MINISTERY OF EDUCATION AND SCIENCE

SAMARA NATIONAL RESEARCH UNIVERSITY
(Samara University)

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DECISION MAKING

Рекомендовано редакционно-издательским советом федерального государственного автономного образовательного учреждения высшего образования «Самарский национальный исследовательский университет имени академика С.П. Королева» в качестве учебного пособия для обучающихся по основной образовательной программе высшего образования по направлению подготовки 38.04.02 Менеджмент

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The tutorial "Decision Making" is recommended for Master's degree students, who raise the skill level. It can also be useful for teachers, specialists of organizations, business representatives and those, who wants to get acquainted with the modern methodology and methods of decision-making. The course presents fundamental aspects of technology and process of developing and making decisions in different systems, considers the modeling in the Decision Theory, interprets tasks, which are focused on the achievement of goals by contributing the formation students` systematic and clear thinking in the university educational process.

This tutorial was created for students enrolled in the Master Program on 38.04.02 Management.

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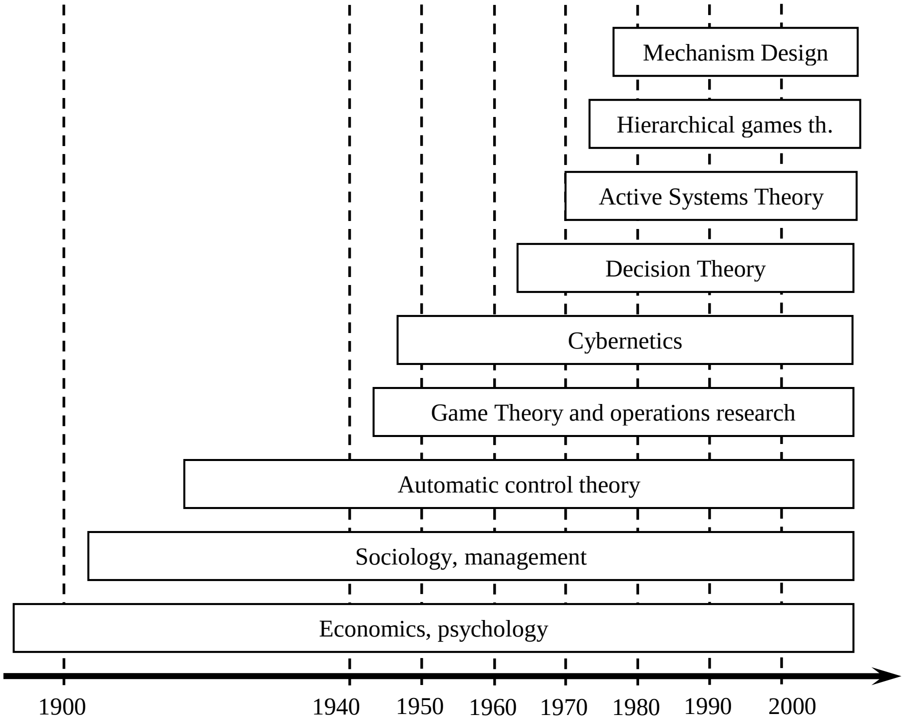
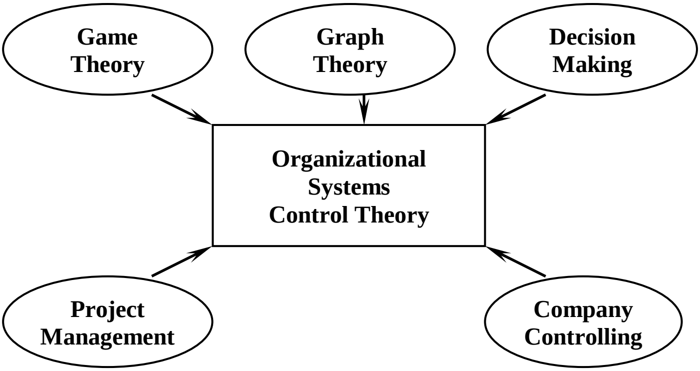
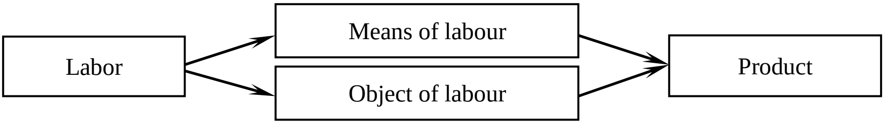
Content

Decision-making in economic-organizing control systems





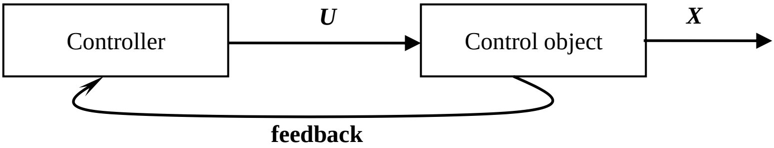
Labor activity scheme:



The model of the organizational control system

Controlling – an effect on the controlled system to provide its desired behavior.

Elementary model of the control system



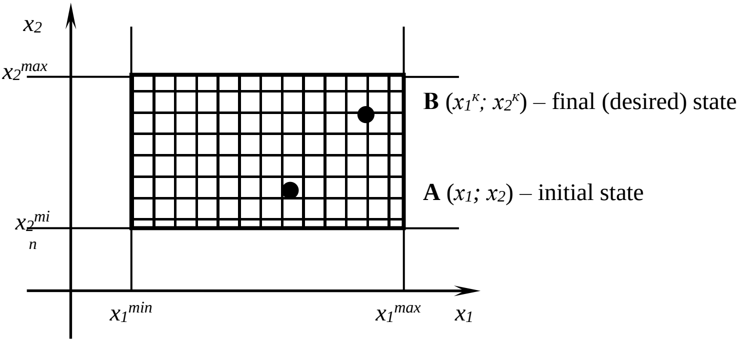
U – control actions vector

X – control object state vector

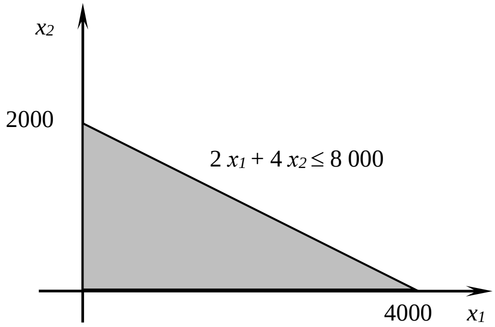
U (U_1, U_2 ... Un) **–** set of the characteristics

Controller – a part of a system, which does not produce something by itself, but gives control actions to the object.

Control object is characterized by the ***region of admissible states*** X (x_1; x_2)



Example. An enterprise produces two types of products using equipment – machine-tools. Output of the “A” production unit needs 2 hours of machine run time, “B” – 4 hours. Valuable fund of the equipment working time = 8 000 hours. How will be the system’s region of admissible states represented?



Aim of the controller is transition of the control object from the initial to the final state.

Objective function (criterion) of the controller – idea of the controller about control object state.

P_1 – price of the production “A” unit.

P_2 – price of the production “B” unit.

$$F(x_1; x_2) = G = P_1 x_1 + P_2 x_2 - Z \rightarrow \max$$

Classification of the control systems

1. **By the character of the relations between object and controller:**
   - ✓ non-feedback control system (without considering information about object state)
   - ✓ feedback control system
2. **By the specifics of the control object**
   - ✓ technical systems (reflexive), i.e. a technical device that identically reacts to the control action
   - ✓ organizational systems

   Presence of a human in the system leads to the variance of the reaction.

$\psi(x)$ – criterion or the objective function

Criterion should fit the following requirements:

1. Quantitativeness
2. Measurability
3. Comparability

The formalization of the decision-making problems

Management tasks statement

Management task is aimed at the achievement of the optimal criterion value:

$\psi(x) \to \max$

$\psi(x) \to \psi_{fix}$

$\psi(x) \to \min$

Object constraint: $x \in X$ (admissible states region)

Mathematical statement of the management decision making task needs two components:

1) Problem criterion
2) Constraints

Mathematical formulation of the management task:

$$\begin{cases} \psi(x) \to \max \\ x \in X \end{cases}$$

As a result of the similar tasks solution we have an optimal decision x^{opt} that satisfies the interests of the controller.

Depending on the control objects, objective functions constraints can have different types.

Constraints

Constraints are caused by 2 factors:

1. external;
2. internal.

External constraints are caused by external environment (demand on goods, price for needed raw materials, procurement quantity that can be provided by input supplier etc.) Controller can not actually affect these constraints.

Internal constraints are determined by object's character, properties, specifics, abilities. Some of internal constraints can be corrected by the controller in permissible limit.

Intercept of external and internal constraints combine into region of admissible states $X = x^{ext} \bigcap x^{in}$.

An enterprise produces two types of products. The cost of the manufacture of each production unit is 3 and 5 rubles, respectively. Amount of the circulating assets of the enterprise is 800 rubles. The price is 10 and 15 rubles, respectively. We need to formulate the statement of the production plan developing task to maximize the receipts.

The mathematical formalization rule:

I. Substantive description of the task.
II. Imposing of the variables, parameters of the function and indexes.
III. Mathematical statement of the task.
IV. Selection of the solution method.
V. Solving and the analysis of the results.

X – the amount of the production output

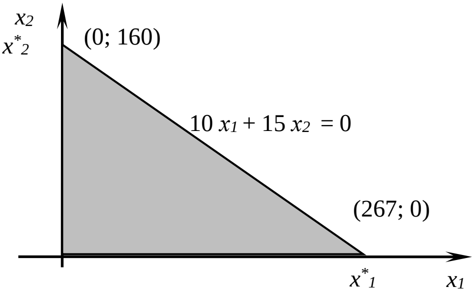
$j = \overline{1,2}$

x_j – amount of the *j*-th type production
P_j – price of the *j*-th type production realization
O – circulating assets
a_j – input normals for the *j*-th type production manufacturing
F – receipt

$$\begin{cases} F = P_1\, x_1 + P_2 x_2 \rightarrow \max \\ a_1\, x_1 + a_2 x_2 \leq O \end{cases}$$

$$\begin{cases} F = 10\, x_1 + 15 x_2 \rightarrow \max \\ 3x_1 + 5x_2 \leq 800 \end{cases}$$

Finding *region of the admissible states* and x_1 and x_2.



$10\, x_1 + 15\, x_2 = 0$
$x_1 = -1,5\, x_2$

	A	B
x_2	*100*	*-100*
x_1	*-150*	*150*

A (160;0) ⇒ F(A) = 10·0 + 15·160 = 2400
B (0; 267) ⇒ F(B)= 10·267 + 15·0 = 2670
$F^{max} = F(B) = 2670$

We need to make dialectic pair of the control system.

External environment – everything that is out of the system and affects it or is under the impact of the system.

Defensible choice of the external environment contributes to the optimal development of the organization.

Criterion – a quantitative characteristic that describes aims of the controller (F).

$$F(u) \to \max\ (\min)$$

$U \in V$, i.e. there are constraints laid on the control actions.

$X = x\ (u)$ – function of the control action.

$$\begin{cases} F(x(u)) \to \max\ (\min) \\ x \in X \\ U \in V \end{cases}$$

Control criteria classification

1. External / internal

External criterion is formed by the external environment effect.

«-» organizational system has no choice

«+» controller is free from decision-making

Internal criterion is generated by the controller of the considered system. For that controller is usually guided by its own conception about aims of the system’s development.

«-» the criterion can be chosen subjectively

The choice of the criterion affects to the finding of the system development way.

2. Mono-criterion

Mono-criteria are characterized by the dimension and physical sense. They are usually explainable and understandable for the control object.

«-» they are compared between each other but they are not convenient in use as absolute values.

The solution of the problem:

1) Introduction of the differential valuations ($x - \Pi$)

2) It is advisable to use non-dimensional valuations $\frac{x - \Pi}{\Pi}$ (relative plan exceed)

If there is no plan Π, the following solutions are possible:

$$\frac{x - x^{-}}{x^{-}} \cdot 100\%$$

where x^{-} – system's result achieved in previous period of functioning.

3. Multicriterial tasks are widely accepted.

Multicriteriality in decision-making problems

In many cases it is impossible to select the single main criterion. The aggregate of the criteria $x = (x_1; x_2 \ldots x_n)$ is needed.

Example 1. We need to develop a system of adding up the results of the world speed skating championship. The sportsmen run 4 distances: 500, 1500, 5000, 10000 meters. We need to define the absolute champion.

There $j = \overline{1,4}$ – number of the distance; $i = \overline{1,n}$ – quantity of the participants; t_{ij} – time spent by *i*–th participant for the *j*-th distance, y_{ij} – she spot participant won.

We need to make unified criterion of the sportsmen's integral valuation determination

The 1st alternate solution

y_{ij} – she spot participant won

$y_{ij}^{*} = n - y_{ij}$

β_j – significance of the distance

The methods of the aggregation into the integral criterion like this have negative feature – absence of the substantial sense.

The 2nd alternate solution

t_{ij} – time spent by *i*–th participant for the *j*-th distance

$\min\limits_{i} t_{ij}$ – winner's time

$\max\limits_{i} t_{ij}$ – outsider's time

$$x_{ij} = \frac{\max\limits_{i} t_{ij}}{t_{ij}}$$

The winner has the shortest time, consequently, $x_{ij} > 1$

The winner has the longest time, consequently, $x_{ij} = 1$

Example 2. A woman is choosing a husband from the 3 candidates using 3 criteria ($j = \overline{1,3}$, $i = \overline{1,3}$):

a) financial position
b) appearance
c) spirituality

The marks each candidate has are:

	Financial position	**Appearance**	**Spirituality**
1	4	6	8
2	6	4	8
3	7	7	4

In multicriterial tasks we have to introduce weight coefficients of the relative significance.

Weight coefficients $\beta_i \in (0;1)$ should fit the criterion of normability: $\sum\beta_i = 1$

$$\sum_i y_{ij}\beta_i \to \text{multicriterial valuation}$$

Formulation of the linear programming problems. Task solution by the graphical interpretation method.

Basic task interpretation:

$$\boxed{\begin{cases} F(x) \to \max \\ x \in X \end{cases}}$$

Task. An enterprise produces sausage of two types: boiled sausage (120 rubles for kilo) and ham sausage (200 rubles for kilo).

Three types of resources are used for production: beef, pork and pea. The stock of beef is 100 kilo, pork – 60 kilo, pea – 30 kilo. Input normals for each resource are in table below:

	Boiled sausage	***Ham sausage***	
Beef	$0{,}7 = a_{11}$	$0{,}2 = a_{12}$	b_1
Pork	$0{,}2 = a_{21}$	$0{,}6 = a_{22}$	b_2
Pea	$0{,}1 = a_{31}$	$0{,}1 = a_{32}$	b_3

Decision maker is to develop optimal plan of production output fitting following criteria:

1) fit the task constraints;
2) provide maximal revenue from production realization.

x – amount of the manufactured production
j – number of the production

$j = \overline{1,2}$ (2 types)
$j = \overline{1,m}$ (m types)
x_j – amount of the j-th type production to be found
i – resource number
$i = \overline{1,3}$ (3 types)
$i = \overline{1,n}$ (n types)
b_i – amount of the i-th type resource
P – production realization price
a – input normals of the resources
a_{ij} –input normals of the i-th type resource for the j-th type production

Criterion – receipt maximization.

