RECONSTRUCTION OF UNCONTROLLED ATTITUDE MOTION OF SMALL STELLITE AIST

V.I. Abrashkin[1], K.E. Voronov[2], A.V. Piyakov[2], Yu.Ya. Puzin[1], V.V. Sazonov[3], N.D. Semkin[2], A.S, Filippov[1], S.Yu. Chebukov[3]

[1]JSC "Space-Rocket Centre "Progress", Samara, Russia

[2]Samara State Aerospace University, Samara, Russia

[3]Keldysh Institute of Applied Mathematics RAS, Moscow, Russia

1. Satellite Aist (stork) was produced by Center CSDB-Progress and was intended for scientific experiments. Specialists of Samara State Aerospace University took part in making its airborne equipment. The satellite was launched 2013.04.21 by means of separation from spacecraft Bion M-1, which was in the almost circular orbit with the altitude 570 km and the inclination 64.9°. The main mode of the satellite flight is undirected. The satellite has the navigation system, which provides measuring its motion parameters and communication with ground controlling means. The real attitude motion of the satellite was reconstructed by processing onboard magnetic field measurements.

2. Motion equations. The satellite is assumed to be a rigid body. To write down the equations of its attitude motion and the relations, used in processing measurement data, we introduce the following four right-hand Cartesian coordinate systems.

The structural system $Oy_1y_2y_3$ is rigidly fixed to the satellite body. The point O is the satellite mass center. The system serves for interpretation of onboard magnetic measurements. The system $Ox_1x_2x_3$ is formed by the satellite principal central axes of inertia. Bellow, components of all vectors regards to this system unless specifically stated to the contrary.

The system $CY_1Y_2Y_3$ is closed to the second equatorial coordinate system of date. Its origin coincides with the Earth center of mass; the plane CY_1Y_2 coincides with the equatorial plane; the axis CY_3 is directed to North Pole. The axis CY_1 is directed approximately to the equinoctial point; the axis is turned from the plane of Greenwich meridian on the mean sidereal time against the Earth rotation. The system $CY_1Y_2Y_3$ is used in the model SGP4 [1], which serves for description the satellite orbital motion. The system is assumed to be inertial one.

The quasi inertial system $OZ_1Z_2Z_3$ serves for description and graphic representation of the satellite attitude motion. The axis OZ_2 is directed along the orbital angular momentum at every instant, the axis OZ_3 is parallel to the nodal line of the satellite osculating orbit. The absolute value of the angular rate of this system did not exceed 5° per day.

We denote the matrix of transition from the system $Ox_1x_2x_3$ to the structural system by $\| a_{ij} \|_{i,j=1}^{3}$, where $a_{ij} = \cos(Oy_i \wedge Ox_j)$. The matrix elements are expressed by functions of the angles γ, α, β, which are defined as follows. The system $Oy_1y_2y_3$ is transformed to the system $Ox_1x_2x_3$ by three sequential rotations: (1) through the angle α around the axis Oy_2, (2) through the angle β around the new axis Oy_3, (3) through the angle γ around the new axis Oy_1, which coincides with the axis Ox_1.

We use the normalized quaternion $\mathbf{Q} = (Q_0, Q_1, Q_2, Q_3)$, $Q_0^2 + Q_1^2 + Q_2^2 + Q_3^2 = 1$ to assign the attitude of the system $Ox_1x_2x_3$ with respect to the system $CY_1Y_2Y_3$. The quaternion transition formula is $(0, Y_1, Y_2, Y_3) = \mathbf{Q} \circ (0, x_1, x_2, x_3) \circ \mathbf{Q}^{-1}$. Here, the points O and C are assumed to be coincident. We assign the attitude of the system $Ox_1x_2x_3$ with respect to the system $OZ_1Z_2Z_3$ by the common Euler's angles ψ, θ, φ. The system $OZ_1Z_2Z_3$ is transformed to the system $Ox_1x_2x_3$ by three sequential rotations: (1) through the angle ψ around the axis OZ_3, (2) through the angle θ around the new axis OZ_1, (3) through the angle φ around the new axis OZ_3, which coincides with the axis Ox_3.

