A Geometric Approach to the Modeling of Critical Phenomena in Combustion Models

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The paper is devoted to the modeling of the critical phenomena in multiscale combustion models. Such models are usually described by singularly perturbed systems of differential equations to reflect the significant distinction in characteristic relaxation times of different physicochemical processes.

The paper proposes an approach for modeling of critical phenomena on the basis of the geometric asymptotic method of integral manifolds and the black swans and canards techniques [1-3]. The interest to critical phenomena is occasioned by not only of safety reason; in many cases namely the critical regime is the most effective in technological processes. The sense of criticality here is as follows. The critical regime corresponds to chemical reaction separating the domains of self-accelerating reactions and the domains of slow safe reactions.

Recall that a canard (or “French duck”) is a trajectory of a singularly perturbed system of differential equations if it move at first along a stable slow integral manifold and then for a while along an unstable slow integral manifold. The slow integral manifold is defined as an invariant surface of slow motions. In combustion models canards simulate the critical regimes when the temperature increases as high as possible but without explosion. A canard trajectory may be considered as the result of gluing stable (attractive) and unstable (repulsive) slow integral manifolds at one point of the breakdown surface, due to the availability of an additional scalar parameter in the differential system. However during a chemical process perturbations are possible. These perturbations can lead to the thermal explosion when the perturbed trajectory deviates from the canard. To solve this problem it is possible to glue the stable and unstable slow integral manifolds at all points of the breakdown surface simultaneously. For this goal an additional function of a vector variable parameterizing the breakdown surface is used. As a result we obtain the continuous stable/unstable integral surface or black swan. This guarantees the safety of chemical regimes, even with perturbations, during a chemical process.

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