



АВТОМАТИЗИРОВАННЫЕ СИСТЕМЫ НАУЧНЫХ ИССЛЕДОВАНИЙ

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MEASUREMENT UNCERTAINTY OF INDIRECT MEASUREMENTS EVALUATION BY MONTE CARLO METHOD

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The measurement results make sense if a quantitative assessment of their accuracy is indicated. Relatively recently, a new quantitative assessment of measurement accuracy, namely, measurement uncertainty, was standardized [1]–[3]. The methods for uncertainty evaluation are given in [4]. The Monte Carlo method (MCM) performs random sampling from probability distribution of the input quantities, and it provides a probability density function for the output quantity as the final result, from which the coverage interval can be determined.

Monte Carlo method was devised as an experimental probabilistic method to solve difficult deterministic problems since computers can easily simulate a large number or experimental trials that have random outcomes.

1. Algorithm of Monte Carlo method for uncertainty estimation. This method basically consists in randomly generate a number M of Monte Carlo trials in where the distribution function of the value of the output quantity Y will be numerically approximated. Then a sample vector of the input quantities can be drawn repeatedly using random number generators. For each input sample vector the corresponding value of the output quantity is calculated by measurement model. The set of the M output sample vectors yields an empirical distribution which can be used to approximate the correct random distribution of the output quantity. It is recommended to use $M \geq 10^6$ to estimate a 95% coverage interval for the output quantity [5].

The following single steps of the Monte Carlo algorithm for uncertainty estimation, shown in Fig. 1 form the formal procedure described in [5], [6]:

- 1) Select the number M of Monte Carlo trials to be made.
- 2) Generate a set of N input parameters $\{x_1, \dots, x_N\}$, which are random variables distributed according to a probability density function assigned to each input parameter. This process should be repeated M times for every input quantity.
- 3) Calculate the corresponding value of quantity Z under measurement using the following model:

$$z_j = f(x_{1j}, \dots, x_{Nj}) \text{ for } j = 1, \dots, M. \quad (1)$$

From this sample it is possible to estimate the probability density function of z .



4) Calculate the mean and the standard deviation from output vector $\{z_1, \dots, z_M\}$ as the measurement result z for Z and its associated standard uncertainty $u(z)$.

5) Sort the output vector in ascending order and determine a coverage interval $[z_L, z_H]$ at coverage probability p [7]:

$$z_L = \text{round}((M+1)\gamma) \quad \text{and} \quad z_H = \text{round}((M+1)(1-\gamma)), \quad (2)$$

where γ is the significance level ($\gamma=0.025$ for 95% coverage probability) and the function $\text{round}(\beta)$ is used to represent the nearest integer to β .

