УДК 519.7

DISTRIBUTION OF CYLINDRICAL WAVES IN THE ELASTIC MEDIA

Сейтим Н.[1], Старинова О. Л.[2], Алипова Б. Н.[1]

[1]International Information Technology University, Almaty, Kazakhstan

[2]Самарский национальный исследовательский университет
имени академика С. П. Королёва

Let in an infinite space there is a cylindrical cavity of radius a. We choose a coordinate system so that the z axis coincides with the axis of the cylinder. Let the harmonic load $p(r,t) = p_0 e^{i\omega t}$ act on the boundary $r = a$ of the elastic region $r \geq a$, where t – time.

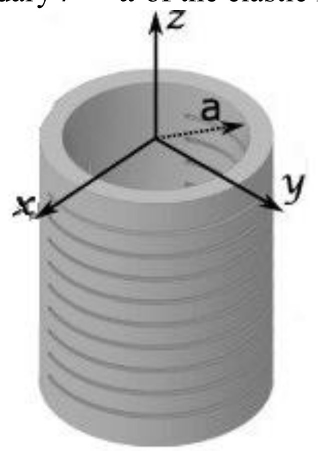

Fig. 1. Cylindrical cavity

Cylindrical waves caused by this load are described by equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{c_1^2}\partial_t^2\right)\ (r,t) = 0 \qquad r = (x_1^2 + x_2^2)^{1/2} \tag{1}$$

To the equation (1) we add the boundary condition

$$\sigma_{rr} = -p_0 e^{i\omega t}. \tag{2}$$

A solution of equation (1), satisfying the boundary condition (2) and the radiation condition at infinity, is the

$$\Phi(r,t) = AH_0^{(1)}(kr)e^{i\omega t},\ \ k = \frac{\omega}{c_1}, \tag{3}$$

Where $H_0^{(1)}$ is the Hankel function of first kind or Bessel functions of the third kind and equals to
$$H_0^{(1)}(kr) = J_\alpha(kr) + iY_\alpha(kr),$$
here J_α, Y_α are Bessel function of first and second kind, respectively.

Noticing that

$$\sigma_{rr} = 2\mu\frac{\partial^2\Phi}{\partial r^2} + \frac{\lambda}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}, \tag{4}$$

We obtain from the boundary condition (2):

$$A = -\frac{p_0}{2\mu\left\{\frac{\partial^2}{\partial r^2}\left[H_0^{(1)}(kr)\right]\right\}_{r=a} - \lambda k^2 H_0^{(1)}(ka)}. \tag{5}$$

So the function (r,t) is thus defined. And we can calculate the displacements and stresses with the help of it:

$$u_r = \frac{\partial\Phi}{\partial r}, \tag{6}$$

$$\sigma_{rr} = 2\mu\frac{\partial^2\Phi}{\partial r^2} + \frac{\lambda}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}, \tag{7}$$

$$\sigma_{\theta\theta} = \frac{2\mu}{r}\frac{\partial\Phi}{\partial r} + \frac{\lambda}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}, \tag{8}$$

$$\sigma_{zz} = \lambda \nabla^2 \Phi. \qquad (9)$$

we get, that

$$u_r = \frac{\partial \Phi}{\partial r} = -kAe^{i\omega t}(J_1(kr) + iY_1(kr)), \qquad (10)$$

$$\sigma_{rr} = 2\mu \frac{\partial^2 \Phi}{\partial r^2} + \frac{\lambda}{c_1^2} \frac{\partial^2 \Phi}{\partial t^2} =$$

$$-2\mu k^2 \left(\frac{(J_0(kr) - J_2(kr)}{2} + i \frac{(Y_0(kr) - Y_2(kr)}{2} \right) - \frac{\lambda}{c_1^2} \left(\omega^2 A H_0^{(1)}(kr) e^{i\omega t} \right), \qquad (11)$$

$$\sigma_{\theta\theta} = \frac{2\mu}{r} \frac{\partial \Phi}{\partial r} + \frac{\lambda}{c_1^2} \frac{\partial^2 \Phi}{\partial t^2} = -kAe^{i\omega t}(J_1(kr) + iY_1(kr)) - \frac{\lambda}{c_1^2} \left(\omega^2 A H_0^{(1)}(kr) e^{i\omega t} \right) \qquad (12)$$

Then we realize all of these formulas of displacements and stresses in the MatLab.

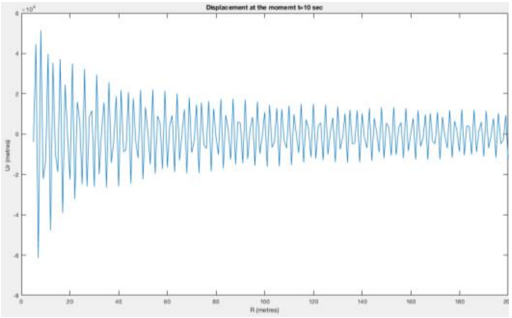

Fig. 2. Displacement of wave at the moment t=10

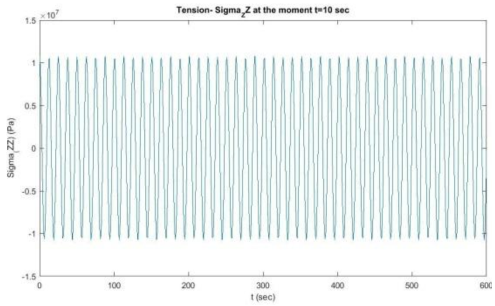

Fig. 3. Tension σ_{zz} at the moment t = 10 sec

On the this graph we can see how changed tension on the each point along the radius.

In conclusion, we calculate tensor displacement and stress for several medium, such as sandstone, granite and steel. To solve this problem, the numerical calculations were made by using Matlab environment.

References

1. Alexeeva L. A., Kupesova (Alipova) B. N, Method of generalized functions for boundary problems of coupled thermoelastodynamics // Journal of Applied Mathematics and Mechanics (of Russian Academy of Sciences), V.65, N 2, 2001, pp. 334-345

2. Alipova B. N. Method of Boundary integral equations (BIEM) and generalized solutions of transient problems of thermoelastodynamics. ICNPAA 2012, 9-15 July, AIP Conference Proceedings, 1493, 39(2012); doi: 10.1063/1.4765466, American Institute of Physics, Vien, Austria, http://dx.doi.org/10.1063/1.4765466, pp.39-46 http://www.scimagojr.com/journalsearch.php?q=26916&tip=sid&clean=0

3. Aytaliev Sh. M., L. A. Alexeyeva, Sh. A. Dildabayev, N. B. Zhanbyrbaev. Method of boundary integral equations.

4. Dortman N. B. (1984). Physical properties of rocks and minerals

5. Kupradze V. D, Gegelia T. G., Bashelishvili M. O., Burchuladze T. V. (1976). Three-dimensional problems Mathematical theory of elasticity and thermoelasticity. - M.: "Science".

6. Novacki V. (1975). Theory of elasticity. "MIR". Moscow.