We use the model SGP4 to assign the satellite orbital motion. The input data for the model are NORAD two line elements [1]. The equations of satellite attitude motion consist of Euler's dynamic equations for the components ω_i of the angular rate and kinematical equations for components of the quaternion $\mathbf{Q}$. We take into account the gravitational torque and the restoring torque from the Earth magnetic field. The equations of attitude motion have the form

$$\begin{gathered}
\dot{\omega}_1 = \mu(\omega_2\omega_3 - \nu x_2 x_3) + p_2 H_3 - p_3 H_2, \\
\dot{\omega}_2 = \frac{1-\lambda}{1+\lambda\mu}(\omega_3\omega_1 - \nu x_3 x_1) + \frac{\lambda}{1+\lambda\mu}(p_3 H_1 - p_1 H_3), \\
\dot{\omega}_3 = -(1-\lambda+\lambda\mu)(\omega_1\omega_2 - \nu x_1 x_2) + \lambda(p_1 H_2 - p_2 H_1), \qquad (1) \\
2\dot{Q}_0 = -\sum_{i=1}^{3} Q_i\omega_i, \quad 2\dot{Q}_i = Q_0\omega_i + \sum_{j,k=1}^{3} e_{ijk} Q_j \omega_k, \\
\lambda = \frac{I_1}{I_2}, \quad \mu = \frac{I_2 - I_3}{I_1}, \quad \nu = \frac{3\mu_E}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}.
\end{gathered}$$

Here, x_i are the components of the vector $\overrightarrow{CO}$; H_i are the components of the Earth magnetic field strength at the point O; I_i are the moments of inertia of the satellite relative to the axes Ox_i; $I_1 p_i$ are the components of the spacecraft magnetic dipole moment; e_{ijk} is Levi-Civita's symbol ($e_{ijk} = 1$ when the indexes i, j, k form an even permutation of numbers 1, 2. and 3; $e_{ijk} = -1$ when the indexes form an odd permutation, $e_{ijk} = 0$ in other cases). The components H_i are specified by the models IGRF and SGP4, as well as the transfer equations from Greenwich system to the system $CY_1Y_2Y_3$ and next to the system $Ox_1x_2x_3$. We use 1000 s as a unit of time and 1000 km as a unit of length when numerical integrate equations (1). Then the units of the other quantities are following: $[\omega_i] = 10^{-3}\text{s}^{-1}$, $[H_i] = 10^{-5}\text{g}^{1/2}\text{cm}^{-1/2}\text{s}^{-1} = 10^{-5}\text{Oe}$, $[p_i] =$ $0.1 g^{-1/2}\text{cm}^{1/2}\text{s}^{-1}$. The units for p_i and H_i are given in the CGSM system.

The variables Q_i are not independent owing to the normalizing condition of the quaternion $\mathbf{Q}$. If $\mathbf{Q}$ satisfies this condition at initial time, then $\mathbf{Q}$ will satisfy it identically owing to properties of the kinematical equations in (1). So it is enough to normalize $\mathbf{Q}$ only at initial time.

The parameters λ, μ, p_i, γ, α, β are assumed to be constant in an interval of data processing, but their values are estimated by processing measurement data along with initial values of a satellite attitude motion, i.e. they are fitted parameters.

3. Data processing. There are two triaxial magnetometers onboard Aist. We denote them magnetometer 1 and magnetometer 2. Magnetic measurements were carried out in separate time segments, which lasted not more than 6 hr. Readings of the magnetometers were digitized for the same instants of time. The distances between neighbor instants were either 5 or 10 s. The measurable components of the magnetic strength are referred to the own magnetometer coordinate systems. The coordinate system of magnetometer 2 is used as structural one. Magnetometer 1 proved to be less precise than magnetometer 2 [2], and we didn't include its measurement in final processing.

