Transformation of a vortex polarized beam in an anisotropic medium

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Abstract. We consider a vector cylindrical polarized beam having a symmetric transverse distribution of the amplitude of the electric field vector of the electric field relative to the axis, incident on planar inhomogeneous anisotropic layer. The beam is presented in the form of a set of plane polarized waves. For each of the waves, based on Maxwell's equations, a matrix solution is obtained for a wave propagating in a gyrotropic medium, as well as a reflection and transmission matrix. The transmitted and reflected beams are restored by the inverse Fourier transform.

1. Introduction

In present time, there is growing interest in cylindrical vector vortex beams. This is due to the intensive development of high-tech applications: optical manipulation [1], laser technologies of nanomaterials [2], optical biosensors [3], high beam focusing [4], and optical communication [5]. To implement new applications, it is important to be able to control the properties of the beam, the papers [6] and [7] are devoted to the study of the spatial distribution and polarization of the beam. An analytical description of Bessel beams in scalar form is presented in paper [8]. A mathematical model for high-order vector beams was presented and implemented in [9]. The integral operator of propagation of electromagnetic waves in an anisotropic crystal was obtained in paper [10] and was used to calculate cylindrical vector beams in [11 - 13]. The formation of cylindrical beams is an urgent task [14]. At present, Russian scientists have obtained and implemented a very effective mathematical model for the propagation of vector beams in anisotropic media [15]. The properties of materials affect the direction of propagation of the phase and group velocity, as well as the localization of electromagnetic waves. Natural and artificial media have many remarkable properties: optical and magnetic anisotropy, gyrotropy, chirality and dispersion. In addition, the localization of waves is affected by heterogeneity.

In this paper, we propose a mathematical apparatus for the matrix calculation of vector beams in inhomogeneous plane anisotropic media.

2. Basic equations

Let us consider the oblique incidence of a beam with a certain transverse amplitude distribution onto a flat inhomogeneous anisotropic layer and, based on the equations of classical electrodynamics, we study its propagation in the layer and reflection at the boundaries. Using the Fourier transform, we represent the incident beam in the form of a set of plane waves. For each of the plane waves, the wave vector has the form: $\vec{k}_0 = \{\alpha k_0, \beta k_0, k_z\}$. Maxwell's equations for the rotors of the fields $\vec{E}$ and $\vec{H}$ for the case of an anisotropic medium, when $\hat{\varepsilon}$ - is the tensor and μ is the scalar, taking into account the material equations:

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \alpha k_0 & \beta k_0 & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = ik_0\mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad (1)$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \alpha k_0 & \beta k_0 & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = ik_0\hat{\varepsilon} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \quad (2)$$

lead to a system of four ordinary differential equations:

$$\frac{d}{dz}\begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix} = ik_0 \begin{pmatrix} -\beta\frac{\varepsilon_{23}}{\varepsilon_{33}} & -\left(\mu - \frac{\beta^2}{\varepsilon_{33}}\right) & -\frac{\alpha\beta}{\varepsilon_{33}} & -\beta\frac{\varepsilon_{31}}{\varepsilon_{33}} \\ -\left(\varepsilon_{22} - \frac{\varepsilon_{23}\varepsilon_{32}}{\varepsilon_{33}} - \frac{\alpha^2}{\mu}\right) & -\beta\frac{\varepsilon_{32}}{\varepsilon_{33}} & \frac{\alpha\varepsilon_{23}}{\varepsilon_{33}} & -\left(\varepsilon_{21} - \frac{\varepsilon_{23}\varepsilon_{31}}{\varepsilon_{33}} + \frac{\alpha\beta}{\mu}\right) \\ \varepsilon_{12} - \frac{\varepsilon_{13}\varepsilon_{32}}{\varepsilon_{33}} + \frac{\alpha\beta}{\mu} & \beta\frac{\varepsilon_{13}}{\varepsilon_{33}} & -\frac{\alpha\varepsilon_{13}}{\varepsilon_{33}} & \left(\varepsilon_{11} - \frac{\varepsilon_{13}\varepsilon_{31}}{\varepsilon_{33}} - \frac{\beta^2}{\mu}\right) \\ -\frac{\alpha\varepsilon_{32}}{\varepsilon_{33}} & \frac{\alpha\beta}{\varepsilon_{33}} & \mu - \frac{\alpha^2}{\varepsilon_{33}} & -\frac{\alpha\varepsilon_{31}}{\varepsilon_{33}} \end{pmatrix} \begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix}. \quad (3)$$

