the angle of attack equal to zero, whose value is 2π. θ_s is Glauert variable that is calculated by the following formula

$$\theta_s = \arccos\left(1 - 2\frac{c_S}{c}\right),$$

where $c = c_s + c_p$, c_S is chord of the slat and c_p is chord of the plate.

Fig. 7 and 8 show plots of the numerical results and engineer results of lift coefficients as functions of deflection angle at zero angle of attack. When the deflection angle ranges from 0 to 15°, the data from the numerical calculation is close to engineer results.

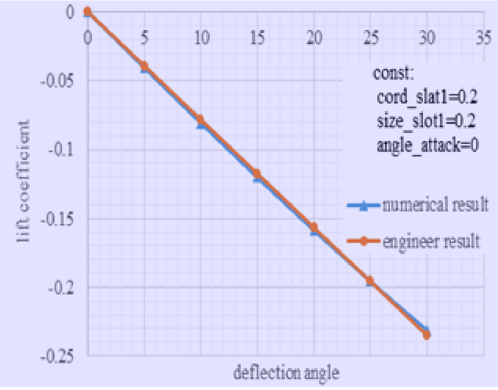

Fig. 7. Comparison of Numerical Results and Engineer Results (Slot Exists)

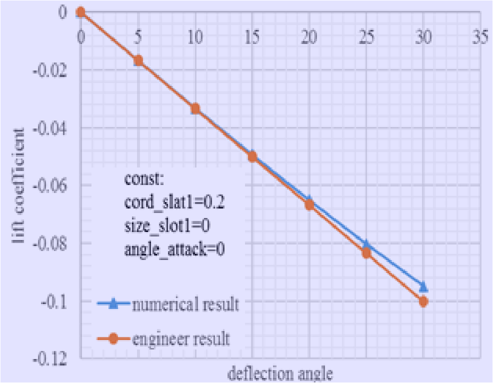

Fig. 8. Comparison of Numerical Results and Engineer Results (Slot Doesn't Exist)

Results

Fig. 9, 10 and 11 respectively show the relationships between lift coefficient and chord of the slat, size of slot and deflection angle. Fig. 9, 10 indicate that the lift coefficient decreases when the chord of slat and size of slot becomes longer, and the decrease speed becomes slower. Fig. 11 shows that the lift coefficient decreases with the increasing of deflection angle. The larger the deflection angle, the quicker the decrease speed is.

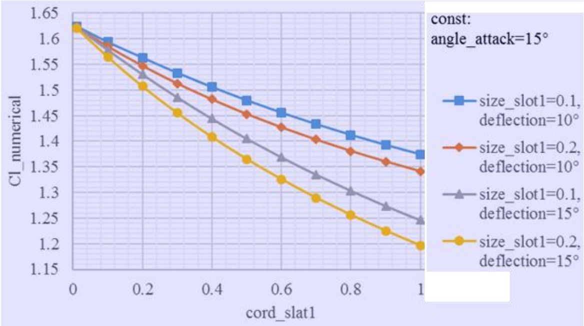

Fig. 9. Lift Coefficient vs Chord of Slat

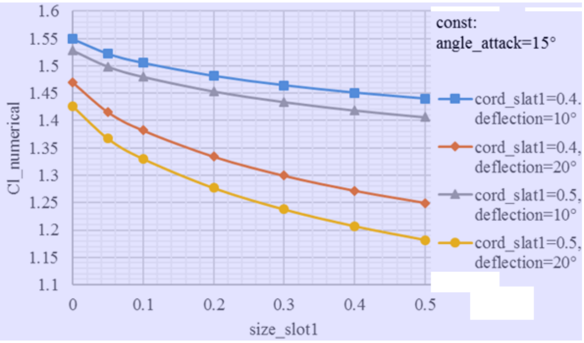

Fig. 10. Lift Coefficient vs Chord of Slot

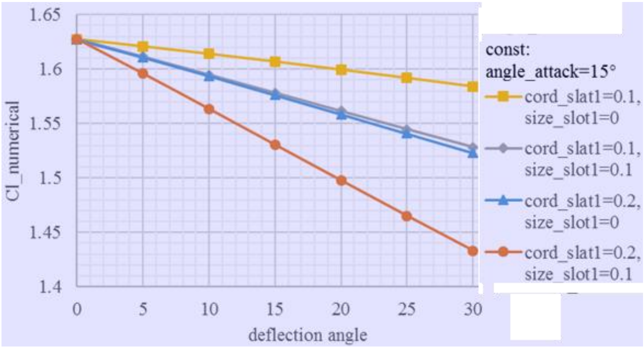

Fig. 11. Lift Coefficient vs Deflection Angle of the Slat

Fig. 12, 13 and Fig. 14 show that lift coefficient is negative when the angle of attack equal to zero because of the exits of the slat, slot and positive deflection angle. Besides, the lift coefficient has a linear relationship with the angle of attack. The derivative of this relationship is a constant value.

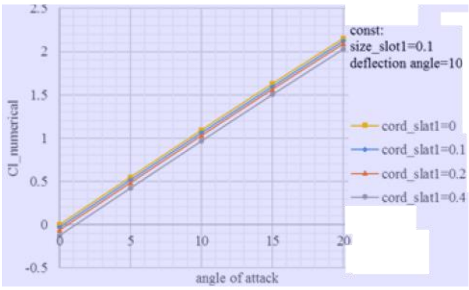

Fig. 12. Lift Coefficient vs Angle of Attack (Chord_slat1 Variable)

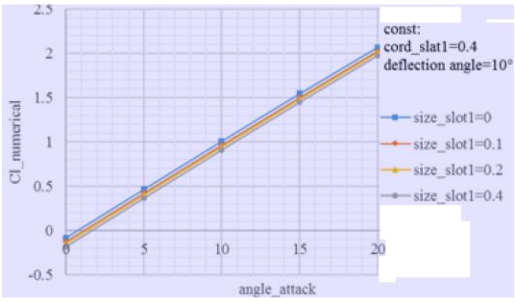

Fig. 13: Lift Coefficient vs Angle of Attack (Size_slot1 Variable)

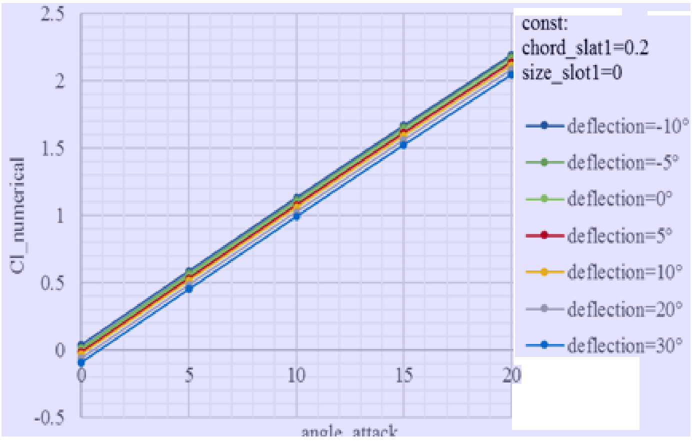

Fig. 14. Lift Coefficient vs Angle of Attack (Deflection Angle Variable)

In conclusion, we can get some important findings. The results show that, within a certain range, control other parameters unchanged, the lift coefficient is positively linear correlated with the angle of attack, and the correlation coefficient does not change with other parameters. The increasing the chord length of the slat, the deflection slat angle and the size of the slot can effectively reduce the lift coefficient at the same angle of attack; thereby increase the critical angle of attack and the maximum lift coefficient of the airfoil.

References

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