$F = x_1 \cdot P_1 + x_2 \cdot P_2 \rightarrow \max$

$$\begin{cases} a_{11} \cdot x_1 + a_{12} \cdot x_2 \leq b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 \leq b_2 \\ a_{31} \cdot x_1 + a_{32} \cdot x_2 \leq b_3 \end{cases}$$

In the general case mathematical model of the linear programming model will be the following:

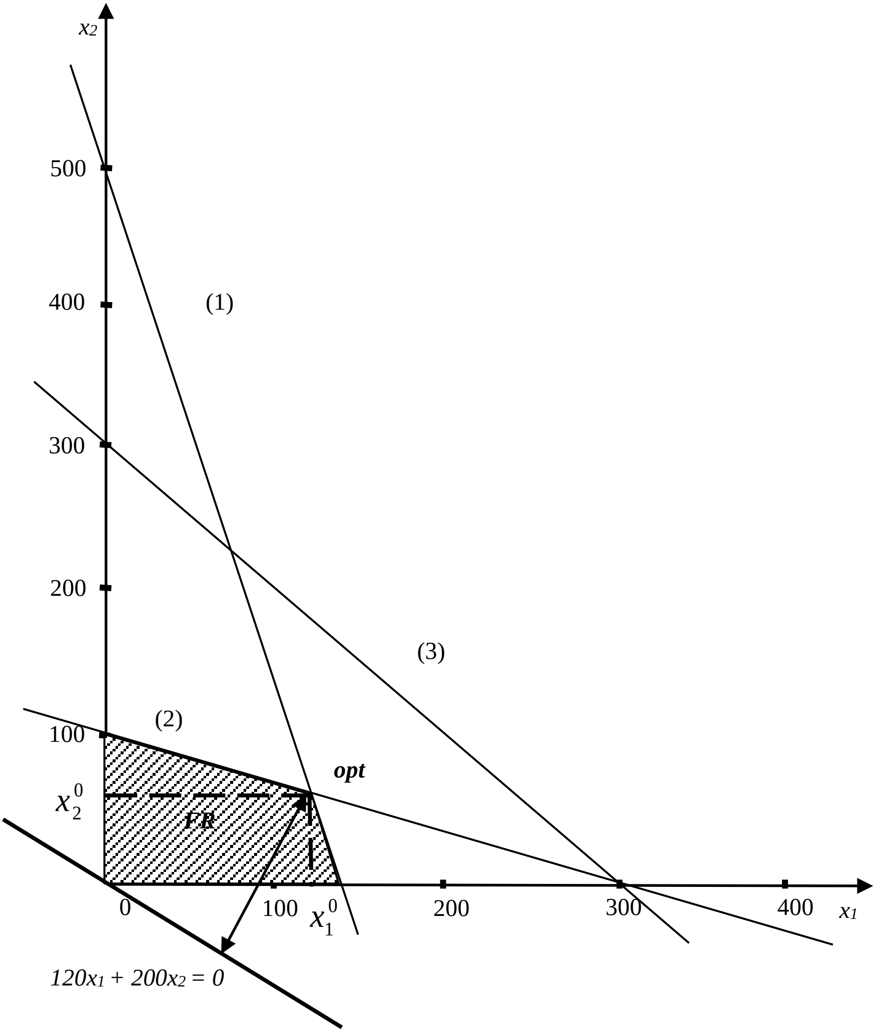
$$\begin{cases} F = \sum_{j=1}^{m} P_j x_j \rightarrow \max \\ \sum_{i=1}^{n} a_{ij} \cdot x_j \leq b_i, i \in \overline{1,n} \end{cases}$$

The criterion and constraints type determines the class of the task.

If the objective function and constraints are linear functions to the first power, the task is **linear programming problem**.

Graphical interpretation method

We plot the coordinate system.



$$\begin{cases} 0{,}7 \cdot x_1 + 0{,}2 \cdot x_2 \leq 100 \\ 0{,}2 \cdot x_1 + 0{,}6 \cdot x_2 \leq 60 \\ 0.1 \cdot x_1 + 0.1 \cdot x_2 \leq 30 \end{cases}$$

In determining the optimal point of the feasible region we are to consider two factors:

1) optimal point lies on the limits of the constraints;

2) optimal point in on the intersection of the constraints (one of the corner points)

For finding optimal point by geometric method we will plot the line corresponding to the zero level of the objective function – revenue (*F* = 0). Point from feasible region which is the most distant from this line will be optimal or maximizing criterion of the task.

$$\begin{cases} 0.7x_1 + 0.2x_2 = 100 \\ 0.2x_1 + 0.6x_2 = 60 \end{cases}.$$

Thus, $x_1^0 = 126.3$, $x_2^0 = 57.9$. Consequently, maximal value of the optimality criterion (revenue) of the task is:

$$F^0 = 120 \cdot 126.3 + 200 \cdot 57.9 = 26736$$

We denote $y_i = b_i - \sum_j a_{ij} x_j$.

The indicator y_i shows the amount of resource of $i-$th type, residuary after optimal decision realization. It is reserve of the particular resource.

$y_1 = 0$

$y_2 = 0$

$y_3 \neq 0$

If $y_i = 0$, we will determine resource as “scarce”. If $y_i > 0$, resource is “non-scarce”.

Management decisions making tasks contain parameters that are determined by the external environment or internal nature of the object.

Parameters include the price, input normals, amount of the storage.

Initial characteristics:

1) management decision
2) characteristics, directly dependent on the management decision: the storage remains, the value of the objective function.

Another situation: there is no storage, i.e. all indicators $y = 0$.

1 kilo of the beef – 100 r.

pork – 150 r.

pea – 20 r.

The enterprise has circulating assets in the amount of 5 000 r. for the productive activity.

We consider profit as a criterion.

$$F = x_1 \cdot P_1 + x_2 \cdot P_2 - x_1 (a_{11} \cdot C_1 + a_{21} \cdot C_2 + a_{31} C_3) -$$
$$- \underbrace{x_2 (a_{12} \cdot C_1 + a_{22} \cdot C_2 + a_{32} \cdot C_3)}_{\textbf{Cost of the production unit}}$$

C_1 – cost of the beef

C_2 – cost of the pork

C_3 – cost of the pea

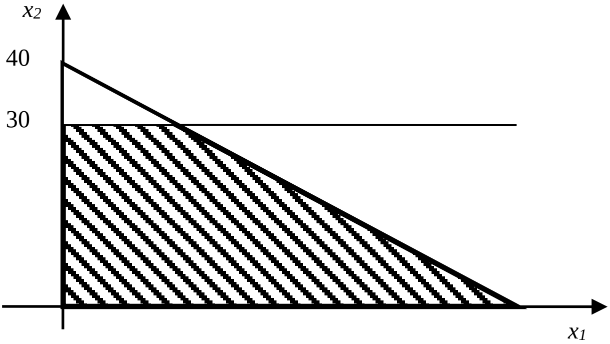
O – amount of the circulating assets

We bring the expressions into the canonical form:

$$\begin{cases} F = x_1 (P_1 - a_{11} \cdot C_1 - a_{21} \cdot C_2 - a_{31} \cdot C_3) + x_2 \underbrace{(P_2 - a_{12} \cdot C_1 - a_{22} \cdot C_2 - a_{32} \cdot C_3)}_{\textbf{profit from the 2}^{\textbf{nd}}\textbf{ production}} \rightarrow \max \\ x_1 (a_{11} \cdot C_1 + a_{21} \cdot C_2 + a_{31} \cdot C_3) + x_2 (a_{12} \cdot C_1 + a_{22} \cdot C_2 + a_{32} \cdot C_3) \le O \end{cases}$$

$$F = x_1 (120 - 0{,}7 \cdot 100 - 0{,}2 \cdot 150 - 0{,}1 \cdot 20) +$$
$$+ x_2 (200 - 0{,}3 \cdot 100 - 0{,}6 \cdot 150 - 0{,}1 \cdot 20) \rightarrow \max$$

$$\begin{cases} F = 18\, x_1 + 78\, x_2 \rightarrow \max \\ 102\, x_1 + 122\, x_2 \le 5000 \end{cases}$$



By the data the demand will be less than 30 kilo.

If the enterprise were paid back the debt in amount of 2000r., the constraint will be the following: $102\, x_1 + 122\, x_2 \le 7000$.

Analysis of management decisions sensitivity

$$\begin{cases} F=\sum_{j=1}^{m} P_j x_j \to \max \\ \sum_{i=1}^{n} a_{ij}\cdot x_j \le b_i, i\in\overline{1,n} \end{cases}$$

We have found the optimal task solution: $\mathbf{x_j^{opt}}$**, F (**$\mathbf{x_0^{opt}}$), $y_i = b_i - \sum_j a_{ij} x_j^{opt}$.

But there can happen internal or external disturbances which have the effect: $\mathbf{\Delta C_j}$; Δa_{ij} ; $\Delta b_i \to \Delta\, x_j^{opt}$; **ΔF (**$\mathbf{x_0^{opt}}$), Δy_i

Problem of the analysis

Δx → ? Δ y (What will be the change?)

Sensitivity coefficient:

$$\alpha = \frac{\partial y}{\partial x}$$

The indicator α shows how will the initial parameter *y* change in case of the initial parameter *x* changing by the unit.

We consider the matrix:

X						B
a_{11}	a_1	a_1	a_1	a_1	a_1	$\le b_1$
a_{21}	a_2	a_2	a_2	a_2	a_2	$\le b_2$
A_k						…
a_{i1}	a_i	a_i	a_i	a_i	a_i	$\le b_i$
$a_{к1}$	$a_к$	$a_к$	$a_к$	$a_к$	$a_к$	$\le b_к$
a_{m1}	a_m	a_m	a_m	a_m	a_m	$\le b_m$

Assertions:

1. In this optimal solution: $x_j \ne 0, j=\overline{1,k}$ (advantageous); $x_j = 0, j=\overline{k+1,m}$ (disadvantageous)

2. The stocks of the resource y_i = 0, if $i = \overline{1, k}$; the stocks of the resource $y_i \neq 0$, if $i = \overline{k+1, n}$.

These assertions are connected between each other, because optimal solution lies on the intersection of two lines.

In the initial matrix we select the nonvacuous matrix $k \times k$ (Aк)

X – row-vector of the nonnull task solutions

B – column-vector

$$A_к \times X = B$$

The expression of the sensitivity in the vector form:

$$\alpha = \frac{\partial X}{\partial B} \text{ – matrix of the sensitivity coefficient}$$

The following indicator determines the sensitivity of the *j*-th variable to the variation of the *i*-th resource:

$$\alpha_j^i = \frac{\partial x_j}{\partial b_i}$$

$$\Delta x_j = a_j^i \cdot b_j .$$

To calculate the α_j^i we differentiate left and right sides of the (1) with respect to B.

$$A_k \cdot \frac{\partial X}{\partial B} = 1$$

We multiply left and right sides by the matrix which is inverse to the sensitivity coefficient matrix A.

$$A_k \cdot A_k^{-1} \cdot \frac{\partial X}{\partial B} = A_k^{-1} .$$

Consequently,

$$\alpha = A_k^{-1} ;$$

$$\alpha_j^i = \frac{\partial x_j}{\partial b_i} .$$

If there is variation of several resources, general reaction of the system will be subject to the additive rule:

$$\Delta x_j = \sum a_j^s \cdot b_s$$

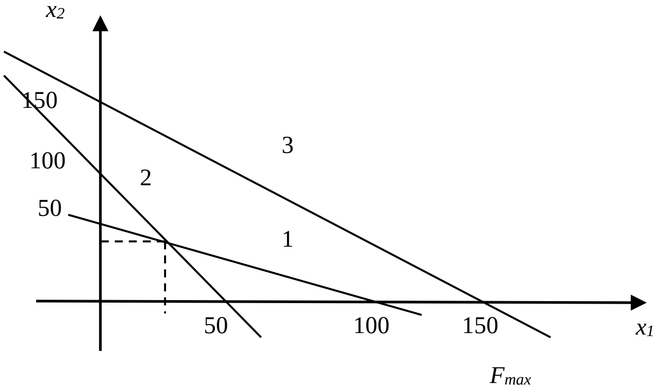
Example.

$$F = 2\,x_1 + 4x_2 \rightarrow \max$$

$$\begin{cases} x_1 + 2x_2 \le 100 \\ 2x_1 + x_2 \le 100 \\ 2x_1 + 2x_2 \le 300 \end{cases} \left| \begin{array}{l} b_1 \\ b_2 \\ b_3 \end{array} \right.$$

$$\begin{cases} \mathbf{x_1^{opt}} = 100/3 \\ \mathbf{x_2^{opt}} = 100/3 \end{cases}$$

$y_1 = 0$
$y_2 = 0$
$y_3 = 500/3$



$$\begin{cases} F = 200 \\ A_k = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \end{cases}$$

$$\begin{cases} x_1 + 2x_2 = b_1 \\ 2x_1 + x_2 = b_2 \end{cases}$$

Let us differentiate the system with respect to b₁.

$$\begin{cases} \dfrac{\partial x_1}{\partial b_1} + 2 \cdot \dfrac{\partial x_2}{\partial b_1} = 1 \\ 2 \cdot \dfrac{\partial x_1}{\partial b_1} + \dfrac{\partial x_2}{\partial b_1} = 0 \end{cases}$$

$$\begin{cases} \alpha_1^1 + 2 \cdot \alpha_2^1 = 1 \\ 2 \cdot \alpha_1^1 + \alpha_2^1 = 0 \end{cases}$$

$$\begin{cases} \alpha_2^1 = -2\cdot\alpha_1^1 \\ a_1^1 + 2\cdot\left(-2\alpha_1^1\right) = 1 \end{cases}$$

$$\begin{cases} \alpha_1^1 = -{}^{1}\!/_{3} \\ a_2^1 = {}^{2}\!/_{3} \end{cases}$$

Thus, $\alpha_1^1 = \dfrac{\partial x_1}{\partial b_1}$ shows how will the variable x change in case of the first resource stock b_1 changing by the unit.

With respect to b_2:

$$\begin{cases} \alpha_1^2 + 2\cdot\alpha_2^2 = 0 \\ 2\cdot\alpha_1^2 + \alpha_2^2 = 1 \end{cases}$$

$$\begin{cases} \alpha_1^2 = -2\cdot\alpha_2^2 \\ -4\cdot\alpha_2^2 + \alpha_2^2 = 1 \end{cases}$$

$$\begin{cases} \alpha_2^2 = -{}^{1}\!/_{3} \\ a_1^2 = {}^{2}\!/_{3} \end{cases}$$

Sensitivity coefficient, relative to non-scarce resource always equals zero.

$$\boldsymbol{\alpha_1^{3} = \alpha_2^{3}} = 0.$$

Resources substitution task

We assume that there appeared disturbances with respect to certain scarce resource Δb_s, thus $\Delta b_s \Rightarrow \Delta x_j$

The j-th production is particularly important for the system. Thus, we need to retain the output of the production on the previous level, i.e. $\Delta x_j = 0$.

$$\Delta x_j = \alpha_j^s\ \Delta b_s$$

This case can be formulated as a **resources substitution task.**

$$\alpha_j^s > 0;\ \Delta b_s < 0 \Rightarrow \Delta x_j < 0.$$

But we have resource b_l, which can be controlled. Thus, $\alpha_j^l > 0$ with varying resource b_l**.**

We need to estimate amount of the *l*–th resource, which allows to compensate incomplete delivery of the resource *S*.

$$\Delta x_j = \alpha_j^l \ \Delta b_l$$

$$\Delta x_j = 0$$

The condition of the interchanging: $\alpha_j^s \cdot \Delta b_s = -\alpha_j^l \cdot \Delta b_l$.

Substitution coefficient:

$$\Delta b_l = \frac{\alpha_j^s}{\alpha_j^l} \cdot \Delta b_s .$$

Next we introduce characteristic of the sensitivity coefficient (of the objective function) to the variation of the stacks of the resources.

$$Z_i = \frac{\partial F}{\partial b_i}, i = \overline{1, n}$$

$$F = \sum C_j x_j$$

$$Z_i = \sum_{j=1}^{n} c_j \frac{\partial x_j}{\partial b_i} = \sum_{j=1}^{n} c_j \alpha_j^i$$

$$\Delta F = Z_i \Delta b_i$$

$$\Delta F = \sum Z_s \Delta b_s$$

Coefficients $\boldsymbol{Z_i}$ for non-scarce resources always more than zero. For non-scarce resources coefficients Z_i equal zero.

Production profitability

$\boxed{\rho = \frac{G}{Z}}$, where G – profit.

