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COMPUTER MODELING OF DYNAMICS OF ELASTIC HALF-SPACE

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In the modeling of seismic waves, solutions of Lamb problems are useful. Lambs pioneering work contained most of the elements that are important for analytical studies on modeling the propagation of seismic waves in an elastic medium. This method is useful in military like testing atomic bombs on the surface or in simulations of disasters such as: fallen of asteroids or accidents in weapon warehouses. The sequence of consequence after concentrate pressure (asteroid) to elastic body(soil). The sequence of events in the formation of the model of falling asteroid to elastic body: a - the initial position of the spherical impactor and the layered target, b - 23 seconds after the impact, the crater's transition cavity reaches a maximum depth of about 19 km, at 115 seconds after the impact, the collapse of the transition cavity due to subsidence of the sides) leads to the formation of a "transitional hill" with a height of up to 5 km, the deep rocks beneath the crater rise above the level of their initial occurrence, r - 200 seconds after the impact, the "transition hill" spreads in the field while the rocks in the depths stopped due to the restoration of the normal value of internal friction, the velocity of near-surface spreading reaches 200 m / s, d - 300 seconds after the impact, the motion is close to stopping, e - 400 seconds after impact, the crater acquires a stable final form.

In our cases, instead of the soil was chosen bulk soils with a density of 1500 kg/m^3 was chosen as an elastic medium. Parameters Lame are equal to λ=1,561.10^4 MPa, μ= 1,0935.10^4 MPa. In our situations the velocity of Longitudinal wave (c_p=500m/s) and Transversal wave is equal to (c_s=270m/s).

Consider the following problem related to a flat deformed state. Suppose that in the x_3 = 0, bounding an elastic half-space $x_3 \geq 0$, there is a load $P(x_1,t) = e^{i\omega t}p(x_1)=P_0$. This load causes a stressed and deformed state in the half-space, longitudinal and transverse waves appear. It is required to solve the wave equations

$$\left(\partial_1^2 + \partial_3^2 - \frac{1}{c_1^2}\partial_t^2\right) \quad = 0, \qquad \left(\partial_1^2 + \partial_3^2 - \frac{1}{c_2^2}\partial_t^2\right)\psi = 0 \quad (1)$$

with boundary conditions

$$\sigma_{33} = (x_1, 0, t) = -e^{i\omega t}p(x_1), \qquad \sigma_{13}(x_1, 0, t) = 0 \qquad (2)$$

Here we have assumed that the load $p(x_1)e^{i\omega t}$ acts perpendicularly to the plane bounding the elastic half-space in the positive direction of the axis x_3. Potentials Φ and ψ are related to displacements and stresses by the following relations

$$u_1 = \partial_1 \quad - \partial_3\psi, \qquad u_3 = \partial_3 \quad + \partial_1\psi, \qquad (3)$$

$$\sigma_{13} = \mu(2\partial_1\partial_3 \quad + \partial_1^2\psi - \partial_3^2\psi),$$

$$\sigma_{33} = 2\mu(\partial_3^2 \quad + \partial_1\partial_3\psi) + \frac{\lambda}{c_1^2}\ddot{\ } . \qquad (4)$$

The wave equations (1) can be solved by applying the integral Fourier transform. It is easy to verify that equations (1) are satisfied by integral expressions

$$= \frac{e^{i\omega t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(\alpha)e^{-\nu_1 x_3}e^{-i\alpha x_1}d\alpha, \qquad (5)$$

$$\psi = \frac{e^{i\omega t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(\alpha)e^{-\nu_2 x_3}e^{-i\alpha x_1}d\alpha, \qquad (6)$$

$$\nu_\mu = (\alpha^2 - k_\mu^2)^{1/2}, \quad \mu = 1, 2.$$

Substituting formulas (5) and (6) into expressions for displacements (4), we obtain

$$u_1 = \frac{e^{i\omega t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (-i\alpha A e^{-\nu_1 x_3} + \nu_2 B\, e^{-\nu_2 x_3}) e^{-i\alpha x_1} d\alpha,$$

$$u_3 = -\frac{e^{i\omega t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\nu_2 A\, e^{-\nu_1 x_3} + i\alpha B e^{-\nu_2 x_3}) e^{-i\alpha x_1} d\alpha. \qquad (7)$$

Substituting formulas (5) and (6) into formulas (4) and using the boundary conditions (2), we obtain a system of two equations

$$2i\alpha\nu_1 A - (2\alpha^2 - k_2^2)B = 0, \;\; (2\alpha^2 - k_2^2)A + 2i\alpha\nu_2 B = -\frac{\tilde{p}(\alpha)}{\mu}, \qquad (8)$$

where

$$\tilde{p}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} p(x_1)\, e^{i\alpha x_1} dx_1.$$

From the solution of the system of equations (8) we have

$$A = -\frac{2\alpha^2 - k_2^2}{\mu N(\alpha)} \tilde{p}(\alpha), \;\; B = -\frac{2i\alpha\nu_1}{\mu N(\alpha)} \tilde{p}(\alpha), \qquad (9)$$

where $N(\alpha) = (2\alpha^2 - k_2^2)^2 - 4\alpha^2\nu_1\nu_2$.

Substituting (9) into relations (7), we obtain in the $x_3 = 0$ the following expressions:

$$u_1(x_1, 0, t) = -\frac{e^{i\omega t}}{\mu\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\alpha\tilde{p}(\alpha)}{N(\alpha)} (2\alpha^2 - k_2^2 - 2\nu_1\nu_2) e^{-i\alpha x_1} d\alpha,$$

$$u_3(x_1, 0, t) = -\frac{e^{i\omega t} k_2^2}{\mu\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{p}(\alpha)}{N(\alpha)} \nu_1 e^{-i\alpha x_1} d\alpha. \qquad (10)$$

Finally, we simulated our tasks in Matlab media and get results to real life situations. A study of the dynamic behavior of a two-dimensional model of an elastic medium under the influence of a vertical concentrated load on a half-space (the Lamb problem) has shown that the presence of viscosity leads to a decrease in high-frequency slowly damped oscillations behind the Rayleigh wave front which are absent on real seismograms. The obtained result indicates the need to take into account the viscosity of the interlayers in the elastic model of rocks when calculating seismic waves.

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