The ODE system (3) for reduces to the system (2) of [16] for the case of oblique incidence of a plane wave in the X0Z plane. The vector $\vec{E}$ for the s- polarized component is parallel to the 0Y axis, and for the p- polarized component it “lies” in the X0Z plane. In the case when $\alpha = 0$ and the ODE system (1) will describe the oblique incidence of a plane wave in the Y0Z plane. In this case, the electric field vector $\vec{E}$ for the s- polarization wave will be directed along the 0X axis, and for p-polarization the vector $\vec{E}$ will lie in the Y0Z plane. For a plasma in an external magnetic field [17], if the field $\vec{H}$ parallel to the axis, $\overrightarrow{0Y}$ dielectric constant tensor has the following form.

$$\hat{\varepsilon} = \begin{pmatrix} 1 - \frac{u}{1 - W^2} & 0 & i\frac{uW}{1 - W^2} \\ 0 & 1 - u & 0 \\ -i\frac{uW}{1 - W^2} & 0 & 1 - \frac{u}{1 - W^2} \end{pmatrix}. \quad (4)$$

For the case when the anisotropic layer is oriented at an arbitrary angle χ to the plane of incidence, we apply the rotation operator about the 0Z axis to a tensor $\hat{\varepsilon}$, of the form (4):

$$\hat{\varepsilon}_{rot} = \mathbf{T}_z(\chi) \cdot \hat{\varepsilon} \cdot \mathbf{T}_z^{-1}(\chi). \quad (5)$$

Application of the rotation operator to the plasma permittivity tensor in an external field (4) brings it to the form:

$$\hat{\varepsilon}_{rot} = \begin{pmatrix} 1 - \frac{u}{1 - W^2}\cos^2(\chi) - u\sin^2(\chi) & \frac{1}{2}\left(\frac{u}{1 - W^2} - u\right)\sin(2\chi) & i\frac{uW}{1 - W^2}\cos(\chi) \\ \frac{1}{2}\left(\frac{u}{1 - W^2} - u\right)\sin(2\chi) & 1 - \frac{u}{1 - W^2}\sin^2(\chi) - u\cos^2(\chi) & i\frac{uW}{1 - W^2}\sin(\chi) \\ -i\frac{uW}{1 - W^2}\cos(\chi) & -i\frac{uW}{1 - W^2}\sin(\chi) & 1 - \frac{u}{1 - W^2} \end{pmatrix}. \quad (6)$$

In short form, system (3) can be written in the form of a differential equation:

$$\frac{d}{dz}\vec{Q} = ik_0\hat{A}\vec{Q}. \quad (7)$$

We introduce the following notation for the coefficients of the matrix $\hat{A}$ of system (3):

$$a = \frac{\varepsilon_{32}}{\varepsilon_{33}} = -\frac{\varepsilon_{23}}{\varepsilon_{33}} = -i\frac{uW}{1-u-W^2}\sin\chi, \tag{8.1}$$

$$d = \frac{\varepsilon_{13}}{\varepsilon_{33}} = -\frac{\varepsilon_{31}}{\varepsilon_{33}} = i\frac{uW}{1-W^2}\cos\chi, \tag{8.2}$$

$$b = \mu - \frac{\beta^2}{\varepsilon_{33}} = \mu - \frac{\beta^2\left(1-W^2\right)}{1-u-W^2}, \tag{8.3}$$

$$f = \mu - \frac{\alpha^2}{\varepsilon_{33}} = \mu - \frac{\alpha^2\left(1-W^2\right)}{1-W^2-u}, \tag{8.4}$$

$$g = \frac{\alpha\beta}{\varepsilon_{33}} = \frac{\alpha\beta\left(1-W^2\right)}{1-W^2-u}, \tag{8.5}$$

$$\tilde{n} = \varepsilon_{22} - \frac{\varepsilon_{23}\varepsilon_{32}}{\varepsilon_{33}} - \frac{\alpha^2}{\mu}, \tag{8.6}$$

$$e = \varepsilon_{11} - \frac{\varepsilon_{13}\varepsilon_{31}}{\varepsilon_{33}} - \frac{\beta^2}{\mu}, \tag{8.7}$$

$$h = \varepsilon_{21} - \frac{\varepsilon_{23}\varepsilon_{31}}{\varepsilon_{33}} + \frac{\alpha\beta}{\mu}. \tag{8.8}$$