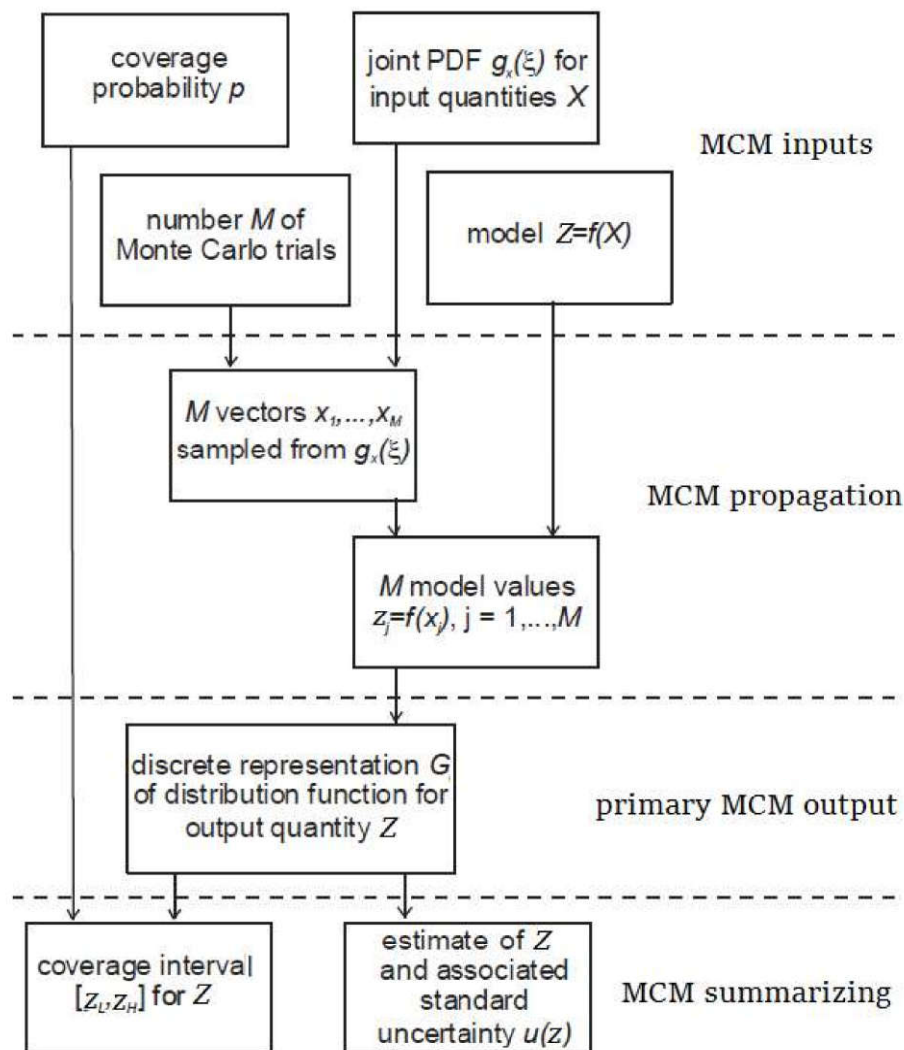


Fig. 1. Schematic flow graph of uncertainty evaluation using MCM [5]

Two examples for the illustration of uncertainty evaluation demonstrate the use of Monte Carlo method. There are software applications that are specifically developed for calculating measurement uncertainties, some of which use the MCM [8]. But it is possible to implement the MCM in general purpose engineering calculation media such as MATLAB, with the aid of which all the following computations were made.

Resistance measurement. In the first example, a resistance measurement using voltmeter and amperometer method (Figure 2) for small resistances (the value of the resistance under test R_x should be much lesser than R_v), in which voltage and current values recorded by a digital multimeters is considered. This is an indirect



measurement where the simple model $R_x = U/I$ is implemented, where R_x is the electrical resistance of the resistor under test, U is the voltage drop on the resistor and I is the current measured by amperometer. The measurement was performed under reference conditions for the instruments. The input resistance of the voltmeter $R_v = 10 \text{ M}\Omega$ and its value is considered in the calculation. The more accurate model for resistance indirect measurement is the following:

$$R_x = U / \left(I - \frac{U}{R_v} \right) \quad (3)$$

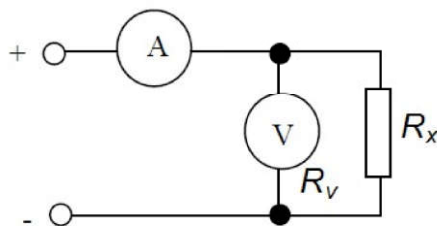


Figure 2. The schema for indirect measurement of an electrical resistance

Table 1 provides the probability distribution parameters assigned to the different input quantities. None of the input quantities are considered to be correlated to any significant extent. From this data, it is possible to use MCM to obtain an estimate \tilde{R}_x , its related standard uncertainty $u(R)$ and the 95% coverage interval.

Table 1. Assigned input quantities and probability distribution for resistance measurement

| Quantity X_i | Estimate x_i | Probability distribution | Uncertainty contribution $u_i(z)$ |
|----------------|----------------|--------------------------|-----------------------------------|
| U | 2.467 V | Uniform | 4.581 mV |
| I | 117.624 mA | Uniform | 68.596 μ A |

The result of measurement for the $20 \text{ }\Omega$ resistance is $20.973 \text{ }\Omega$. The measurement is repeated 150 times without detecting any scatter in the observations. Thus, uncertainty due to limited repeatability does not give a contribution.

