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APPLICATION RESPONSES WEIGHING OPERATION TO PROBLEMS OF DETERMINING THE MEASURING INSTRUMENTS DYNAMIC CHARACTERISTICS

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I. INTRODUCTION

In most data processing tasks are frequently used parametric spectral analysis techniques, which are based largely on the method of least squares:

$$XA + E = Y$$

or

$$\begin{vmatrix} x_{p-1} & x_{p-2} & \cdots & x_1 \\ x_p & x_{p-1} & \cdots & x_2 \\ \vdots & \vdots & \vdots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-p-1} \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{vmatrix} + \begin{vmatrix} e_p \\ e_{p+1} \\ \vdots \\ e_N \end{vmatrix} = \begin{vmatrix} x_p \\ x_{p+1} \\ \vdots \\ x_N \end{vmatrix}$$

and expectation errors

$$e\,\boldsymbol{M}(e) = 0$$

and dispersion

$$\mathbf{M}(e^2) = \sigma^2 \mathbf{I}_n .$$

Then, by the Gauss-Markov theorem, the least squares estimation method is

$$\mathbf{A} = \left(\mathbf{X}^T \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{Y} \tag{1}$$

the most effective (in the sense of least variance) estimate in the class of linear unbiased estimators.

Formulating and solving problems. But in response to the condition of constancy of the variance short record length for the entire length of the recording is not always feasible. The following method can be used to eliminate or reduce the influence of heterogeneity of error in determining the parameters.

In order to reduce the effect of noise and to improve the following methods of parametric spectral analysis of the stability of solutions are developed:

- method of weighted coefficients inversely proportional errors;
- method of weighted coefficients proportional to the instantaneous power of the signal;
- method of weighted coefficients proportional to the amount of units in a sliding window;
- method of weighted coefficients using the estimated or known characteristics of the type of measuring instrument;

- weighing method for low-frequency component;
- method with adaptive weighting.

At the core of these methods is the estimation of the model parameters change agents by solving the system of equations:

$$\begin{vmatrix} x_{p-1} & x_{p-2} & \cdots & x_1 \\ x_p & x_{p-1} & \cdots & x_2 \\ \vdots & \vdots & \vdots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-p-1} \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{vmatrix} + \begin{vmatrix} e_p \\ e_{p+1} \\ \vdots \\ e_N \end{vmatrix} = \begin{vmatrix} x_p \\ x_{p+1} \\ \vdots \\ x_N \end{vmatrix} \quad (2)$$

or in matrix form $\mathbf{XA}+\mathbf{E}=\mathbf{Y}$, with the proviso that the expectation errors $\boldsymbol{e}$ is zero $M(e)=0$, respectively, and the dispersion is $M(e^2)=\sigma^2 I_n$ constant over the entire length of the implementation. Under these conditions the method of least squares is used for the solution of a system (2).

The first stage uses a standard method of least squares and stored vector $\boldsymbol{A}$, an error vector is then computed:

$$\mathbf{E}=\left[e_p, e_{p+1}, e_{p+2}, ..., e_n\right]^T$$

,

$$\mathbf{E}=\mathbf{Y}-\mathbf{XA}.$$

It turns out

$$x_n = -a_1 x_{n-1} - a_2 x_{n-2} - a_3 x_{n-3} - ... - a_p x_{n-p} \quad (3)$$

Further dividing term by term (3), we obtain

$$\frac{x_n}{e_n} = -a_1 \frac{x_{n-1}}{e_n} - a_2 \frac{x_{n-2}}{e_n} - a_3 \frac{x_{n-3}}{e_n} - ... - a_p \frac{x_{n-p}}{e_n}$$

Applying again a standard method of least squares, we obtain the estimates $\boldsymbol{A}$ while minimizing the amount of

$$U(A) = \sum_{i=p}^{N} \left(\frac{1}{e_i} \left(x_n - \sum_{j=1}^{p} a_j x_{i-j} \right) \right)^2 .$$

The idea is as follows. When using a standard least squares method minimizes the amount of

$$U(A) = \sum_{i=p}^{N} \left(\left(x_n - \sum_{j=1}^{p} a_j x_{i-j} \right) \right)^2 ,$$

in which different terms give different statistical contribution due to various dispersions, which leads to inefficiency estimates the least squares method. Weighing each case by a factor $1/e_i$, it is possible to eliminate the heterogeneity, or rather, give greater weight to the observations with a small error.

The second method is based on the method of weighting coefficients in proportion to the instantaneous power signal.

Unlike the first method, this one-step method.