The measurements of magnetometer 2, obtained during a measurement séance, form the set of numbers $t_n, h_i^{(n)}$ $(i = 1, 2, 3;\ n = 1, 2, \ldots, N)$. Here, $h_i^{(n)}$ is a result of measuring the component

h_i of the magnetic strength by magnetometer 2 in its own coordinate system at instant t_n. Following the least squares method, we consider a solution of system (1) to be a reconstruction of the real satellite motion in the interval $t_1 \le t \le t_N$ if it minimizes the functional

$$\Phi = \sum_{i=1}^{3}\left\{\sum_{n=1}^{N}[h_i^{(n)} - \hat{h}_i(t_n)]^2 - N\Delta_i^2\right\}, \quad \Delta_i = \frac{1}{N}\sum_{n=1}^{N}[h_i^{(n)} - \hat{h}_i(t_n)], \quad \hat{h}_i(t) = \sum_{j=1}^{3} a_{ij}H_j(t).$$

Here, Δ_i is the estimate of the unknown constant shift in measurements of the component h_i. The functional has to be minimized over initial conditions of a solution of equations (1) at the instant t_1 and the parameters p_i, λ, μ, γ, α, β. At that, the quaternion $\mathbf{Q}(t_1)$ must be normalized to unity.

4. The example of reconstruction of Aist attitude motion is presented in the plots below. The plots in the upper left part of the next page describe the time dependence of Euler's angles ψ, θ, φ, as well as the angle δ between the Earth magnetic field strength and the estimated magnetic dipole of the satellite. The upper right part of the page contains the plots of the angular rates ω_i. All these plots illustrate the motion of the system $Ox_1x_2x_3$ relative to the quasi inertial coordinate system $OZ_1Z_2Z_3$. The plots in the lower part of the next page illustrate the quality of approximation of measurement data in the framework of our mathematical model. Here, the left plots are the plots of the functions $\hat{h}_i(t)$ and broken lines whose links join in series the points $(t_n, h_i^{(n)} - \Delta_i)$, $n = 1, 2, \ldots, N$. Each pair of the broken line and the plot of its approximating function $\hat{h}_i(t)$ are depicted in the common coordinate system. The corresponding plot and broken lines are in a good agreement. Therefore to illustrate the approximation errors the lower right part of the page contains marks, which depict the points $(t_n, h_i^{(n)} - \hat{h}_i(t_n) - \Delta_i)$.

In this example $t_1 = 20:25:29$ UTC 2013.05.16, $N = 2098$, $t_N - t_1 = 214.4$ min, the standard deviation of errors in the measurement data $h_i^{(n)}$ equals 817γ, $\lambda = 1.226\,(0.00042)$, $\mu = 0.306\,(0.00045)$, $\gamma = 0.024\,(0.0015)$, $\alpha = 0.160\,(0.0017)$, $\beta = -0.191\,(0.0020)$. Hear, the angles are presented in radians; parameters p_i are given in units used at integration of equations (1); standard deviations of estimated parameters are given in brackets. One can find more detail reconstruction results on the Aist attitude motion in [2].

This work was supported by RFBR grant 14-01-00423.