Profitability of the production that was not included into optimal production program is less than profitability of taken.

Example: 2 types of production (A and B) are output; net cost is the same and equal 2; realization price is 4 and 3.

We are to develop production program (determine the output of the productions A and B), which will maximize the profit on condition that we have 100 units of circulating assets.

	A	B
Net cost	2	2
Price	4	3
Output	X_A	X_B

$G = 4X_A + 3X_B - 2X_A - 2X_B = 2X_A + X_B \rightarrow$ **max**
$2X_A + X_B \leq 100$
$X_A = 50$
$X_B = 0$
$F = 100$

$\rho_A = (4\text{-}2) / 2 = 1$
$\rho_B = (3\text{-}2) / 2 = 0,5$

We assume the following plan:

$$\begin{cases} X_A = 49 \\ X_B = 1 \end{cases} \Rightarrow F = 99$$

Thus, output of the production *B* will lead to the deficiency of the 1 unit of profit by the enterprise.

For non-profitable production (that is not included in the plan: $x_j = 0,\ j = \overline{k+1, m}$) we introduce loss unction *V*.

Loss function:

Loss function V_g shows the loss that the enterprise have in case of output of the *g*-th production unit (*g*>*K*, *K* – quantity of the production types in the plan).

Mathematical formulation of the loss function:

$$V_g = \sum_{i=1}^{n} a_{ij} \cdot Z_i - P_g$$

$$Z_i = \frac{\partial F}{\partial b_i}$$

P_g – price of the *g*-th production (objective function coefficient of the variable with the index *g*).

For more profitable production $j = \overline{1, k}$, loss function $V_j = 0$.

For less profitable production ($j > K$) loss function is positive, $V_j > 0$.

Loss function is 4-th output characteristic of the linear programming problem.

First three characteristics: x_j^0, $G(x_j^0)$, ρ_j ; V_j – loss function for the non-profitable production.

Example. $V_g = 4$, $P_g = 20$. If P_g equal 24, the production can be included into the plan.

Analysis of management decisions stability

The notion of the reference basis

Reference basis – situation, in which nomenclature of advantageous and disadvantageous production and also of scarce and non-scarce resources remains the same.

We assume the disturbances with respect to certain scarce resource $\Delta b_{si.}$ This variation leads to the changing of the value of variables x_j:

$$\Delta b_{si} \rightarrow \Delta x_j$$

$$\Delta x_j = \alpha_j^s \Delta b_s$$

If the optimal value of the variable x_j^0 is known, new value of this variable x_j^n is determined by the following expression:

$$x_j^n = x_j^0 + \Delta x_j = x_j^0 + \alpha_j^s \Delta b_s .$$

The condition of the basis invariability is the following: the amount of *j*–th production must be positive: $x_j^n > 0$. If it will equal zero, production will not be included into production program and will become “disadvantageous” instead of “advantageous”. Mathematical formalization of this condition is the following:

$$x_j^0 + \alpha_j^s \Delta b_s > 0 \text{ or } \Delta b_s > -\frac{x_j^0}{\alpha_j^s} .$$

This expression allows us to make analytical evaluation of the changing of Δb_s that will not lead to replacement of the system’s reference basis.

Permitted size of changing of the scarce resource - Δb_s.

If $\alpha_j^s > 0$, then adding of resource *s* will lead to increase of output of the *j* –th production, consequently, in this case changing of the system basis will not happen.

If $\alpha_j^s < 0$, then adding of resource *s* can lead to the changing of basis and amount of *j* –th production output can become equal zero, that mean that production will not be output.

Let us consider non-scarce resource b_i, for which the reserve $y_i \neq 0$ ($i = \overline{k+1, n}$) is calculated by the formula $y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j^o$.

We assume that there appeared disturbances with respect to the stock of scarce resource Δb_s, and it will lead to the changing of the value of the variables Δx_j. In turn, variation of Δx_j will lead to variation of the stocks of the scarce resources Δy_i ($\Delta b_s \rightarrow \Delta x_j \rightarrow \Delta y_i$).

Consequently, there can appear the situation, that Δb_s will lead to the becoming of scarce resource stock equal zero ($\Delta y_i = 0$). That mean that non-scarce resource becomes scarce, that is why the nomenclature of scarce and non-scarce resources has changed and there happened the changing of the system's basis.

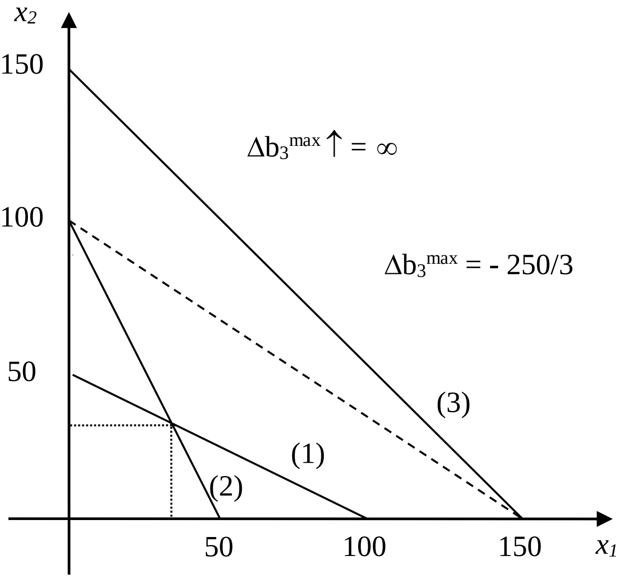
In this case, mathematical formalization of the system basis *invariability condition* will be the following:

$$y_i^n = y_i + \Delta y_i > 0$$

Example. Objective function: $\boldsymbol{F = 2x_1 + 3x_2 \rightarrow}$ **max**. Constraints:

$$\begin{cases} x_1 + 2x_2 \leq 100 \left| b_1 \right. \\ 2x_1 + x_2 \leq 100 \left| b_2 \right. \\ x_1 + x_2 \leq 150 \;\left| b_3 \right. \end{cases}$$

b_3 – non-scarse resource



$$x_1^0 = 100/3$$
$$x_2^0 = 100/3$$
$$F = 500/3$$

Reference basis: both productions are advantageous

$b_1,\ b_2$ – scarce resources;

b_3 – non-scarce resource.

We have disturbances with respect to the first resource Δb_1;

$$\alpha_1^1 < 0;\ \alpha_2^1 > 0$$

$$\Delta b_1^{\max} = 100$$

$$x_1 + 2x_2 \le 100$$

The stock of the first resource has grown, and the line (1) will move parallel up to the point $x_2 = 100$.

But we have constraint $x_1 = 50 \Rightarrow$ using constraint (1) we find x_2:

$$x_1 + 2x_2 \le 100 \Rightarrow x_2 = 25$$

Thus, we have opt (50;0) or (0;25).

Task: calculate the admissible values of b_2, b_3; sensitivity coefficients $\alpha_j^{\ i}$; z_j; calculate admissible limits of the objective function coefficients variation: $2x_1 + 3x_2$.

Δb₂: The stock of the second resource has grown $\Rightarrow$ line (2) $\Rightarrow x_1 = 100$.

$$\Delta b_2^{\max} = -50$$

$$2x_1 + x_2 \le 100 \Rightarrow x_1 = 25 \Rightarrow \text{opt } (0;\ 50) \text{ or } (25;0)$$

∂b₁ :

$$\begin{cases} \dfrac{\partial x_1}{\partial b_1} + 2\cdot\dfrac{\partial x_2}{\partial b_1} = 1 \\ 2\cdot\dfrac{\partial x_1}{\partial b_1} + \dfrac{\partial x_2}{\partial b_1} = 0 \end{cases}$$

$$\begin{cases} \alpha_1^1 + 2\cdot\alpha_2^1 = 1 \\ 2\cdot\alpha_1^1 + \alpha_2^1 = 0 \end{cases}$$

$$\begin{cases} \alpha_2^1 = -2\cdot\alpha_1^1 \\ a_1^1 + 2\cdot\left(-2\alpha_1^1\right) = 1 \end{cases}$$

$$\begin{cases} \alpha_1^1 = -\,{}^1\!/_3 \\ a_2^1 = {}^2\!/_3 \end{cases}$$

$$\alpha_1^3 = \alpha_2^3 = 0$$

∂b$_2$:

$$\begin{cases} \alpha_1^2 + 2\cdot\alpha_2^2 = 0 \\ 2\cdot\alpha_1^2 + \alpha_2^2 = 1 \end{cases}$$

$$\begin{cases} \alpha_1^2 = -2\cdot\alpha_2^2 \\ -4\cdot\alpha_2^2 + \alpha_2^2 = 1 \end{cases}$$

$$\begin{cases} \alpha_2^2 = -{}^1\!/_3 \\ a_1^2 = {}^2\!/_3 \end{cases}$$

$$Z_1 = \frac{\partial(2x_1 + 3x_2)}{\partial b_1} = 2\alpha_1^1 + 3\alpha_2^1 = 2\cdot\left(-{}^1\!/_3\right) + 3\cdot\left({}^2\!/_3\right) = {}^4\!/_3$$

$$Z_2 = 2\alpha_1^2 + 3\alpha_2^2 = 2\cdot\left({}^2\!/_3\right) + 3\cdot\left(-{}^1\!/_3\right) = {}^1\!/_3$$

$$Z_3 = 0$$

DECISION-MAKING UNDER UNCERTAINTY

Uncertainty in decision-making tasks

Basic decision-making task statement:

$$\begin{cases} F(x) \to \max \\ x \in X \end{cases}$$

We denote that a – certain parameters of the task.

In real practice the decision is made in conditions, when there is no information about these parameters.

Having no accurate information, DM conceives about admitted region of the parameters: $a \in A$.

The wider is A, the worse is situation and the higher is degree of uncertainty.

Uncertainty should be removed.

Uncertainty removal methods

Guaranteed result method

This method means that DM considers the worst hypothesis about value of the parameters a.

To be more specific we consider a parameter a_i which has positive sense (for example, production price: the higher is price, the better it is for the firm) and characterized by the admitted region:

$$a_i^{min} \leq a_i \leq a_i^{max}.$$

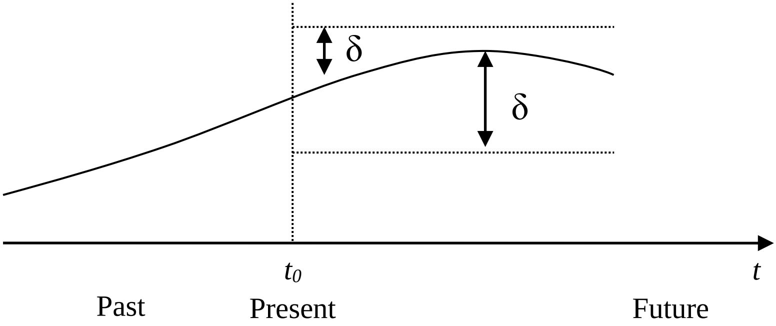
If the guaranteed result method is used, the DM considers the guaranteed (the worst) valuation of the parameters, i.e. $a_i^r = a_i^{min}$

Advantage: the method is easy to understand and implement

Disadvantage: the method leads to deficiency of the probable utility, i.e. the method is inefficient.

Statistical modeling method

Statistical modeling method connotes that decision-maker (DM) does not have accurate information about future values of the needed parameters. But DM has statistics (retroinformation) about earlier values of these parameters.



The variation of the parameters is characterized by the certain trend (upward or downward movement).

Thus, we need to plot the function that will describe the dependence of the parameter on t:

$$a = a\ (t)$$

$$a = \alpha + \beta * t \text{ – linear function}$$

where α and β – concrete numerical coefficients that are calculated by the least-squares method.

a^m – value of the parameter that calculated by this regression model:

$$a^m = \alpha + \beta \cdot t$$

Every model has an error.

Error is determined by the mean-square deviation:

$$\delta = \sqrt{\frac{\sum (a_e - a_m)^2}{N}} = \sqrt{\frac{\sum (a_e - \alpha - \beta \cdot t)^2}{N}}$$

a^e – expert values

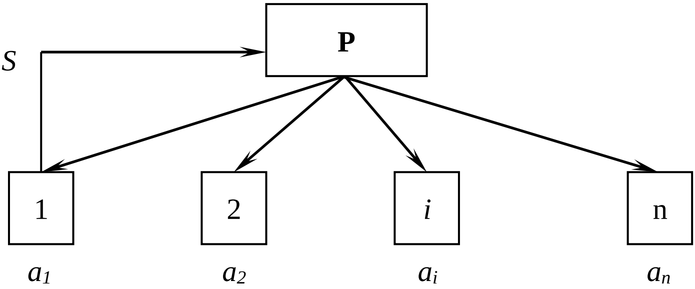
a^m – values got from the model

N – quantity of the time periods or quantity of the measurements.

Disadvantage: these statistical analysis methods are implemented only in case of slowly slowly varying processes.

Data forming method

We consider a system that consist of a center (principal) – decision maker and a set of elements that are characterized by the certain parameters:



In this scheme *a* – parameter that characterizes the element (each element has its own parameter).

DM has no accurate information about values of these parameters and requests the elements to give the information about these values.

S – valuation that element gives DM as a parameter *a* value.

Ideally, $a = S$, but ganarally it is not.

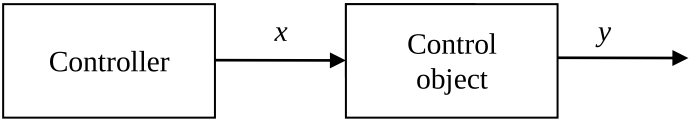
But DM has perception about the admitted region of these valuations:

$$a^{min} \leq S \leq a^{max}$$

Advantage: this is a way to uncertainty removal in decision-making

Disadvantage: there can be deliberate distortion of the valuations, thus, the principal can hardly get the accurate information.

Behavior of the all system generally depends on the elements' aims.



Organizational systems are hierarchical. The hierarchy has subordination levels.

As basic model we consider we consider two-level model because it is:

1) quite simple;

2) allows modeling of the system functioning processes;

3) results of the analysis of functioning can be the basic for the summarizing of conclusions for more complicated organizational systems.

The principal develops control action *x* on the basis of criterion.

Control action of the principal is aimed at the optimization of the functioning processes of the all system.

On basis of *x* the active elements begin to operate and achieve results *y*, i.e. *y* – result of the activity.

$$\begin{cases} F(x) \to \max \\ x \in X \end{cases}$$

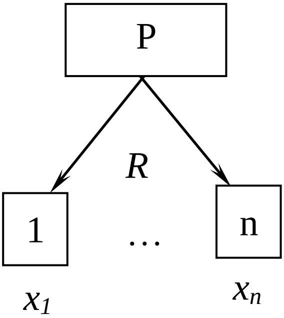
The principal develop plan values and target. (i.e. management decisions). x^0 – desired for the principal state of the system.

Ideally, $x^0 = y$. We introduce the function of two variables: $f\,(x,y)$ – formalized description of the active elements' aims.

At the stage of the plan targets realization the active elements start to transform plans *x* through the lenses of their interests, i.e. according to their objective function.

For the normal (optimal) organizational system functioning DM needs to solve the problem of the co-ordination of the interests and the objective functions of all participants of the interaction.

Example. The center (principal) plans production targets for *n* single-type enterprises. System as a whole should output *R* production units. We need to develop plans of the production output for each element of the system. At that principal needs to minimize general costs from the plan realization.



We denote that the costs are described the following way:

$$z_i = \frac{x_i^2}{2r_i} \text{ – costs function}$$

x_i – production output of the each enterprise
r_i – efficiency indicator of the *i*-th enterprise (comprehensive indicator: qualification level, technical equipment level…)

Formalization of the task:

$$\begin{cases} F = \sum_{i=1}^{n} \frac{x_i^2}{2r_i} \to \min \\ \sum_{i=1}^{n} x_i = R \end{cases}$$

Indicator *F* describes costs.