The essence of the method is to minimize the amount of:

$$U(A)=\sum_{i=p}^{N}\left(s_i\left(x_n-\sum_{j=1}^{p}a_j x_{i-j}\right)\right)^2,$$

where $\boldsymbol{s_i}$ is calculated by the following formula:

$$s_i=\left|x_i\right| / \sum_{j=p}^{N}\left|x_j\right|.$$

This weighting makes it possible to take into account the reports is proportional to the amplitude.

The responses from the sensors have a short length/duration, and mostly it is very difficult to ensure that at different $\boldsymbol{N}$ standard deviation were the same. Therefore, the use of such weighing makes it possible to enhance the data with large and small to ease data reporting values.

The third method is based, as the second, weighting coefficients on, but unlike this method, weighting factors are as follows:

$$s_i=\sum_{j=0}^{p-1}\left|x_{i-j}\right| / \sum_{j=p}^{N}\left|x_j\right|.$$

The fourth method is based on the fact that the response is modeled by the sum of damped sinusoids. The first part of the procedure - normal use of the Prony-method. After finding the parameters of damped sinusoid apply the least squares method again, the assessment is minimized by the weighted coefficients in proportion to the absolute low-frequency component, which makes it possible to take into account the most heavily initial reports and peak values.

The use of weighting operation reduce the impact of noise and, most importantly, reduce the model order.

Description of the developed proposals. Designed weighing methods are based on a three-step procedure. The first step is to find a standard assessment of the vector $\boldsymbol{A}$ of parameters of the mathematical model for the relations (1) with the modified Prony-method. In the second step it is calculated error vector $\mathbf{E}=\mathbf{Y}-\mathbf{XA}$, the results of which were weighed. In the third step, taking into account the results of the weighing is carried out again to find the vector $\boldsymbol{A}$ of parameters of the mathematical model. Application developed by weighting methods can improve the stability of finding a mathematical model parameters means estimates measurements to reduce its order, and reduces the influence of samples with low signal/noise ratio. The proposed methods of weighting applied not directly to the data readings, and the equations of normal systems, there is rationing of equations in the process of applying the method of least squares. This is essential when using the methods of the weighted coefficients gets proper selection response duration.

In applying the method of least squares on the Gauss-Markov theorem, evaluation of response options are most effective (in the sense of least variance) estimates in the class of linear unbiased estimates provided constant variance throughout the length of the data that responses with a short recording length is not always feasible. To eliminate or reduce the impact of data errors heterogeneity suggested a method of weighted coefficients inversely proportional errors.

Using the standard method of least squares for a short duration of response is based on the minimization of the discrepancies in the different terms which give different statistical contribution due to various dispersions, which leads to inefficiency estimates the least squares method. It is shown that, weighing each observation by a factor $1/e_i$ can be eliminated heterogeneity, giving greater weight to the observations with a small error.

The first stage uses a standard least squares method for computing estimates $\boldsymbol{A}$ of the systems of equations (1).

In the second stage error vector $\mathbf{E} = \left[e_p, e_{p+1}, e_{p+2}, ..., e_n \right]^T$ is calculated by the relations $\mathbf{E} = \mathbf{Y} - \mathbf{XA}$, on the basis of which carried out the valuation equation for the current error e_n, if it exceeds a predetermined threshold:

$$\frac{x_n}{e_n} = a_1 \frac{x_{n-1}}{e_n} + a_2 \frac{x_{n-2}}{e_n} + a_3 \frac{x_{n-3}}{e_n} + ... + a_p \frac{x_{n-p}}{e_n}.$$

Further, using the method of least squares estimation vector calculating refined by minimizing the amount of:

$$U_1(\mathbf{A}) = \sum_{i=p}^{N} \left(\frac{1}{e_i} \left(x_n - \sum_{j=1}^{p} a_j x_{i-j} \right) \right)^2.$$

Discussion of research results.

The proposed method is preferably used for responses to low signal/noise ratio in the presence of errors. This method allows to determine the dynamic characteristics of measuring instruments, not overstating the order of a mathematical model, while reducing the impact of noise and errors. The simulation results (Figure 1) show that the signal/noise ratio 5/1 and the unit slips mathematical model parameters are determined with an accuracy of 1%, and the response error of approximation (see Figure 1, c) is not more than 2-3%. The disadvantage of this method is its labor intensity associated with the need to perform a three-step procedure.