In the dielectric constant tensor defined by formula (6) $\varepsilon_{12} = \varepsilon_{21}$, the first element of the third row of the matrix $\hat{A}$ is $\varepsilon_{12} - \frac{\varepsilon_{13}\varepsilon_{32}}{\varepsilon_{33}} + \frac{\alpha\beta}{\mu}$ also equal h. Then the matrix $\hat{A}$ of system (3) takes the form:

$$\mathbf{A} = \begin{pmatrix} i\beta a & -b & -g & i\beta d \\ -c & -i\beta a & i\alpha a & -h \\ h & i\beta d & -i\alpha d & e \\ i\alpha a & g & f & i\alpha d \end{pmatrix}. \tag{9}$$

The eigenvalues of the matrix $\hat{A}$ of system (3) have the form:

$$\lambda_{1,2,3,4} = \pm\sqrt{p_1 \pm \frac{1}{2}\sqrt{p_1^2 - 4p_2}}. \tag{10}$$

In the formula (10), the following notations are used:

$$p_1 = \frac{1}{2}\left(bc + ef - 2gh - \left(\alpha d + \beta a\right)^2\right), \tag{11.1}$$

$$p_2 = \left(ce - h^2\right)\left(bf - g^2\right) + \left(2adh - ea^2 - cd^2\right)\left(b\alpha^2 + 2gh\alpha + f\beta^2\right). \tag{11.2}$$

The solution for the projections of the electric and magnetic field intensity vectors E_y and H_y will be find in the form of functions:

$$E_y = Ae^{ik_0\int_0^z \lambda_1(\xi)d\xi} + Be^{k_0\int_0^z \lambda_2(\xi)d\xi}, \tag{12}$$

$$H_y = Ce^{ik_0\int_0^z \lambda_3(\xi)d\xi} + De^{k_0\int_0^z \lambda_4(\xi)d\xi}. \tag{13}$$

The upper limit in the integrals z corresponds to the plane z = const, and the lower limit corresponds to the interface z = 0. Then the projections H_x and E_x are determined from the first and third equations of the ODE system (3) as follows:

$$\begin{cases} -ik_0bH_x + i^2d\cdot k_0\beta E_x = \dfrac{dE_y}{dz} - i^2k_0\beta aE_y + ik_0gH_y, \\ i^2k_0\beta dH_x + ik_0eE_x = \dfrac{dH_y}{dz} + i^2d\cdot k_0H_y - ik_0hE_y. \end{cases} \tag{14}$$

Or

$$\begin{cases} -bH_x + id\cdot\beta E_x = (\lambda_1 - i\beta a)Ae^{ik_0\int_0^z\lambda_1(\xi)d\xi} - (\lambda_1 + i\beta a)Be^{-ik_0\int_0^z\lambda_1(\xi)d\xi} + g\left(Ce^{k_0\int_0^z\lambda_3(\xi)d\xi} + De^{-ik_0\int_0^z\lambda_3(\xi)d\xi}\right), \\ i\beta dH_x + eE_x = -h\left(Ae^{ik_0\int_0^z\lambda_1(\xi)d\xi} + Be^{-ik_0\int_0^z\lambda_1(\xi)d\xi}\right) + (\lambda_3 + i\alpha d)Ce^{k_0\int_0^z\lambda_3(\xi)d\xi} - (\lambda_3 - i\alpha d)De^{-k_0\int_0^z\lambda_3(\xi)d\xi}. \end{cases} \quad (15)$$

The determinant Δ of the main matrix of the system of linear algebraic equations (15) is equal to:

$$\Delta = \beta^2 d^2 - be. \quad (16)$$

According to the Cramer rule, projections H_x and E_x and are calculated by the formulas:

$$H_x = \frac{\Delta_1}{\Delta},\ E_x = \frac{\Delta_2}{\Delta}. \quad (17)$$

We write the fundamental matrix of the solution:

$$\hat{Y}(z) = \begin{pmatrix} \hat{Y}_{11}(z) & \hat{Y}_{12}(z) \\ \hat{Y}_{21}(z) & \hat{Y}_{22}(z) \end{pmatrix}. \quad (18)$$