The task in case of the complete awareness of the principal

The tasks with distinct from the unity powers are **non-linear programming problems**.

Task: solve the task using Lagrangian method.

$$L(x_i, \lambda) = \sum_{i=1}^{n} \frac{x_i^2}{2r_i} - \lambda \cdot \left(\sum_{i=1}^{n} x_i - R \right)$$

$$\begin{cases} \frac{\partial L}{\partial x_i} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \end{cases}$$

$$\begin{cases} \dfrac{x_i}{r_i} - \lambda = 0 \\ \sum\limits_{i=1}^{n} x_i - R = 0 \end{cases}$$

$$x_i = \lambda\, r_i$$

$$\lambda \cdot \sum_{i=1}^{n} x_i = R \rightarrow \lambda = \frac{R}{\sum\limits_{i=1}^{n} r_i}$$

$$x_i^* = \frac{r_i}{\sum\limits_{i=1}^{n} r_i} \cdot R$$

The last expression is the optimal decision in case of the complete awareness of the principal, i.e. if the principal knows real values of r_i..

This mechanism – R$_i$ – is called *proportional allocation mechanism*.

<u>Properties of the proportional allocation mechanism Ri</u>

1) The procedure of planning and allocation is continuous and monotonic with respect to the indicator of the elements' efficiency.

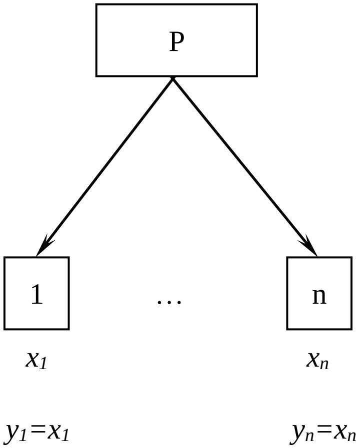
2) If the element gets certain amount of the resource, the element can get any less amount of the resource.

3) If the amount of the resources allocated between the elements has grown, each element can get not less amount of the resource, than earlier.

The principal usually have no information about values of r_i → there appears uncertainty, i.e. incomplete awareness.

Removal of the principal’s incomplete awareness by the guaranteed result method

Example.



x – plan target
y – actual results
$r_1 = 2$; $r_2 = 6$. $R = 80$.

If the principal is informed about actual values of r_i:

$$\left\{\begin{aligned} x_1 &= \frac{2}{2+6}\cdot 80 = 20 \\ x_2 &= \frac{6}{2+6}\cdot 80 = 60 \\ F &= \frac{20^2}{2\cdot 2}+\frac{60^2}{2\cdot 6} = 400 \end{aligned}\right.$$

If the principal is not informed about actual values of r_i but informed about size of their variations, we can use guaranteed result method.

$$\begin{cases} 1 \le r_1 \le 3 \\ r_1^g = 1 \\ 4 \le r_1 \le 8 \\ r_2^g = 4 \\ x_1 = \dfrac{1}{1+4} \cdot 80 = 16 \\ x_1 = \dfrac{4}{1+4} \cdot 80 = 64 \\ F = \dfrac{16^2}{2 \cdot 2} + \dfrac{64^2}{2 \cdot 6} = 64 + 341{,}3 = 405{,}3 \end{cases}$$

Thus, increasing costs in resources allocation using guaranteed result method is a kind of payment for the incomplete awareness about elements' condition.

Removal of the principal's incomplete awareness using data forming method

$$d_i \le S_i \le D_i$$

Using ability to effect by their messages on the decisions taken by principal; the elements try to give the information that will lead to the most advantageous for them decision. That mean that the information provided by the elements can be inadequate.

This distortion is called **information manipulation effect.**

Initial data:

$d_1 = 1$; $d_2 = 4$

$D_1 = 3$; $D_2 = 8$

$$\begin{cases} F = \sum_{i=1}^{n} \dfrac{x_i^2}{2s_i} \to \min \\ \sum_{i=1}^{n} x_i = R \end{cases}$$

Optimal solution:

$$x_i^* = \frac{s_i}{\sum_{i=1}^{n} s_i} \cdot R$$

If the element will overestimate its valuations ↑S_i, it will lead to growth of ↑x_i, but we need to consider strategies of all elements.

To determine S_i, which will be given by the elements, we need to know the elements' objective functions.

Elements' objective function is profit maximization. We denote element's objective function as f_i , production price per unit as P, production output as x_i.

Cost function is $\frac{x_i^2}{2r_i}$.

Thus, the element's objective function is the following:

$$f_i = P \cdot x_i - \frac{x_i^2}{2r_i} \to \max .$$

Thus,

$$\frac{\partial f_i}{\partial x_i} = P - \frac{2x_i}{2r_i} = 0 .$$

Consequently, the optimal for the element plan is determined by the following expression:

$$x_i^e = P \cdot r_i$$

The initial data is: $P = 15$, $r_1 = 2$, $r_2 = 6$, $R = 80$.

The general plan optimal for elements is $\sum x_i^e = 15 \cdot 2 + 15 \cdot 6 = 120 > R = 80 \Rightarrow$ the elements need to adjust to the conditions to maximize their profit.

$$\sum P \cdot r_i > R = \frac{\sum s_i}{\sum s_i} \cdot R$$

There can appear 3 situations of the system's equilibrium position:

1) $V > R \Rightarrow S_i = D_i$
2) $V < R \Rightarrow S_i = d_i$
3) $V = R \Rightarrow S_i = r_i$

Nash Equilibrium

Nash Equilibrium – a stable state of a system involving the interaction of different participants, in which no participant can gain by a unilateral change of strategy (S_i) if the strategies of the others remain unchanged.

Advantage: if we determine the Nash Equilibrium, we can predict the system's behavior

Disadvantages: this equilibrium is not single (i.e. we need additional assumptions and conditions)

This equilibrium is not resistant deviation of two or more participants.

Anonymous decision-making mechanism – mechanism that is symmetrical with respect to the elements' permutation (for example, election).

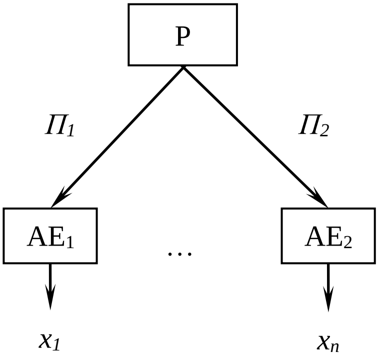
Assertions:

1) All anonymous resources allocation mechanisms are equivalent between each other, i.e. in case of the similar elements' preferences the mechanisms lead to getting the same amount of the resources.

2) All anonymous resources allocation mechanisms are equivalent to the proportional allocation mechanism, thus, henceforward we may consider the proportional allocation mechanism.

Statement of the economical interests co-ordination tasks

There are two types of the interactions in organizational systems: horizontal (between colleagues) and vertical (chief – subordinate).



The principal solves the problem of the elements' activity controlling.

The task formalization:

$$\begin{cases} F(\pi) \to \max \\ \pi \in \Pi \end{cases}$$

π – system's state, desired for the principal

x_i – actual system's state

Element's strategy is a choice of x_i.

Formalized **model of the elements' decision-making** can de represented the following way:

$$\begin{cases} f_i(x_i;\pi_i) \to \max \\ x_i \in X_i \end{cases}$$

If the objective functions of the principal and active elements differ (not coordinated), then actual results x_i can be inappropriate to the whole system's optimum because of the elements' activity.

In this case we are to solve the problem of the coordination of the principal's and active elements' interests.

1. One is approaches to the coordination of interests is using **coordinated planning methods.** This method is formalized the following way:

$$\begin{cases} F(\pi) \to \max \\ \pi \in \Pi \\ \pi \in \arg\max f_i(x_i;\Pi_i) \end{cases}$$

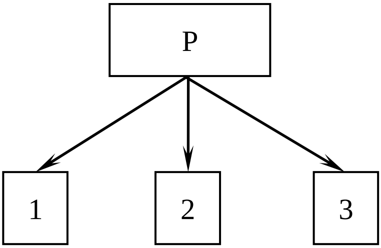
The last condition $\pi \in \arg\max f_i(x_i;\Pi_i)$ means that the plan should give the active element maximal value of his objective function.

Disadvantage: attempt to follow active element's interests can lead to inefficiency of the system as a whole.

2. Parametrical approach

We assume that there is one parameter common for all the system's members and the principal have economical and juridical power to vary this parameter. Thus, one or another value of the parameter is appointed and the principal as a meta-player that defines "rules of the game" can make coordination of the economical interests of the interaction participants.

Example.



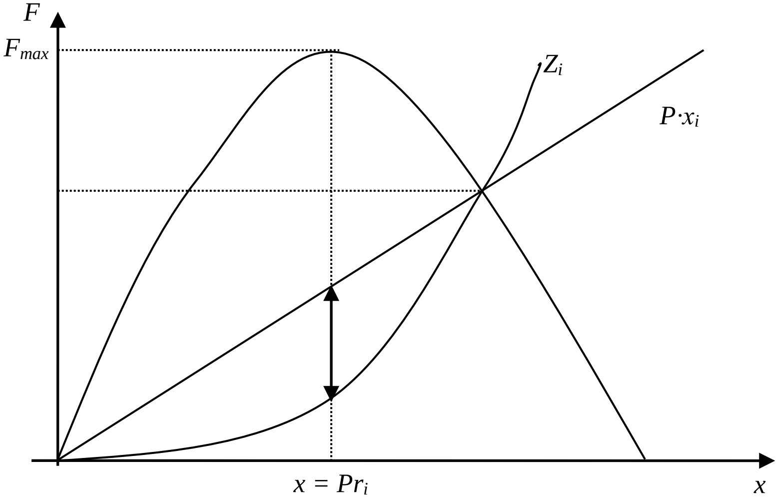
$$\begin{cases} F = \sum_{i=1}^{n} \frac{x_i^2}{2r_i} \to \min \\ \sum_{i=1}^{n} x_i = R \end{cases}$$

$x_i^{opt} = \frac{r_i}{\sum_{i=1}^{n} r_i} \cdot R$ – proportional allocation principle. (1)

Initial data: r_1= 1; r_2 = 2; r_3 =3; R= 180; P = 40.
Thus, x_1^o = 30; x_2^o = 60; x_3^o = 90.

$$f_i = P \cdot x_i - \frac{x_i^2}{2r_i}$$

The action desired for the elements (see figure below): $x_i^* = P \cdot r_i$ => x_1^* = 40; x_2^* = 80; x_3^* = 120.



We assume that real values of r_i are not known to the principal. The center uses data forming method:

$$d_i \le S_i \le D_i$$

The choice of S_i is a strategy of the i-th active element at the stage of planning.

The model of the planning in case of using data forming method:

$$x_i = \frac{s_i}{\sum_{i=1}^{n} s_i} \cdot R \qquad (2)$$

If the elements will overestimate their valuations, their plan will grow:

$$\frac{\partial x_i}{\partial s_i} > 0 \, .$$

If d_i= 1; D_i = 5 => S_1^n = S_2^n=S_3^n =D_1=D_2=D_3=5 =>x_1= x_2= x_3=60.

We note, that obtained x_1 value is more than x_1^* = 40. We are to calculate the optimal valuation S_1^{opt} for the first active element. This value is determined from the expression (2).

$$40 = \frac{s_i}{\sum_{i=1}^{n} s_i} \cdot 180$$

$$s_i^{opt} = 2.85$$

Thus, x_1 = 40; x_2 = x_3= 70.

Is there any parameter, common for all elements? **This is price.**

In certain cases it is appropriate to introduce intraproductive mechanisms and to appoint the intercompany prices.

Intercompany price – a price used into the system and is not relative to the market prices.

We denote V – productive plan desired for the elements.

$$V = \sum_{i=1}^{n} x_i^* = \sum_{i=1}^{n} P \cdot r_i = P \cdot \sum_{i=1}^{n} r_i$$

The problem is that V does not equal R.

In case of this inequality the center introduces intercompany prices P_c such that:

$$x_i^* = P_c \cdot r_i$$

$$V = R = P_c \cdot \sum r_i$$

Thus, $P_c = \frac{R}{\sum r_i}$ (3) – formulation of the intercompany prices forming mechanism

As $x_i = \frac{s_i}{\sum_{i=1}^{n} s_i} \cdot R$ – planning rule, $P_c = \frac{R}{\sum r_i} = \frac{R}{\sum s_i}$ – pricing mechanism (4).

(4) – the model of the coordinated parametrical controlling mechanism.

Thus, if $V > R \to x_i \uparrow \to S_i \uparrow \to P_c \downarrow \to x_i^* \to S_i \downarrow$.

Mechanism (4) is a coordinated controlling mechanism. It is proved that if *n* is sufficiently great (n > 12), this mechanism gives the reporting of the reliable valuations (requests) by the elements in the Nash Equilibrium state.

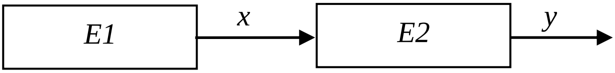
Use of this mechanism givel ability to solve the **aggregation problem.**

Aggregation problem means that all the elements can be substituted by the single element, which action is a sum of all their actions and which type is sum of their types (efficiency indicators).

Organization of the controlling in horizontal systems

The practice of the market economy often requires organization of the “horizontal” economical interaction. That means that there is no certain principal (center) that undertakes functions of “metaplayer” and determines game directive. Subjects of the interaction are to find interconsistent compromise of interaction.

We decide a model of the system that consists of two productive elements *E1* and *E2*.



The first element produces semimanufactures in amount of x and sells it to the second, that, in turn, produces commodity output in amount of y and sells it at the market price of P_2.

The problem is to determine the equitable price P_1.

Features to be considered:

1. The price is a result of the negotiations between first and second elements.
2. Both first and second elements are monopolists.

We consider that *E1* has costs determinated by function $z_1 = a_1 \cdot x$ ($a_1 -$ input normals). The costs of *E2* are described by the function

$z_2 = a_2 \cdot y + P_1 \cdot x$ (a_2 – input normals without accounting of buying semis). We assume that the objective functions of *E1* and *E2* represent their profit ("gain" – $G_{1,2}$). Obviously, feasible region with respect of P_1 is determined by following constraints:

$$G_1 = P_1 x - a_1 x \geq 0$$

$$G_2 = P_2 y - a_2 y - P_1 x \geq 0$$

Consequently, the size of the price value that will satisfy both elements is:

$$\frac{(P_2 + a_2) \cdot y}{x} \geq P_1 \geq a_1 .$$

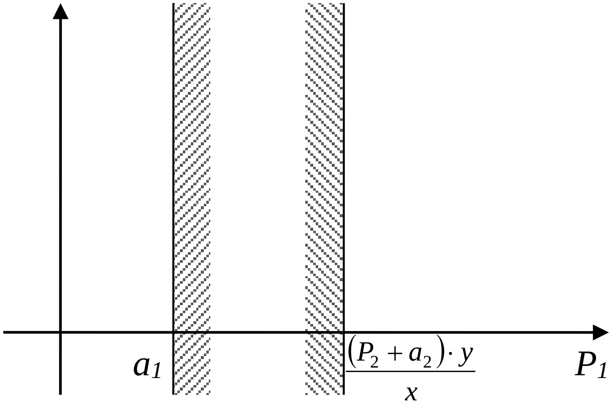
There is the ratio $y = j \cdot x$ that shows part of the semimanufacture x in finished product y.

We take as example the following initial data: $z_1 = 2 \cdot x$; $z_2 = 3 \cdot y + P_1 \cdot x$, $j = 0{,}5$. The second element sells production for the price $P_2 = 10$. Thus, objective functions of the elements will be:

$$G_1 = P_1 x - 2x \to \max$$

$$G_2 = 10 \cdot 0{,}5 \cdot x - 3 \cdot 0{,}5 \cdot x - P_1 x = 3.5x - P_1 x \to \max$$

The feasible region of the task is:



Determine the equitable price P_1 needs economically justified approach.