The development of this method are one-step methods of weighted coefficients. The method of the weighted coefficients proportional to the instantaneous amplitude of the signal, the essence of which is to minimize the amount of:

$$U_2(\mathbf{A}) = \sum_{i=p}^{N} \left(s_i \left(x_n - \sum_{j=1}^{p} a_j x_{i-j} \right) \right)^2,$$

where $\boldsymbol{s_i}$ is calculated by the following formula:

$$s_i = \left|x_i\right| / \sum_{j=p}^{N} \left|x_j\right|.$$

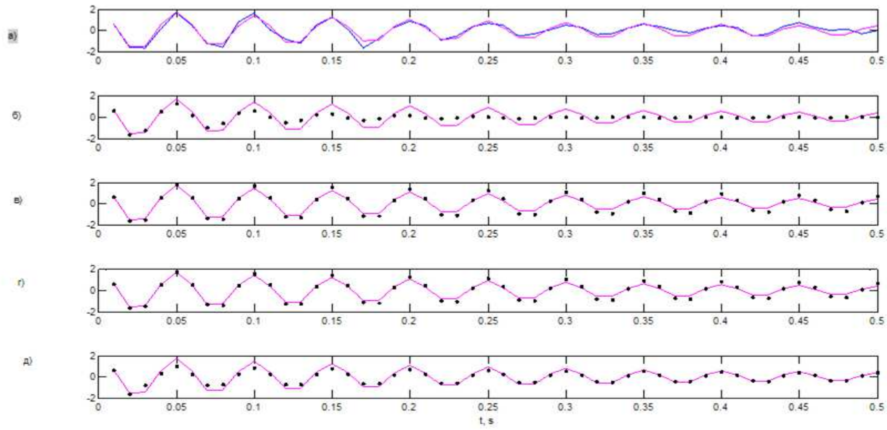

Figure 1. The results of the application of weighting methods: a – signal and noise signal; b – the results of processing with the use of modernized method Prony; c – use of the method of weighted coefficients; d – the application of the method of weighted coefficients proportional to the instantaneous amplitude of the signal; e – the use of a method that uses a coefficient proportional to the amount of modules in response sliding window

This weighting makes it possible to take into account in proportion to the amplitude of the samples (see Figure 1, d). The main advantage of this method is the ability to select the optimal response duration. At the same time, this method is more sensitive to errors. The simulation results confirm the ability of a method to weaken the influence of the "tail" of the response to determine the parameters of the model. The method allows to determine the parameters of up to 0.5-1%, and the response of the approximation error does not exceed 2%.

In the method of the weighted coefficients, which are proportional to the sum of the modules in a sliding window, weighting coefficients are normalized by the following formula:

$$s_i = \sum_{j=0}^{p-1} \left|x_{i-j}\right| / \sum_{j=p}^{N} \left|x_j\right|.$$

In this method, the response is submitted in the form of an autoregressive model of order ***p***, and weighting the selected equation involves only ***p*** previous samples, which is calculated based on the initial reference value. This approach using integrated assessments to reduce the model order, reduces the impact of near misses and the effect of the trend, present in the response, making computing more sustainable results. The simulation showed that this method for a signal/noise ratio of 5/1, allows you to define the parameters of a mathematical model to within 0.5-1.5% (see Figure 1, e.), and the response approximation error does not exceed 2%.

A method for weighing a low frequency component based on the Prony-method is applied, through which the search parameters damped sinusoids. Then, using the method of least squares estimates obtained by the weighted coefficients are minimized in proportion to the instantaneous values of the module low-frequency component that makes it possible to take into account the initial and peak values of samples.

The effect of weighting methods is shown in Figure 1, which shows that the application of the developed methods allows to obtain more accurate results than the Prony-method.

It is developed a method with adaptive weighting, which comprises applying the two-step procedure. In the first step use the standard method of least squares. Residues are calculated in the second step and all the samples for which a large residues are replaced with the values calculated using the autoregressive approximation and least squares procedure is repeated. The main advantage of the developed adaptive method is the stability of its solutions and the independence of the individual errors. In the simulation, the error in determining the parameters do not exceed 0.5-1%, and the determination of response error is 3%.

Thus, the use of the developed methods based on weighing, would greatly reduce the effects of blunders and response areas with high noise

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РАЗРАБОТКА ЭКСПЛУАТАЦИОННОЙ ДОКУМЕНТАЦИИ В ЕДИНОМ ИНФОРМАЦИОННОМ ПРОСТРАНСТВЕ ПРЕДПРИЯТИЯ

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Рассматриваются переход на интерактивную электронную эксплуатационную документацию на ракетно-космические изделия, разрабатываемые в едином информационном пространстве предприятия.

Жизненный цикл изделия, интерактивное электронное техническое руководство, информационные технологии, эксплуатационная документация