The results of calculation stage that were obtained using 1100 trials are as follows:

- Standard uncertainty $u(R) = 0.3585 \text{ }\Omega$.
- Expanded uncertainty $U(R) = 0.717 \text{ }\Omega$.
- 95% confidence interval = $[20.902, 21.0454] \text{ }\Omega$.

The MCM approach gives the same value of R_x as the conventional standard method described in the Guide to the Expression of Uncertainty in Measurement (GUM) [4] though the coverage interval deduced by MCM turned out to be approximately 14% narrower than that of the GUM method. This difference is due to the Gaussian approximation made in the GUM method instead of the uniform distribution assumed by the MCM in this situation.

DC power measurement. In the second example, an indirect DC power measurement using voltage and current values recorded by digital multimeters is



considered. For DC voltages and currents, power is simply the voltage across a pure resistance multiplied by the current through this element: $P = UI$.

Connection of the instruments and the resistor for DC power measurement is the same as the connection for the resistance measurement (Figure 2). If we consider the input resistance of the voltmeter R_v , the model will have the form:

$$P = UI - \frac{U^2}{R_v}. \quad (4)$$

None of the input quantities are considered to be correlated to any significant extent.

The digital multimeters were used for this measurement. The input resistance of the voltmeter is $R_v = 10 \text{ M}\Omega$. Table 2 provides the probability distribution parameters assigned to the different input quantities. The result of DC power measurement is 2.160 W.

Table 2. Assigned input quantities and probability distribution for DC power measurement

| Quantity X_i | Estimate x_i | Probability distribution | Uncertainty contribution $u_i(z)$ |
|----------------|----------------|--------------------------|-----------------------------------|
| U | 14.75 V | Uniform | 0.08 V |
| I | 146.468 mA | Uniform | 0.29 mA |

The results of calculation stage that were obtained using 1100 trials are the following:

- Standard uncertainty $u(R) = 0.0111 \text{ W}$.
- Expanded uncertainty $U(R) = 0.0222 \text{ W}$.
- 95% confidence interval = $[2.1380, 2.1824] \Omega$.

In this example the distribution of the output quantity is trapezoidal. Difference in the 95% coverage interval established by MCM and GUM is minimal.

Conclusions. Monte Carlo method is practical numerical method for evaluating uncertainty of measurement in practice. Evaluation of measurement uncertainty of indirect measurements by Monte Carlo method and its relation to the GUM has been described. The implementation in MATLAB of the MCM has been illustrated by two application examples.

Hence, the described MCM is well suited for uncertainty calculation and should be applied in cases where the assumptions defined in the standard method of the GUM cannot be successfully applied.

References

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ПРИМЕНЕНИЕ КЕПСТРАЛЬНОГО АНАЛИЗА И ДЕКОМПОЗИЦИИ НА ЭМПИРИЧЕСКИЕ МОДЫ В ЗАДАЧАХ ОБНАРУЖЕНИЯ И ОЦЕНКИ ПСИХОЭМОЦИОНАЛЬНЫХ РАССТРОЙСТВ ЧЕЛОВЕКА ПО РЕЧИ

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В основе обнаружения и оценки психоэмоциональных расстройств человека по речи лежит важное правило: патофизиологические механизмы развития психических расстройств строятся на принципах взаимодействия нервной и речеобразующей систем организма; расстройства нервной системы активируют каскад механизмов, влияющих на функционал органов речеобразующей системы. Из данного правила можно сделать вывод, что психоэмоциональные расстройства «зашифрованы» в речевых сигналах в виде информативных параметров [1].

Точность обнаружения и оценки психоэмоциональных расстройств напрямую зависит от качества предварительной обработки и последующего анализа информативных параметров речевых сигналов. Корректная обработка речи определяется точностью измерения информативных характеристик (амплитудно-временных, спектрально-частотных и кепстральных). Главной причиной больших погрешностей при измерении информативных