1) equal profit principal: $G_1 = G_2$

$$P_1 x - a_1 x = P_2 jx - a_2 jx - P_1 x$$

$$P_1 = \frac{P_2 j - a_2 j + a_1}{2}$$

$$P_1 = \frac{10 \cdot 0{,}5 - 3 \cdot 0{,}5 + 2}{2} = 2{,}75$$

2) equal profitability principal: $\rho_1 = \rho_2$

$$\rho_1 = \frac{P_1 x - a_1 x}{a_1 x} = \frac{P_1 - a_1}{a_1}$$

$$\rho_2 = \frac{P_2 jx - a_2 jx - P_1 x}{a_2 jx + P_1 x} = \frac{P_2 j - a_2 j - P_1}{a_2 j + P_1}$$

$$\frac{P_1 - a_1}{a_1} = \frac{P_2 j - a_2 j - P_1}{a_2 j + P_1}$$

$$(P_1 - a_1)(a_2 j + P_1) = a_1 (P_2 j - a_2 j - P_1)$$

$$P_1 a_2 j - a_1 a_2 j + P_1^2 - a_1 P_1 = a_1 P_2 j - a_1 a_2 j - a_1 P_1$$

$$P_1^2 + P_1 a_2 j - a_1 P_2 j = 0$$

Positive solution of the quadratic equation is:

$$P_1 = \frac{-a_2 j + \sqrt{a_2^2 j^2 + 4 a_1 P_2 j}}{2}$$

$$P_1 = \frac{-3 \cdot 0.5 + \sqrt{3^2 0.5^2 + 4 \cdot 2 \cdot 10 \cdot 0.5}}{2} \approx 2.5$$

3) principal of the normative profitability allocation

We assume that the first enterprise is less profitable than the second. We denote basic profitability values ρ_1^b and ρ_2^b. Contracting parties decide to maintain existing levels of profitability.

The coefficient K shows how ρ_1^b is less than ρ_2^b.

$$K = \frac{\rho_2^b}{\rho_1^b}$$

$$\rho_2^b = K \cdot \rho_1^b$$

$$\frac{P_1 - a_1}{a_1} \cdot K = \frac{P_2 j - a_2 j - P_1}{a_2 j + P_1}$$

$$K \cdot P_1^2 + [a_1 + K \cdot (a_2 \cdot j - a_1)] \cdot P_1 + j \cdot [a_1 \cdot a_2 \cdot (1 - K) - P_2 \cdot a_1] = 0$$

We denote $K = 2$.

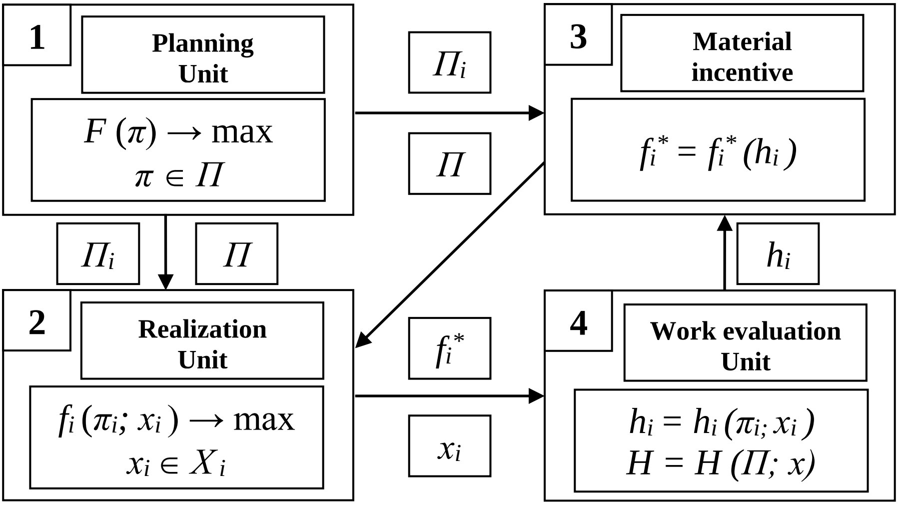
$$2\cdot P_1^2+\left[2+2\cdot(3\cdot0.5-2)\right]\cdot P_1+0.5\cdot\left[2\cdot3\cdot(-1)-10\cdot2\right]=0$$

$$2\cdot P_1^2+P_1-13=0$$

$$P_1=\frac{-1+\sqrt{1+104}}{4}\approx 2.3$$

Description of the organizational systems functioning models.

Mechanism of organizational systems functioning



Whole combination of this units makes the mechanism of organizational systems functioning. The management science mean having ability to analyze existing mechanisms.

1. Planning unit → the planning function is being realized.

In this unit the plan target π_i for each *i*-th productive element and a system as a whole ∏ is being developed.

2. Active element starts to work and at the stage of **realization** makes the decision about his actions.

x_i – the actual strategy that each active element chooses at the stage of plan realization

f_i – local objective function of the active element AE_i

π_i – plan target

X_i – admissible states region

3. Material incentive Unit

The important problem is a goal-setting. In organizations there are 3 common systems of executives’ behavior motivation.

1) Methods of the *administrative* pressure (orders, punishment…)

2) Methods of the *moral* impact – effect on psyche, public opinion (especially positive).

Both administrative and moral methods have one disadvantage – they are hardly formalized.

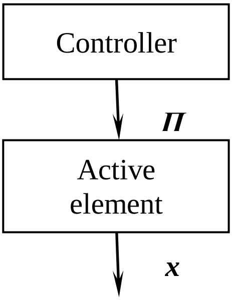
3) Methods of the *material* incentive. The controller provides executive with financial incentives for actions desired for principal. Advantages of the material incentive methods:

– quantitativeness
– formalizability
– correspondence with the present

f_i^* – material stimulus that *i*-th productive element gets; this reward depends on how effective each element made his work.

4. Work evaluation unit

At the first stage we consider the following structure of the organizational system:



In this unit two procedures are processed:1 – evaluation of the each active element work efficiency; 2 – evaluation of the whole system's work efficiency.

We consider that the Controller have developed the plan Π. Active element chooses strategy x according to his own aims. Thus, how can we evaluate the active element's activity?

The first type of the indicators that allow us to evaluate the work of the active element is **absolute indexes.**

Absolute indexes are indicators that have substantial (or economical) sense and dimension (for example, tones of gasoline in refining).

In a number of cases using absolute indexes is not suitable, because we need to know not only absolute results, but also dynamics.

The second type of indicators is **relative indexes.**

Plan is the state of the system, desired for principal.

$x_i^* = \frac{x_i}{\pi_i}$ – relation of the actual value of the indicator to the planned one (part of the plan execution)

We can multiply this expression by 100% is possible to have the percent of the plan execution.

The relative index can be connected not only with a plan. We can calculate index which shows how much better the elements worked against previous period.

$$x_i^* = \frac{x^i}{x^i(-)}.$$

In the expression below $x^i(-)$ is a value of the x^i indicator in previous period.

The main advantage of the relative indexes – non-dimensity.

In complicated organizational systems index *x* is characterized by big set of indicators to avoid rough simplification. But if broad spectrum of indicators, it is hard to compare them.

Requirements for the efficiency indicators:

1. *Occurrence of substantial sense*
2. *Quantitativeness*
3. *Measurability*
4. *Comparability*

We can use different characteristics, for example, part of the plan exceed/shortfall, the average of several indicators, relation to the normative values:

$$x_i^* = \frac{x_i}{\pi_i}$$

$$x^* = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$x_i^* = \frac{x^i}{N^i}$$

In case of having several indicators of work, we need to develop total integral index.

As an example we consider aircraft construction enterprise “Aviastar”.

There are three indicators used in this enterprise:

x_1 – amount of the production (standard hours)

x_2 – quality (% of nondefective production)

x_3 – culture of production and industrial safety: здесь проставляется балл [0 …10].

Next step we make normalization of this absolute indicators.

$$x_1^* = \frac{x_1}{\pi_1}$$

$$x_2^* = \frac{x_2}{100}$$

$$x_3^* = \frac{x_3}{10}$$

Next we need to make several combinations. The simplest integral index is a sum of three mentioned normalized indexes (the “winner” has the largest value). For more profound and exact evaluation we need to introduce coefficients β_i of the relative significance of the i-th indicator. There are constrains laid on these coefficients:

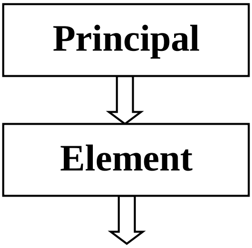
$$\begin{cases} 0 \le \beta_i \le 1 \\ \sum_{i-1}^{n} \beta_i = 1 \end{cases}.$$

Thus, the integral criterion will be the following:

$$\psi = \sum_{i=1}^{n} \beta_i \cdot x_i^*$$

Example 2. We considered developing a system of adding up the results of the world speed skating championship in the chapter “Multicriteriality in decision-making problems” (p. 6).

Basic incentive schemes



δ – incentive function;

$F(\delta(\bullet), y) = H(y) - \delta(\bullet)$ – principal’s gain function;

δ(•) depends on the variety of parameters

$H(y) - \delta(y) = \delta(y) - C(y)$

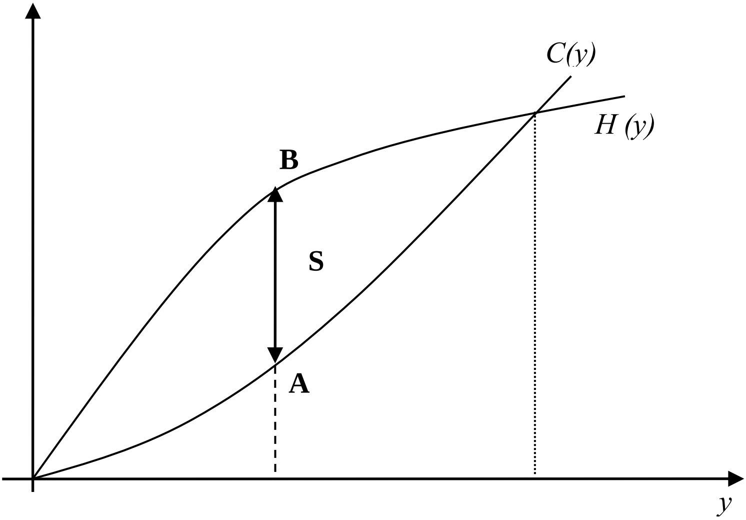
We assume that H is nonnegative in each action y and has its maximum if $y \neq 0$.

C (y) – element’s cost function. C(y) is nonnegative, not decreasing and possesses the zero value if y=0.

$$P(\delta(\bullet)) = \operatorname*{Argmax}_{y\geq 0}(\delta(y) - C(y))$$

The element will choose the action from the set of actions that maximize his objective function.

We determine the admissible states region:



S – the compromise region of the incentive task

The most advantageous point for principal is **A**, for the active element – **B**.

Let us assume that the principal uses incentive scheme with the complicated dependence of the active element's remuneration on its actions.

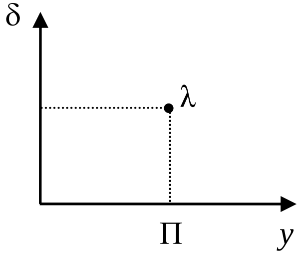
Assertion. It is enough for principal to use the scheme of the class in which the incentive function is non-nil in one point, i.e. center can use *compensatory* incentive scheme.

$$\begin{cases} \delta(y,\Pi) = \lambda, y = \Pi \\ \delta(y,\Pi) = 0, y \neq \Pi \end{cases}$$

All complicated schemes are ineffective.

Basic functions of the financial incentive

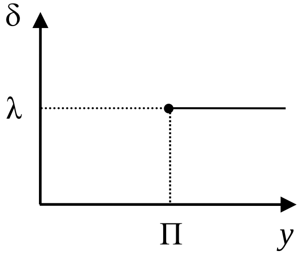
1. *Compensatory incentive scheme* (mentioned above)



Π – plan

This scheme aims the element at the exact execution of the plan.

2. *Spasmodic incentive scheme*

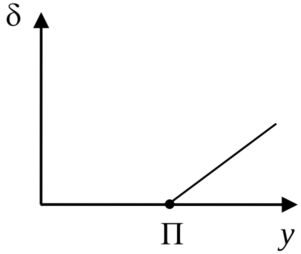


$$\delta = \begin{cases} \lambda, y \geq \Pi \\ 0, y < \Pi \end{cases}$$

This scheme does not aim the element at the overfulfilment of the production plan

The first and second incentive schemes belong to the *right mechanisms* class, because they aim the element at the execution of the plan and no more.

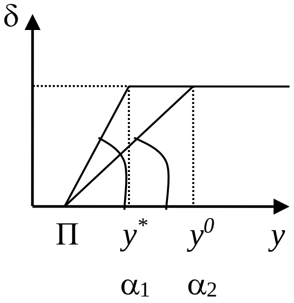
3. *Proportional (linear) incentive scheme*

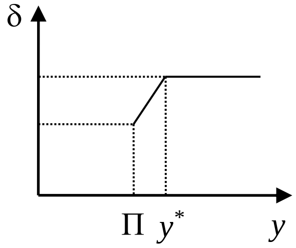


$$\delta = \begin{cases} \alpha\,(y - \Pi),\ y \geq \Pi \\ 0,\ y < \Pi \end{cases}$$

α – reward norm

4–5. Combined incentive schemes





Task. We consider the brigade of two workers that output single-type production. Their plans equal Π_1 and Π_2, respectively. Every production unit gives one ruble to the wages fund (*WF*).

$WF = x_1 + x_2$, where $x_{1,2}$– amount of the production, output by the workers.

$$f_1 = f_2 = \frac{x_1 + x_2}{2}$$ –scheme of payment.

We denote workers' maximal amount of the production output as A_1 and A_2 . Thus, their labor comfortability function is $(A_i - x_i)$.

Their labor satisfaction function is the following:

$$f_1 = f_2 = \frac{x_1 + x_2}{2} \cdot \left(A_i - x_i\right)$$

The initial data is the following:

$$\Pi_1 = 9, \Pi_2 = 10$$

$$A_1 = 20, A_2 = 22.$$

We make the decisions table:

x_2 x_1	9	10	11	12
10	104,5 114			
11	110 110	105 115,5		
12			103,5 115	
13				

System's equilibrium position

Pseudo-optimal point

Conclusion:

1) System of the leveling pay will not lead to the growth of the productivity of labor.

2) System of the leveling pay is used in the centralized system of planning

Piece-rate system

$$S_i = x_i$$

We consider the same labor comfortability function, thus the worker's objective function will be the following:

$$f_i = x_i\,(A_i - x_i)$$

$$\frac{\partial f_i}{\partial x_i} = A_i - 2x_i = 0$$

$$x_i = \frac{A_i}{2}$$

System's equilibrium position

$$\begin{pmatrix} x_1^o = 10 \\ x_2^o = 11 \end{pmatrix}$$

DECISION-MAKING IN WEAKLY-FORMALIZABLE SYSTEMS

1. Decisions table

We assume that there is a set of the probable management decisions. Every decision leads to a certain fixed gain of the decision-maker. But the amount of this gain is determined by the conditions that will appear while the decision is being realized.

Generally, **decisions table** has the following form:

Situation						
j \ i	1	2	…	i	…	n
1						
2						
…						
j				$\boldsymbol{a_{ij}}$		
…						
m						

$\boldsymbol{a_{ij}}$ – gain after the decision $\boldsymbol{j}$ in the situation $\boldsymbol{i}$**.**

After forming the decisions table we need to elaborate the criteria which allow to define the management decisions concretely.

Wald Criterion

Analogue of the guaranteed result method.

$$F = \max_j \min_i a_{ij}$$

For each probable future situation we select the decision that **maximizes minimal gain**.