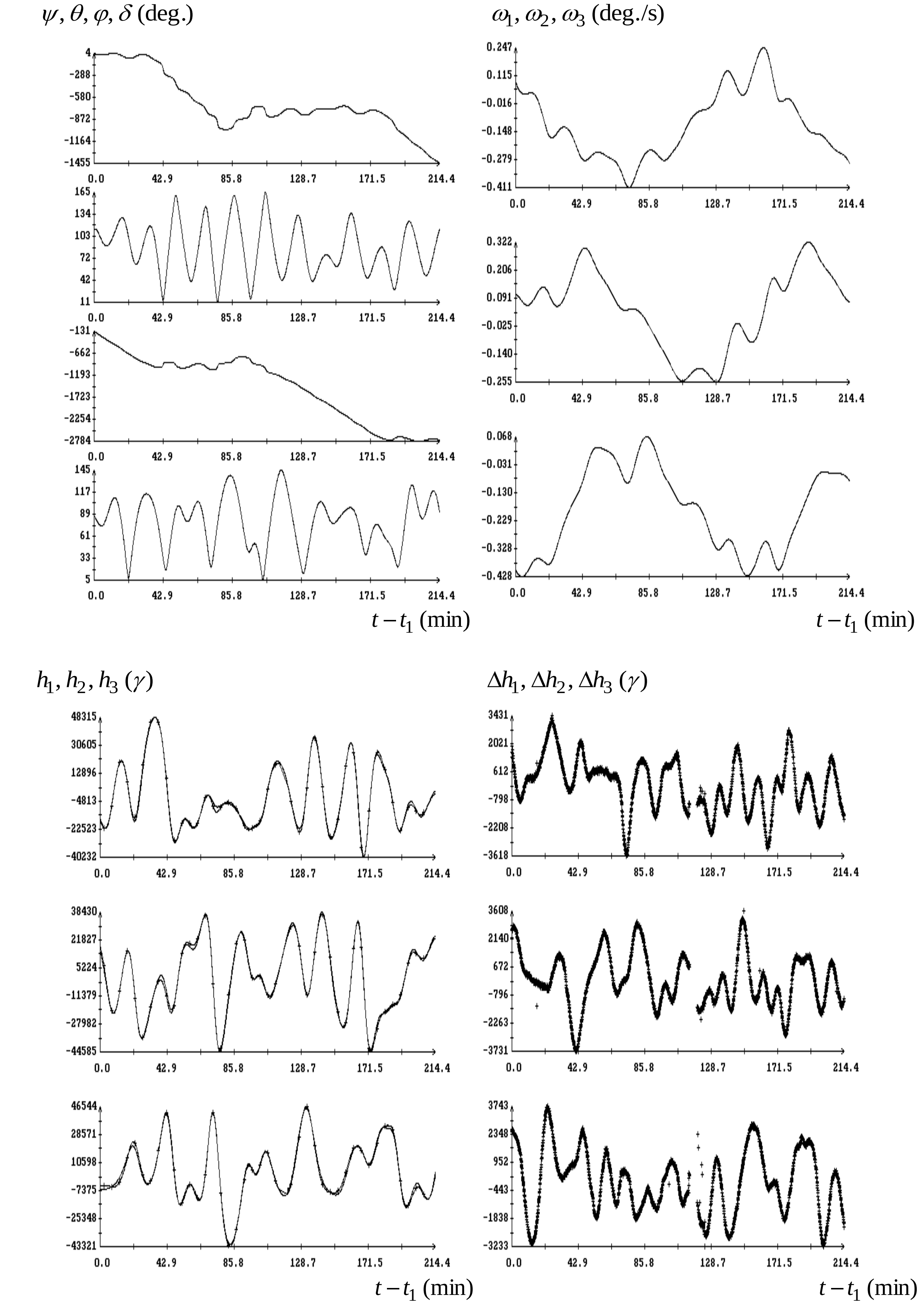

Fig. 1Example of reconstruction of Aist attitude motion and approximation of the measurement data.

References

1. Hoots F.R., Roehrich R.L. Models for propagation of NORAD element sets. Spacetrack report No. 3. 1988.

2. Abrashkin V.I., Voronov K.E., Piyakov I.V., Puzin Yu.Ya., Sazonov V.V., Semkin, N.D., Filippov A.S., Chebukov S.Yu. Determination of the spacecraft Aist attitude motion on measurements of the Earth magnetic field. Preprints of Keldysh Institute of applied mathematics, 2014, No. 17.