Matrices $\hat{Y}_{11}(z)$, $\hat{Y}_{12}(z)$, $\hat{Y}_{21}(z)$, $\hat{Y}_{22}(z)$ have the form:

$$\hat{Y}_{11}(z) = \frac{1}{\Delta(z)}\begin{pmatrix} Ae^{ik_0\int_0^z\lambda_1(\xi)d\xi} & Be^{-k_0\int_0^z\lambda_1(\xi)d\xi} \\ A\left(e(\lambda_1 - i\beta a) + i\beta d\cdot h\right)e^{ik_0\int_0^z\lambda_1(\xi)d\xi} & -B\left(e(\lambda_1 + i\beta a) - i\beta d\cdot h\right)e^{-k_0\int_0^z\lambda_1(\xi)d\xi} \end{pmatrix}, \quad (19.1)$$

$$\hat{Y}_{12}(z) = \frac{1}{\Delta(z)}\begin{pmatrix} 0 & 0 \\ C\left(eg - i\beta d(\lambda_3 + i\alpha d)\right)e^{ik_0\int_0^z\lambda_3(\xi)d\xi} & D\left(eg - i\beta d(\lambda_3 - i\alpha d)\right)e^{-k_0\int_0^z\lambda_3(\xi)d\xi} \end{pmatrix}, \quad (19.2)$$

$$\hat{Y}_{21}(z) = \frac{1}{\Delta(z)}\begin{pmatrix} 0 & 0 \\ A\left(bh - i\beta d(\lambda_1 - i\beta a)\right)e^{ik_0\int_0^z\lambda_1(\xi)d\xi} & B\left(bh + i\beta d(\lambda_1 + i\beta a)\right)e^{-k_0\int_0^z\lambda_1(\xi)d\xi} \end{pmatrix}. \quad (19.3)$$

$$\hat{Y}_{22}(z) = \frac{1}{\Delta(z)}\begin{pmatrix} Ce^{ik_0\int_0^z\lambda_3(\xi)d\xi} & De^{-k_0\int_0^z\lambda_3(\xi)d\xi} \\ -C\left(b(\lambda_3 + i\alpha d) + i\beta ad\right)e^{ik_0\int_0^z\lambda_3(\xi)d\xi} & D\left(b(\lambda_3 - i\alpha d) + i\beta ad\right)e^{-k_0\int_0^z\lambda_3(\xi)d\xi} \end{pmatrix}. \quad (19.4)$$

The obtained FMR allows us to calculate the Cauchy matrix by the formula:

$$\hat{N}(z,0) = \hat{Y}(z)\hat{Y}^{-1}(0). \quad (20)$$

Matrix $\hat{Y}(0)$ has the form:

$$Y(0) = \frac{1}{\Delta(0)}\begin{pmatrix} \hat{Y}_{11}(0) & \hat{Y}_{12}(0) \\ \hat{Y}_{21}(0) & \hat{Y}_{22}(0) \end{pmatrix}. \quad (21)$$

$$\hat{Y}_{11}(0) = \frac{1}{\Delta(0)}\begin{pmatrix} A & B \\ A\left(e(0)(\lambda_1(0) - i\beta a(0)) + i\beta d(0)\cdot h(0)\right) & -B\left(e(0)(\lambda_1(0) + i\beta a(0)) - i\beta d(0)\cdot h(0)\right) \end{pmatrix}, \quad (22.1)$$

$$\hat{Y}_{12}(0) = \frac{1}{\Delta(0)}\begin{pmatrix} 0 & 0 \\ C\left(e(0)g(0) - i\beta d(0)(\lambda_3(0) + i\alpha d(0))\right) & D\left(e(0)g(0) - i\beta d(0)(\lambda_3(0) - i\alpha d(0))\right) \end{pmatrix}, \quad (22.2)$$

$$\hat{Y}_{21}(0) = \frac{1}{\Delta(0)}\begin{pmatrix} 0 & 0 \\ A\left(b(0)h(0) - i\beta d(0)(\lambda_1(0) - i\beta a(0))\right) & B\left(b(0)h(0) + i\beta d(0)(\lambda_1(0) + i\beta a(0))\right) \end{pmatrix}, \quad (22.3)$$

$$\hat{Y}_{22}(0)=\frac{1}{\Delta(0)}\begin{pmatrix} C & D \\ -C\left(b(0)\left(\lambda_3(0)+i\alpha d(0)\right)+i\beta a(0)d(0)\right) & D\left(b(0)\left(\lambda_3(0)-i\alpha d(0)\right)+i\beta a(0)d(0)\right)\end{pmatrix}. \quad (22.4)$$

When calculating the matrix the values of variables are used in the case of an inhomogeneous medium of quantities $a(z)$, $b(z)$, $c(z)$, $d(z)$, $e(z)$, $f(z)$, $g(z)$, $h(z)$, which are functions of z, when the variable z is zero. If the layer is symmetric, then at its boundaries $a(z_0)=a(0)$, $b(z_0)=b(0)$, $c(z_0)=c(0)$, $d(z_0)=d(0)$, $e(z_0)=e(0)$, $f(z_0)=f(0)$, $g(z_0)=g(0)$, $h(z_0)=h(0)$.