Maximization of the average

$$F = \max \frac{\sum_{i=1}^{n} a_{ij}}{n}$$

The controller makes the decision $\boldsymbol{j}$**, which maximizes the average gain**

Probabilistic approach

We make the assumption that situations are not equiprobable.

$$0 \le p_i \le 1$$

The indicator p_i determines the probability of the i-th scenario.

$$F = \max_j \min_i \left[\sum_{i=1}^{n} p_i a_{ij} \right]$$

$$\sum_{i=1}^{n} p_i = 1$$

2. Expert examinations method

The method is generally used in evaluation problems.

There are *n* objects. The objects can be ranked by the range of their significance or value. To do this we involve *m* experts. They are to fill in the following table:

Objects						
j / i	1	2	…	*i*	…	*n*
1						
2						
…						
j				$\boldsymbol{a_{ij}}$		
…						
m						

$\boldsymbol{a_{ij}}$ – numerical score of the *i*-th object by the ***j*-th expert**.

$$\varphi_i = \sum_{j=1}^{m} a_{ij}$$

Criterion is a sum of the scores that object got by the experts valuation..

$$0 \le \boldsymbol{a_{ij}} \le \boldsymbol{A_{ij}}$$

0:10 0:100

The main disadvantage is that in case of the large quantity of objects the experts valuations can be contradictory.

Paired valuations method

The experts fill the following table with valuations:

i	**1**	**2**	**3**
1		1	3
2	1		
3			

K – column
S – row

At the intersection of the row S and column K we indicate the index of the object preferred by expert.

Next we sum the preferences of each object for all experts and then we rank the objects by descending of the preferences.

GUIDE FOR THE WORKSHOPS

Theme 1. Feasible region in management tasks

Introduction

Criteria. Realization of management decisions leads to different results. To compare quality of different management decisions, we should have an ability to evaluate their results. Totals of management decisions are evaluated using *efficiency criteria*, or *optimality criteria*. Optimality criterion is a mathematical expression (model) of decision's goal that allows quantitative evaluation of goal achievement measure. "Criterion" is defined in the dictionary as "principle or standard by which something may be judged or decided". Management decision which is the best by chosen optimality criterion, i.e. decision that provides needed extreme value (maximal or minimal), is called *optimal decision (optimal control).*

We should notice that "optimal control" relative not absolute concept. There cannot be absolutely optimal control, every optimal control can be the best only in concrete narrow sense that was determined by optimality criterion. One concrete control that is optimal by one criterion can be not optimal, "bad" by another criterion.

In management Decision Theory a criterion is a tool for decisions' quantitative evaluation, their comparison and selection of the best (optimal). Consequently, criterion should fit the following requirements:

4. *Quantitativeness*
5. *Measurability*
6. *Comparability*

Any complex object of management decision is characterized by many indicators. Generally these indicators are not equal ranking: some of them are accessory, weakly related with the aims of control, therefore slightly affecting on decision-making. Another, on the contrary, are main indicators that directly express management aims and determinatively affect the decision-making. Obviously, the latter indicators should be criteria of choosing optimal decision.

Whereas optimality criterion value depends on indicators describing object properties, used resources etc., that is why optimality criterion is also usually called *criterion function* (or *objective function*, or *effectiveness function)*. Let us denote *criterion function* as $F(x)$, where $x = (x_1, x_2, ..., x_i, ..., x_n)$ $n-$ control object state vector.

Constraints. Feasible region. Right statement and solution of optimizing task is impossible without constraints, always peculiar to control object and caused by its physical, economical or another properties. Complex of constraints data is *region of admissible states (RAS)* of the control object.

Control object in general case is not isolated. It functionates in external environment. Therefore, constraints in decision-making are determined by two factors:

– external;

– internal.

External constraints are caused by external environment, for example, demand on goods, price for needed raw materials, procurement quantity that can be provided by input supplier etc. Decision maker can not actually affect these constraints. Denote $x^{exte} = (x_1^{ext}, x_2^{ext}, ..., x_j^{ext}, ..., x_n^{ext})$ – vector of external constraints.

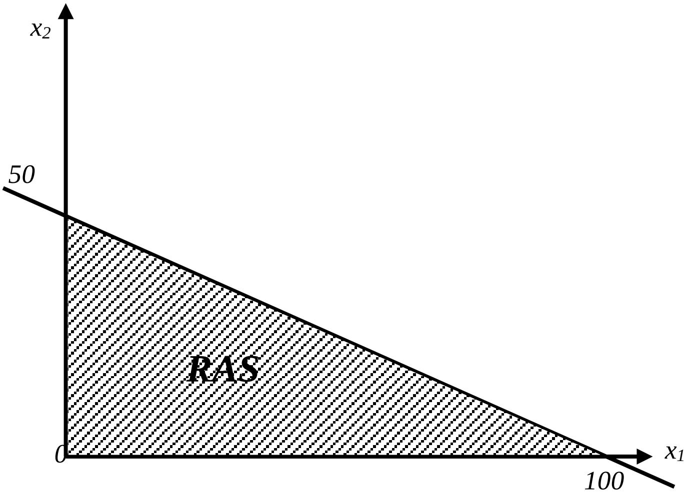
Internal constraints are determined by object’s character, properties, specifics, abilities. Some of internal constraints can be corrected by decision maker in permissible limit. We denote vector of internal constraints $x^{in} = (x_1^{in}, x_2^{in}, ..., x_j^{in}, ..., x_n^{in})$

Intercept of external and internal constraints combine into region of admissible states $X = x^{ext} \bigcap x^{in}$.

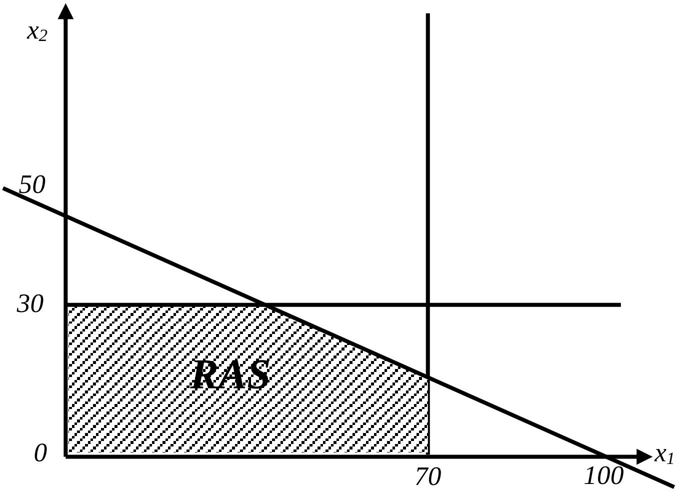
Obviously, optimization task solution need constraints of possible control object states $x \in X$.

Example.

An enterprise produces two types of products. It uses a single resource (money). A controller generates plans to products release. The cost of manufacture of each production unit is 1 and 2 monetary units, respectively. The amount of working capital is 100 currency units. We give a graphical interpretation of the region of admissible states.



We denote internal constraints caused by production capabilities. The maximum permissible output of the first product is 70 units, the second is 30. How does the region of admissible states change in this case?



Task

Industrial technological object (petroleum refinery) produces gasoline which is characterized by a quality indicator (initial boiling point). State standard value of bubble point is 35ºC. The actual value of this parameter should be less. The volume of production index associated with the quality of the following equation: $y = a - bt$, where $y -$ the volume of output, $t -$quality indicator (initial boiling point), a, b – the numerical coefficients. The task is:

1. Plot the region of admissible states if $a = 287.5$ and $b = 2.5$.

2. We denote that controller gives a shop floor productivity scheme: $\pi = 100$. How does the region of admissible states change in this case?

Theme 1 (continuation). Optimization of mixing semis (in refining)

Example.

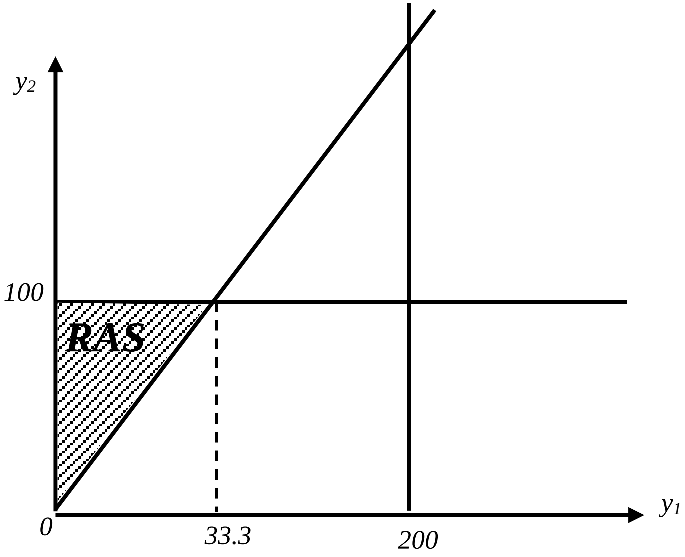
There are two types of gasoline intermediate products with octane levels of quality. The first intermediate product octane is $O_1 = 86$, the second is $O_2 = 98$. The octane of the mixture is described by the additive expression: $O^{mix} = \frac{O_1 y_1 + O_2 y_2}{y_1 + y_2}$, where y_1 and y_2 – the amount of the first and second intermediate products.

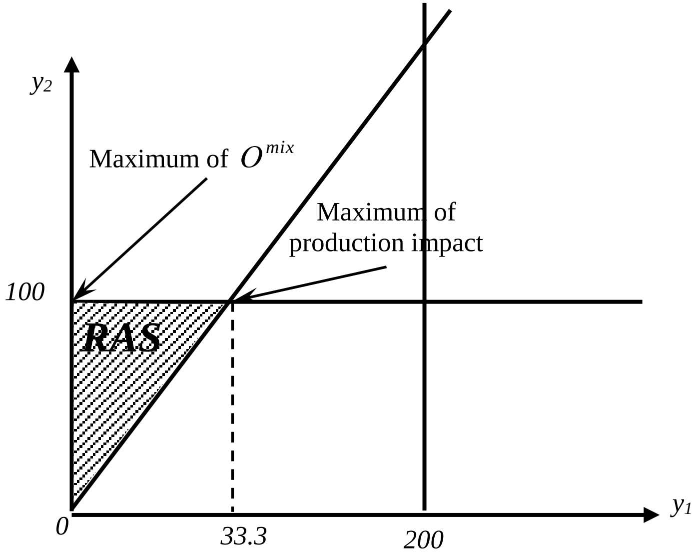
Let us plot the region of admissible states if formulation of mixture is $O^{mix} \geq 95$.

Let us find the optimal decision of the gasoline mixture by the following criteria:

a) maximum of production impact;

b) maximum of O^{mix}.





Task

A petroleum refinery has three types of gasoline intermediate products, all are restricted. The intermediate product quality is characterized by octane: 74, 80 and 98. The task is to mix intermediate products into two integrated products with octane 76 and 92 maximizing the criterion of profit. The octane of the mixture is described by the additive expression. The stocks and prices of intermediate products, integrated products prices are given in the table below:

Intermediate products (octane)	***Stocks***	***Price***
74	100	2
80	150	7
98	50	12
Integrated products (octane)	***Production impact***	***Price***
76	?	6
92	?	11

Theme 2. Formulation and solution of optimization decision-making tasks. Geometric and economic interpretation.

Introduction

The optimization task of management decision making of can have definite mathematical statement. The task can be represented the following way.

We consider a process of management decision making. The result depends on the control object state vector and several nonrandom fixed parameters that are absolutely known to decision maker. Control object state vector can be represented as $n-$dimensional vector $x=(x_1,x_2,...,x_j,...,x_n)$. There are constraints imposed on vector components. These constraints are conditioned by physical and economical meaning of the task. The efficiency of the control is characterized by numeral optimality criterion F.

To be more specific we would like to consider that we have to find optimal decision of manufacturing firm controlling by the optimality criterion of revenue maximization. We consider that organization outputs n production types. We denote production output as $x_j, j=\overline{1,...,n}$, price as c_j. Producing takes m types of raw materials (or resources). Resources stocks are denoted as $b_i, i=\overline{1,...,m}$. Also we denote that a_{ij} means input normals of $i-$th resource needed for producing one unit of production of $j-$th type.

The task is to find optimal production output that provides maximal profit of the organizational system.

The mathematical formalization of the task will be the following:

$$\begin{cases} F=\sum_{j=1}^{n} c_j x_j \to \max \\ \sum_{i=1}^{m} a_{ij}\cdot x_j \le b_i, i\in\overline{1,m} \\ x_j \ge 0, j\in\overline{1,...,n} \end{cases} \qquad (2.1)$$

The task (2.1) is often called *linear programming problem*. This type of task means that objective function and constraints are linear functions of the variables x. Other mathematical programming tasks that can not be represented as a model (2.1) are *nonlinear programming problems.*

The solution of the task includes optimal production output x_j^0 and maximal value of the objective function $F^0 = F(x_j^0)$.

There is huge variety of linear programming problems solution methods. The most multipurpose and prevailing is simplex-method. This method is well-developed and made in form of standard software that belong to mathematical software for modern computers. Simplex-method can be used for solution of every linear programming problem. Advantage of this method is ability to find exact solution for the finite number of steps.

In linear statement of management decision making problem we need to take into account that linear (especially determinate) description of the problem is rough approximation of real problem. More detailed analysis of the problem often allows to find nonlinear and stochastic phenomena. A linear phenomenon refers to one in which there is no direct proportionality between cause and effect.

We would like to consider graphical interpretation of the linear programming problem at the example.

Example.

An enterprise produces sausage of two types: boiled sausage (120 rubles for kilo) and ham sausage (200 rubles for kilo). Three types of resources are used for production: beef, pork and pea. The stock of beef is 100 kilo, pork – 60 kilo, pea – 30 kilo. Input normals for each resource are in table below:

	Boiled sausage	***Ham sausage***
Beef	0,7	0,2
Pork	0,2	0,6
Pea	0,1	0,1

Decision maker is to develop optimal plan of production output fitting following criteria:

3) fit the task constraints;

4) provide maximal revenue from production realization.

According to denoted marking we will have following variables in task formalization (2.1):

x_1, x_2 – production output of first (boiled sausage) and second (ham sausage) production,

$c_1 = 120, c_2 = 200$ – unit price of first and second production,

$b_1 = 100, b_2 = 60, b_3 = 30$ – storage of resources (beef, pork, pea),

$\alpha_{11} = 0{,}7$, $\alpha_{12} = 0{,}2$ – beef input normals for first and second production, respectively,

$\alpha_{21} = 0{,}2$, $\alpha_{22} = 0{,}6$ – pork input normals for first and second production, respectively,

$\alpha_{31} = 0{,}1$, $\alpha_{32} = 0{,}1$ – pea input normals for first and second production, respectively.

All things considered, mathematical formalization of the task will be the following:

$$F = 120x_1 + 200x_2 \to \max$$

$$\begin{cases} 0{,}7x_1 + 0{,}2x_2 \le 100 \\ 0{,}2x_1 + 0{,}6x_2 \le 60 \\ 0{,}1x_1 + 0{,}1x_2 \le 30 \\ x_1, x_2 \ge 0 \end{cases} \tag{2.2}$$

In co-ordinates x_1, x_2 we construct feasible region (FR) of the task (2.2), that mean intersection of all the constraints of the task (see Figure 2.1).

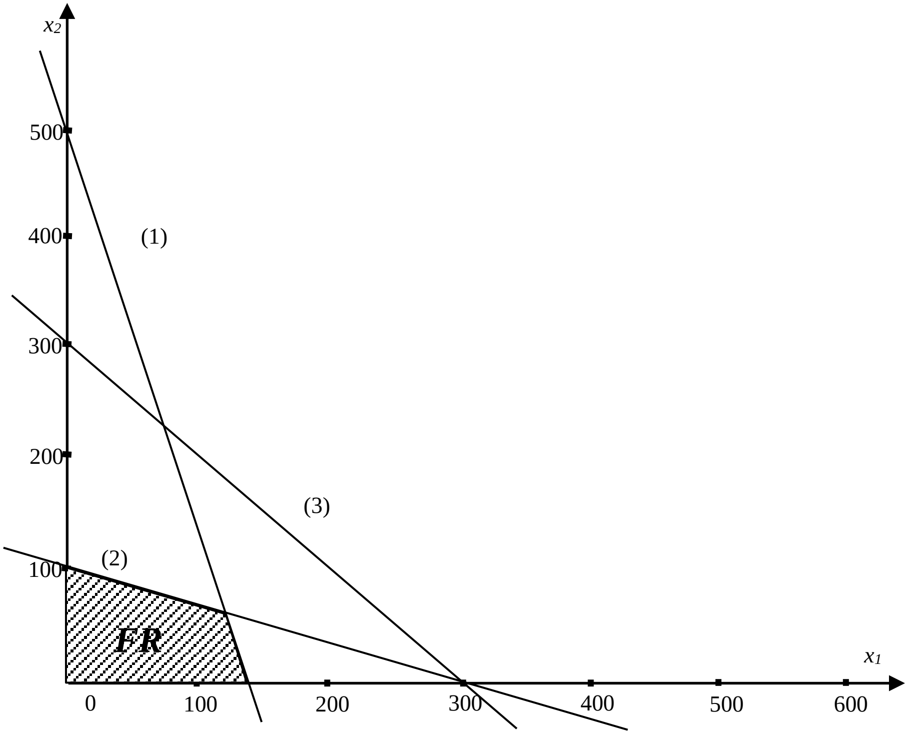


Figure 2.1 Geometric interpretation of feasible region

We would like to note that every point from the feasible region can be the solution of the task, that mean that it will fit all the constraints. But it will not certainly be optimal by the criterion. Optimal solution is always on the intersection of the constraints (corner point). The solution is not necessarily the only one. There can be situations of infinite set of solutions when every point of the line segment is optimal.

For finding optimal point by geometric method we will additionally plot the line corresponding to the zero level of the objective function – revenue (see Figure 2.2). Point from feasible region which is the most distant from this line will be optimal or maximizing criterion of the task.

As we can see in the figure 2.2, optimal point is formed by intersection of first and second constraints. The co-ordinates of the point are found as a decision of the combined equations:

$$\begin{cases} 0.7x_1 + 0.2x_2 = 100 \\ 0.2x_1 + 0.6x_2 = 60 \end{cases}.$$

Thus, $x_1^0 = 126.3$, $x_2^0 = 57.9$. Consequently, maximal value of the optimality criterion (revenue) of the task is:

$$F^0 = 120 \cdot 126.3 + 200 \cdot 57.9 = 26736$$

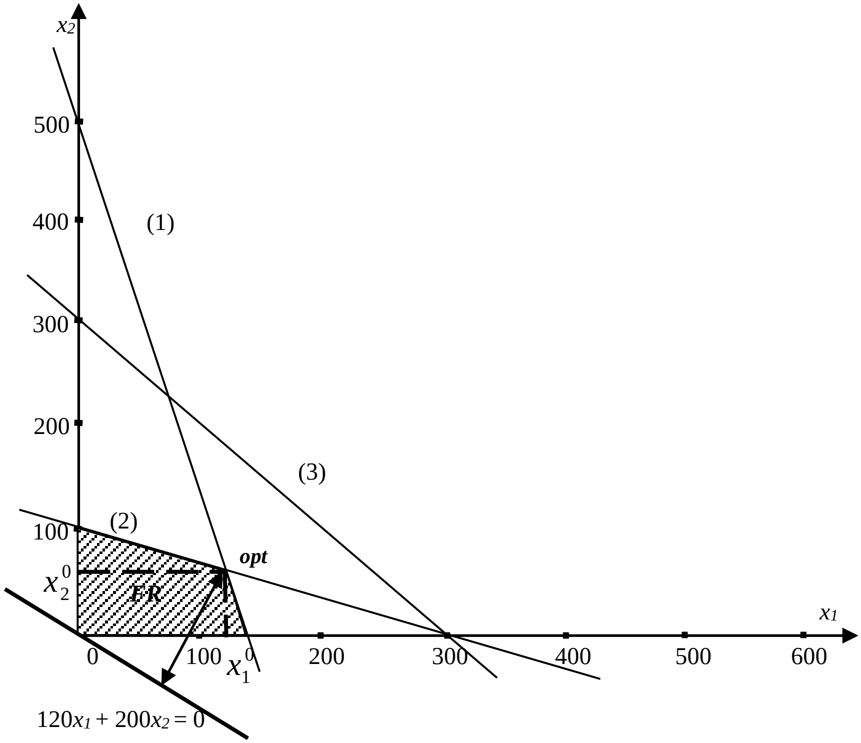


Figure 2.2 Optimal point determining using graphical method

Task.

An enterprise outputs two types of products. Market price of every production unit is 5 monetary units. Producing needs three types of resources. The stocks of the resources are 100, 100 and 150 units, respectively. Input normals of the first resource for producing one unit of the first и second types are 2 и 1 respectively, second resource – 1 and 3 respectively, third resource – 1 and 1. State and solute optimization problem of revenue maximization using geometrical method.

Theme 3. Analysis of management decisions sensitivity. The task of the resources substitution.

Introduction

In real life during management decision realization, in our case – optimal production program, there can happen disturbances of the system parameters, caused by external and internal factors. These disturbances lead to changing of optimal values of the problem's variables (production output) and objective function (profit). That is why we need to solute problem of evaluation of these disturbances' effect on management decision. Then we formulate activities basing on this problem. Decision maker is to make these activities in new conditions.

For solution of this problem we would like to use the mathematical apparatus of sensitivity theory.

Denote that we solve the linear programming problem:

$$\begin{cases} F = \sum_{j=1}^{n} c_j x_j \to \max \\ \sum_{j=1}^{n} a_{ij} \cdot x_j \le b_i \end{cases}$$

where c_j, a_{ij}, b_i – parameters of the model.

We would like to assume that the optimal task solution is found, that mean that optimal values of output characteristics – the variables x_i^o and the objective function F^o – are determined.

At that, for some types of the production $x_j^0 (j = 1, k) \ne 0$, for other $x_j^0 (j = k+1, n) = 0$. We would like to identify production, for which $x_i^o > 0$, as «advantageous»; production, for which $x_i^o = 0$ as «disadvantageous».

We take into account the characteristic of reserves of resources $y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j^o$, which shows the amount of resource of i – th type, residuary after optimal decision realization.

If $y_i = 0$, we will determine resource as "scarce". If $y_i > 0$, resource is "non-scarce".

We would like to evaluate effect of changing of $i-$th resource stock on output characteristics of the task. To do this, we take into account sensitivity coefficients $\alpha_j^i = \dfrac{\partial x_j}{\partial b_i}$, which show how will the value of $j-$th variable change if the stock of $i-$th resource will change by one unit. It is defended in sensitivity theory that these coefficients are other from zero for “scarce” resources and equal zero for «non-scarce».

Sensitivity coefficients $z_i = \dfrac{\partial F}{\partial b_i} = \sum_{j=1}^{n} c_j \dfrac{\partial x_j}{\partial b_i} = c_j \sum_{j=1}^{n} \alpha_j^i$, show how will the value of the objective function change if the stock of $i-$th resource will change by one unit.

Example.

We would like to conduct the sensitivity analysis of the decision for of the variation of system parameters at the following numerical illustration. We denote the profit maximization as an objective function and raw resources stock as constraints.

$$F = 2x_1 + 2x_2 \to \max$$

$$\begin{cases} x_1 + 2x_2 \le 100 \\ 2x_1 + x_2 \le 100 \\ 2x_1 + 2x_2 \le 300 \end{cases}\left|\begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix}\right.$$

The optimal task solution is: $x_1^o = \dfrac{100}{3}, x_2^o = \dfrac{100,}{3} F^o = \dfrac{400}{3}$. Since $x_1^o, x_2^o > 0$, consequently, both first and second types of production are “advantageous”. Let us determine reserves of the resources.

Since $y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j^o$, consequently, $y_1 = 0, y_2 = 0, y_3 = \dfrac{500}{3}$.

That is why we conclude that the first and second resources are “scarce”, third is “non-scarce”. Since sensitivity coefficients for “non-scarce” resource equal zero, consequently, $\alpha_1^3 = \alpha_2^3 = 0, z_3 = 0$. To determine remaining coefficients we will exclude third inequality from the system of constraints; we will change other two inequalities into strict equality and denote right sides of equations as b_1 and b_2. Thus,

$$\begin{cases} x_1 + 2x_2 = b_1 \\ 2x_1 + x_2 = b_2 \end{cases}$$

We differentiate this system with respect to b_1:

$$\begin{cases} \frac{\partial x_1}{\partial b_1} + 2\frac{\partial x_2}{\partial b_1} = 1 \\ 2\frac{\partial x_1}{\partial b_1} + \frac{\partial x_2}{\partial b_1} = 0 \end{cases}$$

Taking into account the expression $\alpha_j^i = \frac{\partial x_j}{\partial b_i}$,

$$\begin{cases} \alpha_1^1 + 2\alpha_2^1 = 1 \\ 2\alpha_1^1 + \alpha_2^1 = 0 \end{cases}$$

Thus $\alpha_1^1 = -\frac{1}{3}, \alpha_2^1 = \frac{2}{3}$.

Analogously, having differentiated this system with respect to b_2, we find $\alpha_1^2 = \frac{2}{3}, \alpha_2^2 = -\frac{1}{3}$.

Next we need to calculate the sensitivity coefficients of objective function to variations of “scarce” resources.

Since $z_i = \frac{\partial F}{\partial b_i} = \sum_{j=1}^{n} c_j \frac{\partial x_j}{\partial b_i} = \sum_{j=1}^{n} c_j \alpha_j^i$, consequently

$$z_1 = c_1\alpha_1^1 + c_2\alpha_2^1 = 2(-\frac{1}{3}) + 2(\frac{2}{3}) = \frac{2}{3}$$

$$z_2 = c_1\alpha_1^2 + c_2\alpha_2^2 = 2(\frac{2}{3}) + 2(-\frac{1}{3}) = \frac{2}{3}$$

We assume that the stock of the first resource is increased by 30 units. How will it effect on management decision or, to be exact, on optimal production program and profit? We will consider sensitivity coefficients α_1^1 and α_2^1.

Since $\alpha_1^1 = -\frac{1}{3}$, consequently, if the stock of the first resource increases by 30 units, optimal output of the first production will decrease by $30 \cdot \frac{1}{3} = 10$ units.

Since $\alpha_2^1 = \frac{2}{3}$, consequently, if the stock of the first resource increases by 30 units, optimal output of the second production will increase by $30 \cdot \frac{2}{3} = 20$ units.

Since the sensitivity coefficient $z_1 = \frac{2}{3}$, consequently, if the stock of the first resource increases by 30 units, maximal value of the profit will increase by $30 \cdot \frac{2}{3} = 20$ units.

In a similar way, we can make sensitivity analysis of the optimal decision to variation of the stocks of other resources.

Task.

Using task data (see Theme 2, Task) calculate the sensitivity coefficients α_j^i and z_i.

Theme 4. Analysis of management decisions stability.

Introduction

Management decisions stability is usually denoted in optimization problems as invariability of the system's reference basis. In the context of the task considered in coursework reference basis is a situation, in which nomenclature of advantageous and disadvantageous production and also of scarce and non-scarce resources remains the same.

We consider general-theoretical approach for the problem of the examination of the system's basis stability. We assume that there appeared disturbances with respect to certain scarce resource Δb_s. This variation leads to the changing of the value of variables x_j: thus, $\Delta x_j = \alpha_j^s \Delta b_s$.

If the optimal value of the variable x_j^0 is known, new value of this variable x_j^n is determined by the following expression:

$x_j^n = x_j^0 + \Delta x_j = x_j^0 + \alpha_j^s \Delta b_s$.

The condition of the basis invariability is the following: the amount of *j*–th production must be positive: $x_j^n > 0$. If it will equal zero, production will not be included into production program and will become "disadvantageous" instead of "advantageous". Mathematical formalization of this condition is the following:

$$x_j^0 + \alpha_j^s \Delta b_s > 0 \text{ or } \Delta b_s > -\frac{x_j^0}{\alpha_j^s}. \qquad (4.1)$$

Let us define concretely above-stated condition. Evaluation of the decision stability's dependence by x_j^0 on variations if b_s is mostly determined of the sign of α_j^s.

If $\alpha_j^s > 0$, then there is the following result from the condition (4.1):

$$\Delta b_s^{\max} = \infty$$

$$\Delta b_s^{\min} = -\frac{x_j^0}{\alpha_j^s}.$$

If $\alpha_j^s < 0$, then there is the following result from the condition (4.1):

$$\Delta b_s^{\min} = -\infty$$

$$\Delta b_s^{\max} = -\frac{x_j^0}{\alpha_j^s}.$$

Substantially this can be commented the following way. If $\alpha_j^s > 0$, then adding of resource *s* will lead to increase of output of the *j* –th production, consequently, in this case changing of the system basis will not happen. If $\alpha_j^s < 0$, then adding of resource *s* can lead to the changing of basis and amount of *j* –th production output can become equal zero, that mean that production will not be output.

Let us consider non-scarce resource b_i, for which the reserve $y_i \neq 0$ is calculated by the formula $y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j^o$. Let us assume that there appeared disturbances with respect to the stock of scarce resource Δb_s, and it will lead to the changing of the value of the variables Δx_j. In turn, variation of Δx_j will lead to variation of the stocks of scarce resources Δy_i ($\Delta b_s \to \Delta x_j \to \Delta y_i$). Consequently, there can appear the situation, that Δb_s will lead to the becoming of scarce resource stock equal zero ($\Delta y_i = 0$). That mean, that non-scarce resource becomes scarce, that is why the nomenclature of scarce and non-scarce resources has changed and there happened the changing of the system's basis. In this case, mathematical formalization of the system basis invariability condition will be the following:

$$y_i^n = y_i + \Delta y_i > 0 \qquad (4.2)$$

Example.

Let us conduct the analysis of the system basis stability for the following model:

$$F = 2x_1 + 2x_2 \to \max$$

$$\begin{cases} x_1 + 2x_2 \le 100 \\ 2x_1 + x_2 \le 100 \\ 2x_1 + 2x_2 \le 300 \end{cases} \begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix}$$

Optimal solution of the task is: $x_1^o = \frac{100}{3}, x_2^o = \frac{100,}{3} F^o = \frac{400}{3}$.

Reserves of the resources are: $y_1 = 0, y_2 = 0, y_3 = \frac{500}{3}$. Thus, the system's basis includes two "advantageous" productions, two scarce resources (1st and 2nd) and one non-scarce resource (3rd).

Let us determine size of changing of the scarce resources b_1 and b_2 stocks, which will not lead to the changing of the system's basis:

$$\Delta b_{1,1} = -\frac{x_1^0}{\alpha_1^1} = -(-)\frac{100 \cdot 3}{3} = 100\,,\ \Delta b_{1,2} = -\frac{x_2^0}{\alpha_2^1} = -\frac{100 \cdot 3}{3 \cdot 2} = -50\,.$$

Consequently, if stock of the first resource will increase by 100 units or decrease by 50, the changing of the system's basis will happen (see Figure 4.1). In first case, the first production will become "disadvantageous", in second case – second production.

Analogously, for the second resource:

$$\Delta b_{2,1} = -\frac{x_1^0}{\alpha_1^2} = -\frac{100 \cdot 3}{3 \cdot 2} = -50\,,\ \Delta b_{2,2} = -\frac{x_2^0}{\alpha_2^2} = -(-)\frac{100 \cdot 3}{3} = 100\,.$$

Consequently, if stock of the second resource will decrease by 50 units or increase by 100, the changing of the system's basis will happen (see Figure 4.1). In first case, the second production will become "disadvantageous", in second case – the first production.

In case of increase of the third (non-scarce) resource stock, changing of the system's basis will not happen (see Figure 4.1), in case of decreasing by a certain value Δb_3, resource becomes scarce. According to the expression (4.1), $\Delta b_3 = -y_3 = -\frac{500}{3}$.