Coefficients of the Cauchy matrix for a thickness layer z_0

$$\hat{N}(z_0,0)=\begin{pmatrix} n_{11}(z_0,0) & n_{12}(z_0,0) & n_{13}(z_0,0) & n_{14}(z_0,0) \\ n_{21}(z_0,0) & n_{22}(z_0,0) & n_{23}(z_0,0) & n_{24}(z_0,0) \\ n_{31}(z_0,0) & n_{32}(z_0,0) & n_{33}(z_0,0) & n_{34}(z_0,0) \\ n_{31}(z_0,0) & n_{42}(z_0,0) & n_{43}(z_0,0) & n_{44}(z_0,0)\end{pmatrix}, \quad (23)$$

has the form:

$$n_{11}(z_0,0)=\cos\left(k_0\int_0^{z_0}\lambda_1(\xi)d\xi\right)-\beta\frac{\alpha d\left(abe-2bhd+\alpha\beta^2d^2\right)+ib(ae-hd)\lambda_3(z_0)}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_1(\xi)d\xi\right). \quad (24.1)$$

$$n_{12}(z_0,0)=\frac{b\left(i\alpha d-\lambda_3(z_0)\right)}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_1(\xi)d\xi\right). \quad (24.2)$$

$$n_{13}(z_0,0)=-i\frac{\alpha\beta^2d^3g+b\left(-i\alpha d+\lambda_3(z_0)\right)\left(ieg+\lambda_3(z_0)\right)}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_1(\xi)d\xi\right). \quad (24.3)$$

$$n_{14}(z_0,0)=-\frac{d^2\alpha\beta}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_1(\xi)d\xi\right). \quad (24.4)$$

$$n_{31}(z_0,0)=\frac{-h\left(be-\beta^2d^2\right)}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_3(\xi)d\xi\right). \quad (24.5)$$

$$n_{32}(z_0,0)=\frac{i\beta d}{\lambda_1(z_0)\left(d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)\right)}\sin\left(k_0\int_0^{z_0}\lambda_3(\xi)d\xi\right). \quad (24.6)$$

$$n_{33}(z_0,0)=\cos\left(k_0\int_0^{z_0}\lambda_3(\xi)d\xi\right)-\frac{\beta^2\lambda_2(z_0)d}{d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)}\sin\left(k_0\int_0^{z_0}\lambda_3(\xi)d\xi\right). \quad (24.7)$$

$$n_{34}(z_0,0)=\frac{e}{d^3\alpha\beta^2+be\left(\alpha d+i\lambda_3(z_0)\right)}\sin\left(k_0\int_0^{z_0}\lambda_3(\xi)d\xi\right). \quad (24.8)$$

The coefficients of the second row n_{21}, n_{22}, n_{23}, n_{24} as well as the coefficients of the fourth row n_{41}, n_{42}, n_{43}, n_{44} are quite large, therefore, they are not given in this paper.

The Cauchy matrix (23) found in this paper opens up the possibility of realizing the calculation of reflection and reflection for beams and plane waves arbitrarily incident on the medium. For each of the plane wave incident at different angles on a plane inhomogeneous gyrotropic layer, the reflection and transmission matrices are calculated:

$$\hat{R}=\begin{pmatrix} R_{pp} & R_{ps} \\ R_{sp} & R_{ss}\end{pmatrix}, \quad (25)$$

$$\hat{T}=\begin{pmatrix} T_{pp} & T_{ps} \\ T_{sp} & T_{ss}\end{pmatrix}. \quad (26)$$

For each of the plane wave having their own amplitude, phase, and polarization, the transmission and reflection matrices of light allow us to calculate the amplitude, polarization, and phase of transmitted and reflected waves. The field structure and polarization of transmitted and reflected beams is restored by applying the inverse Fourier transform.

3. References

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