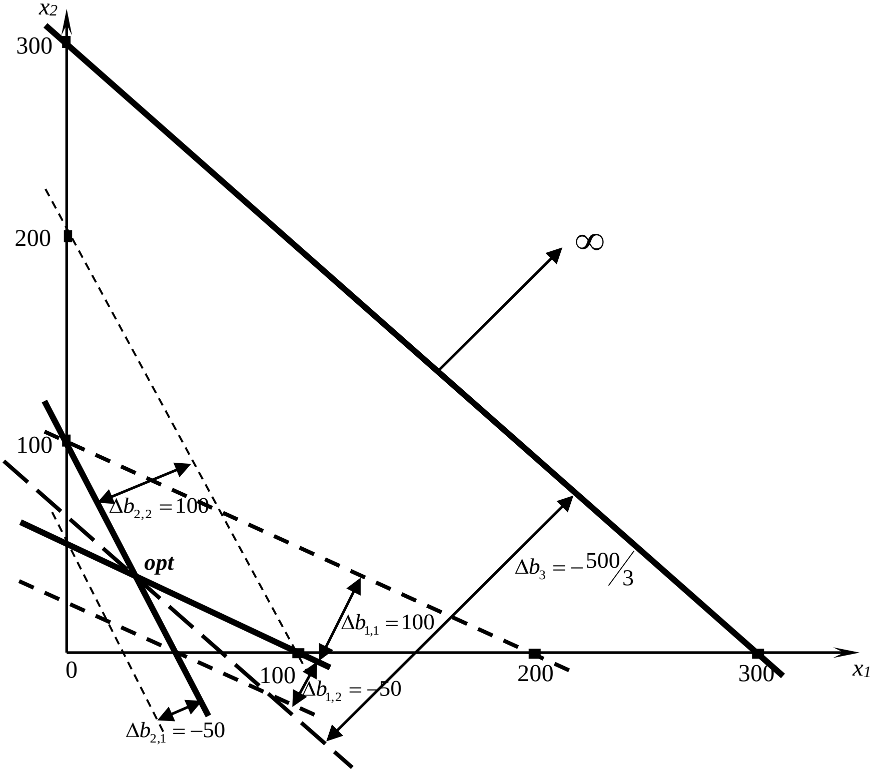


Figure 4.1 Graphical interpretation of the system's basis stability

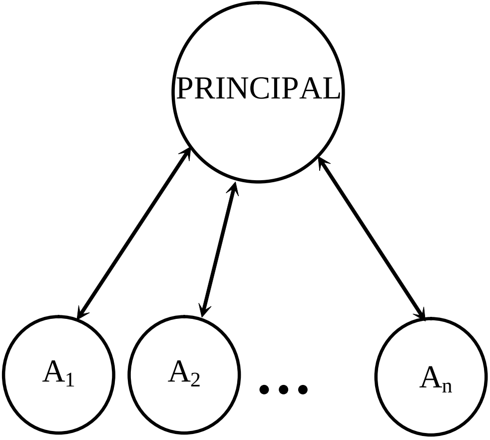
Task.

Using task data (see Theme 2.3, Task) calculate marginal variations of the resources stocks $\Delta b_1, \Delta b_2, \Delta b_3$, which lead to the changing of the system's basis.

Theme 5. Business game. Simulation modeling of the two-level organizational system functioning under uncertainty.

Introduction

Let us consider organizational system which is consist of Principal and *n* agents or business units (A).



Every agent is characterized by efficiency index r_i and cost function z_i. Indicator r_i characterizes working efficiency of $i-$th agent. This indicator is determined by several factors: automation, mechanization, using of resource-saving technologies, industrial engineering quality etc. We would like to denote that costs of $i-$th agent are described by the following model:

$$z_i = \frac{x_i^2}{2r_i}, \tag{5.1}$$

where $x_i -$ scope of work made by $i-$th agent.

The problem of Principal is to allocate resources between agents in scope of R and minimize total costs of the system. That means that he Principal is solving following task:

$$\begin{cases} F = \sum_{i=1}^{n} \frac{x_i^2}{2r_i} \to \min \\ \sum_{i=1}^{n} x_i = R \end{cases} \tag{5.2}$$

If Если real values of efficiency indexes r_i (*the case of certainty*) are known to Principal, the solution of the task (5.2) will be the proportional allocation law:

$$x_i^0 = \frac{r_i}{\sum_i r_i} R \tag{5.3}$$

In *case of uncertainty*, which mean that the Principal does not exactly know values of r_i, but the size of these indicators changing is known ($d_i \le r_i \le D_i$), the problem is to be solved using data forming method. That mean that every agent gives the Principal the assessment of its own efficiency index «s_i». This assessment should fit the criterion: $d_i \le s_i \le D_i$. Thus the following allocation law will be optimal for principal:

$$x_i^0 = \frac{s_i}{\sum_i s_i} R \tag{5.4}$$

We would like to analyze the strategy of agents' behavior under uncertainty. We denote that their objective function is profit maximization:

$$f_i = Px_i - \frac{x_i^2}{2r_i} \to \max \tag{5.5}$$

where P– price of unit of work.

Analysis of the function (5.5) extremum shows the decision which is optimal for agent:

$$x_i^* = P \cdot r_i \tag{5.6}$$

The expression (5.6) allows to find production volume that maximizes agent's profit and,consequently, agent will try to get from Pricipal exactly this amount of work. The agents' strategy is a choice of assessment value s_i. In this task we can determine situations of the Nash Equilibrium. ***Nash Eqilibrium*** *is a stable state of a system involving the interaction of different participants, in which no participant can gain by a* *unilateral change of strategy if the strategies of the others remain unchanged.*

Obviously, the total amount that all agents “want” to get is determined by the following expression:

$$V = \sum_{i=1}^{n} x_i^* = P\sum_{i=1}^{n} r_i \qquad (5.7)$$

Depending on relation of *R* and *V* we can define the following situations of equilibrium:

1) $R < V$: agents “want” to get more amount of work that the Principal can supply. Consequently, according to the expression (5.4) agents will try to overestimate their assessments s_i. The Nash equilibrium situation will be: $s_i^{eq} = D$.

2) $R > V$: the reverse situation for the first. The Nash equilibrium situation will be: $s_i^{eq} = d$.

3) $R = V$: for maximization of their objective functions value the agents will give real assessments of r_i. Consequently, Nash equilibrium will be: $s_i^{eq} = r_i$.

Task.

According to the instructor’s directions, you are to realize a business game which allows to model the functioning of organizational system under uncertainty.

Theme 6. Multicriteriality in controlling.

Introduction

In real practice of the organization controlling there are usual situations, when the object condition and efficiency of functioning can be characterized by different indicators (indexes), each of them determines particular properties of the object and achievement of targets in different aspects. Besides, these indicators have diverse substantial and economical sense, different dimensionality; their absolute values can heavily differ. That is why there emerges a problem of "compression" of the initial indexes to an integral criterion that will be the quantitative measure of the system functioning efficiency.

The first issue of the problem is normalization of the initial indexes. We can use different methods:

$$x_i^* = \frac{x_i}{\pi_i}$$

$$x_i^* = \frac{x^i}{N^i} \qquad (6.1)$$

$$x_i^* = \frac{x^i}{x^i(-)}$$

where x_i – real value of the i – th indicator, π_i – planned value of the i – th indicator, N^i – normative value of i – th indicator, $x_i(-)$ – value of i – th indicator in previous period.

Taking into account the structure of models (6.1), we can see that normalized indicator values x_i^* are nondimensional and able to vary in size of «1».

Integral criterion can be formed the following way:

$$F = \sum \beta_i \cdot x_i^* \qquad (6.2)$$

where β_i – coefficient of the relative significance of i – th object of evaluation.

Example.

The work of an enterprise is described by the following indicators:

– profit (millions rubles) – x_1

– production profitability (%) – x_2

– average workers' salary (hundreds rubles per month) – x_3.

The values of the mentioned indicators for the previous period are known: $x_1(-)=120$, $x_2(-)=18$, $x_3(-)=8{,}5$.

The normalized indicators will be: $x_1^* = \dfrac{x_1}{120}, x_2^* = \dfrac{x_2}{18}, x_3^* = \dfrac{x_3}{8{,}5}$

.

The coefficients of the relative significance of every indicator are: $\beta_1 = 0.4$, $\beta_2 = 0.2$, $\beta_3 = 0.4$.

The indispensable condition is $\sum\limits_i \beta_i = 1$.

We consider two enterprises. The problem is to evaluate which of them works more effective in respect of criterion *F*.

Given data for the calculation

	$x_1(-)$	$x_2(-)$	$x_3(-)$	x_1	x_2	x_3	β_1	β_2	β_3
1st enterprise	120	18	8,5	140	17	7	0,4	0,2	0.4
2nd enterprise	120	18	8,5	110	18	8	0,4	0,2	0,4

Let us calculate the value of integral criterion for both enterprises.

$$F_1 = \frac{140}{120}\cdot 0{,}4 + \frac{17}{18}\cdot 0{,}2 + \frac{7}{8{,}5}\cdot 0{,}4 = 0{,}98$$

$$F_2 = \frac{110}{120}\cdot 0{,}4 + \frac{18}{18}\cdot 0{,}2 + \frac{8}{8{,}5}\cdot 0{,}4 = 0{,}94$$

Conclusion: in respect of criterion *F* the first enterprise works more effective.

Task.

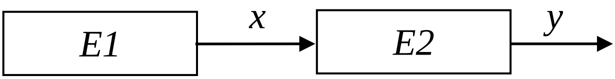
The world championship in speed skating (men) is going. There are four distances (500 m, 1500 m, 5000 m, 10000 m). There take part «*n*» sportsmen. Every $i-$th sportsmen on the $j-$th distance shows the time t_{ij}. How can we give title of the absolute world champion, i.e. sportsmen who is the best in respect of aggregate of all results?

Theme 7. Coordinated controlling mechanisms in horizontally-organized systems

Introduction

The practice of the market economy often requires organization of the “horizontal” economical interaction. That means that there is no certain principal (center) that undertakes functions of “metaplayer” and determines game directive. Subjects of the interaction are to find interconsistent compromise of interaction.

We decide a model of the system that consists of two productive elements *E1* and *E2*.



The first element produces semimanufactures in amount of x and sells it to the second, that, in turn, produces commodity output in amount of y and sells it at the market price of P_2. That is why the question appears: at what price P_1 should the semimanufactures be sold?

We consider that *E1* has costs determinated by function $z_1 = a_1 \cdot x$ (a_1 – input normals). The costs of *E2* are described by the function $z_2 = a_2 \cdot y + P_1 \cdot x$ (a_2 – input normals without accounting of buying semis). We assume that the objective functions of *E1* and *E2* represent their profit (“gain” – $G_{1,2}$). Obviously, feasible region with respect of P_1 is determined by following constraints:

$$G_1 = P_1 x - a_1 x \geq 0$$

$$G_2 = P_2 y - a_2 y - P_1 x \geq 0$$

Consequently,

$$\frac{P_2 y + a_2 y}{x} \geq P_1 \geq a_1 .$$

Task 1.

Using given data $P_2 = 250$, $x = 50$, $y = 25$, $a_1 = 2$, $a_2 = 1$, find P_1, which fit the following criteria:

– equal profit principal;
– equal profitability principal;
– principal of the normative profitability allocation.

Task 2.

You are to propose other pricing principals.

Theme 8. Incentives in organizational systems (in healthcare).

Introduction

One of the basic instruments of the stimulation of organization elements activity is the financial incentive for the labor (salary, bonus). We would like to consider basic provisions connected with remuneration of labor in medical institutions. We consider institutions that are financed from the state budget. But the amount of means is not enough for normal functioning of the institution and for adequate remuneration of labor. As a result in past years being legalized, contract forms of medical services started to develop. That is why the problem of remuneration of labor for the contact medical services became important.

The structure of the income forming and allocation can be represented by the following scheme (Figure 8.1).

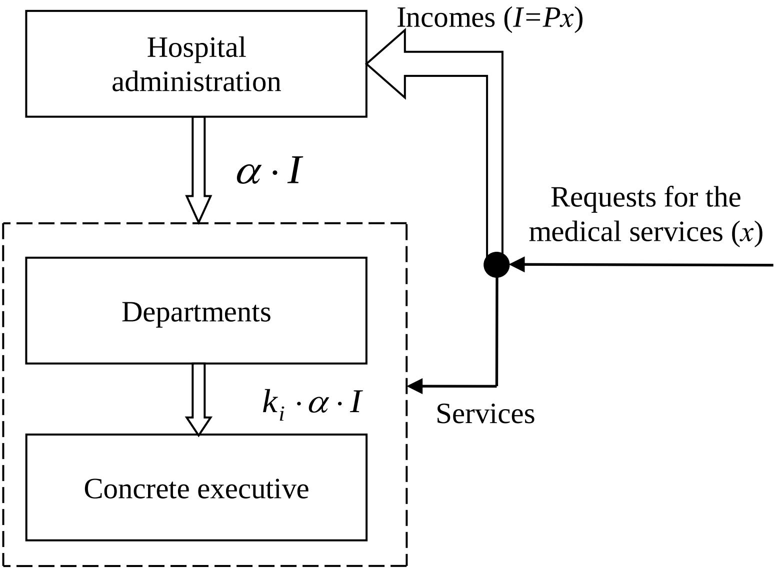


Figure 8.1 Income allocation scheme

In the figure above x – quantity of requests for medical services; P – price of the service unit; I – income; α – standard of the remuneration of labour fund (RLF) forming ($0 \le \alpha \le 1$); k_i – efficiency coefficient (participation in labor) of i – th worker/executive; y_i – amount t of the executed works of i – th executive.

There are two problems of developing incentive scheme in considered case. The first is rule and order of forming k_i coefficient. The second is selecting concrete value of α.

We offer to realize the algorithm of forming k_i by typical method of labor input ratio:

$$k_i = \frac{y_i}{\sum y_i} \tag{8.1}$$

Selection of the concrete value of α by the hospital administration depends on how is the conflict of interests between administration and executives arranged. The interest of executives in maximization of α, that determines remuneration of labour fund. The interest of administration is minimization of α and giving more finances for solving general-system problems. The trading (or searching for compromise) with respect of α should be made by analysis of the executive's behavior. We need to build a model of executive's decision-making on y_i selection. That mean that we are to understand essence of the executive's behavior motivation, to forecast their activity using this understanding, and, consequently, to forecast achieved results in case of our incentives scheme.

We assume that objective function of principal's (hospital administration) is the following:

$$F = (1-\alpha)\cdot P \cdot \sum y_i \tag{8.2}$$

Executive's objective function is the following:

$$f_i(y_i) = f_i^*(y_i) - c_i(y_i) \tag{8.3}$$

where $f_i^*(y_i) -$ earnings of the $i-$th executive, $c_i(y_i) -$ price equivalent of the executive's costs which are connected with the result y_i achievement.

Earnings of the $i-$th executive will be determined by the following principle:

$$f_i^*(y_i) = \frac{k_i}{\sum k_i}\alpha P \sum y_i = \frac{y_i}{\sum y_i}\alpha P \sum y_i = \alpha y_i P \tag{8.4}$$

The function of the costs price equivalent of $i-$th executive can be described the following way:

$$c_i(y_i) = \omega_0 y_i + \omega_1 y_i^2 \tag{8.5}$$

Thus, objective function of the executive will be the following:

$$f_i(y_i) = \alpha Py - \omega_0 y_i - \omega_1 y_i^2 \qquad (8.6)$$

Task.

Given data: $P = 400$, $\omega_0 = 25$, $\omega_1 = 0{,}17$, $\alpha = 0{,}2$.

1. Determine the optimal strategy of the $i -$ th executive (y_i^0).

2. Plot the graph $y_i^0 = y_i^0(\alpha)$.

Solve the problem of choosing α that optimize function $F(\alpha)$ with allowance of constraint, that executive elements will follow their optimal strategies with respect to criterion $f_i(y_i)$.

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