Three-dimensional wave model in coastal marine systems for forecasting wave impact on shore protection and coastal structures

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Abstract. The article is devoted to the research of wave processes on shore protection and coastal structures using mathematical model of wave processes. The model presented in the article gives more realistic description of the physical wave process near the coastline. The practical significance of the numerical algorithms and program complex for their implementation is the possibility of their application in the study of hydrophysical processes in the coastal water systems.

1. Introduction

The construction and study of three-dimensional wave models in coastal marine systems makes it possible to use 3D models for studying hydrophysical processes in coastal water systems, as well as for assessing the hydrodynamic effects on shore protection structures and coastal structures in the presence of gravity surface waves. Waves propagate faster on wide profiles of the slopes of the shelf, the speed can be affected by flows along the shelf and the reverse movements of the lower slope. Large amplitudes and sharp changes in the topography of the coast cause distortion of the wave modes. Waves affect the movement of the shelf and slope on a daily scale and the width of the shelf, they are important for predicting changes in the geometry of the shelf and slope due to the effects of tides [1]. Coastal waves are modeled on the basis of a parabolic equation taking into account the effects of refraction, diffraction and wave destruction. The numerical model has been experimentally tested on coastal waves, coastal currents, sediment transport and the morphological development of the beach around a breakwater from a large-scale sediment transport facility at the US Army Corps of Engineering Research and Development Center [2].

Chinese scientists from Southwestern Jiaotong University Bo Huang, Bin Zhu, Shengai Tsui, Lunliang Duan, Zavei Zhang conducted experimental and numerical simulations of the impact of wave forces on coastal bridges. As a result of experimental tests and numerical simulations, the wave impact on the coastal bridge induced by a hurricane was studied. After the experiments, 2D numerical model is used to study the wave action, in which the Reynolds-averaged Navier-Stokes equations in combination with the k-ω turbulence model are used to model the waves. The VOF method is used to track the interface between air and water [3].

This implementation relies on a dynamic grid that deforms to replicate wave motion, and is integrated into wsiFoam: a multi-regional communication strategy applied to two-phase Navier-Stokes solvers [4]. The combination of the dynamic boundary method with a grid of several regions counteracts the increase in computational costs, which is inherent in modeling using dynamic regions.

This approach leads to the creation of a high-performance computational strategy for wave generation, which can be used to accurately and efficiently model the generation, propagation, and interaction of waves with fixed structures and floating bodies [5]. It is necessary to continue construction and stretch the coast protection section, which leads to an increase in anthropogenic, engineering load, to a decrease in the stability of the surrounding landscapes.

2. Statement of 3D wave hydrodynamics problem

Spatially inhomogeneous three-dimensional mathematical model of the wave hydrodynamics of a shallow water body includes [6-8]:

– equations of motion (Navier - Stokes):

$$u_t' + uu_x' + vu_y' + wu_z' = -\frac{1}{\rho}P_x' + \left(\mu u_x'\right)_x' + \left(\mu u_y'\right)_y' + \left(\nu u_z'\right)_z',$$
$$v_t' + uv_x' + vv_y' + wv_z' = -\frac{1}{\rho}P_y' + \left(\mu v_x'\right)_x' + \left(\mu v_y'\right)_y' + \left(\nu v_z'\right)_z', \quad (1)$$
$$w_t' + uw_x' + vw_y' + ww_z' = -\frac{1}{\rho}P_z' + \left(\mu w_x'\right)_x' + \left(\mu w_y'\right)_y' + \left(\nu w_z'\right)_z' + g;$$

– continuity equation:

$$\rho_t' + \left(\rho u\right)_x' + \left(\rho v\right)_y' + \left(\rho w\right)_z' = 0, \quad (2)$$

where $\mathbf{V} = \{u, v, w\}$ is the velocity vector of the water flow of a shallow water body; ρ is the density of the aquatic environment; P is the hydrodynamic pressure; g is the gravitational acceleration; μ,ν are coefficients of turbulent exchange in the horizontal and vertical directions; $\mathbf{n}$ is the normal vector to the surface describing the boundary of the computational domain.

Add boundary conditions to system (1)-(2):

– the entrance (left border): $\mathbf{V} = \mathbf{V}_0,\ P_\mathbf{n}' = 0,$

– the bottom border: $\rho\mu\left(\mathbf{V}_\tau\right)_\mathbf{n}' = -\boldsymbol{\tau},\ \mathbf{V}_\mathbf{n} = 0,\ P_\mathbf{n}' = 0,$

– the lateral border: $\left(\mathbf{V}_\tau\right)_\mathbf{n}' = 0,\ \mathbf{V}_\mathbf{n} = 0,\ P_\mathbf{n}' = 0,$

– the upper border: $\rho\mu\left(\mathbf{V}_\tau\right)_\mathbf{n}' = -\boldsymbol{\tau},\ w = -\omega - P_t'/\rho g,\ P_\mathbf{n}' = 0,$ (3)

– the surface of the structure: $\rho\mu\left(\mathbf{V}_\tau\right)_\mathbf{n}' = -\boldsymbol{\tau}, w = 0,\ P_\mathbf{n}' = 0,$

where ω is the intensity of evaporation of a liquid, $\mathbf{V}_\mathbf{n}$, $\mathbf{V}_\tau$ are the normal and tangential component of the velocity vector, $\boldsymbol{\tau} = \{\tau_x, \tau_y, \tau_z\}$ is the vector of tangential stress. Figure 1 shows the geometry of the water body.

Let $\boldsymbol{\tau} = \rho_a Cd_s |\mathbf{w}|\mathbf{w}$ is the vector of tangential stress for the free surface, $Cd_s = 0{,}0026$, $\mathbf{w}$ is the wind velocity relative to water, ρ_a is the atmosphere density, Cd_s is the dimensionless surface resistance coefficient, which depends on wind speed. Let us set the tangential stress vector for the bottom taking into account the movement of water as follows: $\boldsymbol{\tau} = \rho Cd_b |\mathbf{V}|\mathbf{V}$, $Cd_b = gk^2/h^{1/3}$, where $k = 0{,}04$ is the group roughness coefficient in the Manning formula, considered in the range of 0,025 – 0,2; h=H+η is the depth of the water area, [m]; H is the depth to the undisturbed surface, [m]; η is the elevation of the free surface relative to the geoid (sea level), [m].

Let use the approximation that allows to build a non-uniform in depth vertical turbulent exchange coefficient on the basis of measured pulsations of the water flow velocity [9]:

$$\nu = C_s^2 \Delta^2 \frac{1}{2}\sqrt{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2}, \quad (4)$$

where C_s is the dimensionless empirical constant, determined on the basis of the calculation of the attenuation process of homogeneous isotropic turbulence; Δ is the characteristic scale of the grid;

$\overline{U},\overline{V}$ are the time-averaged ripple components of the velocity of the water flow in the horizontal direction.

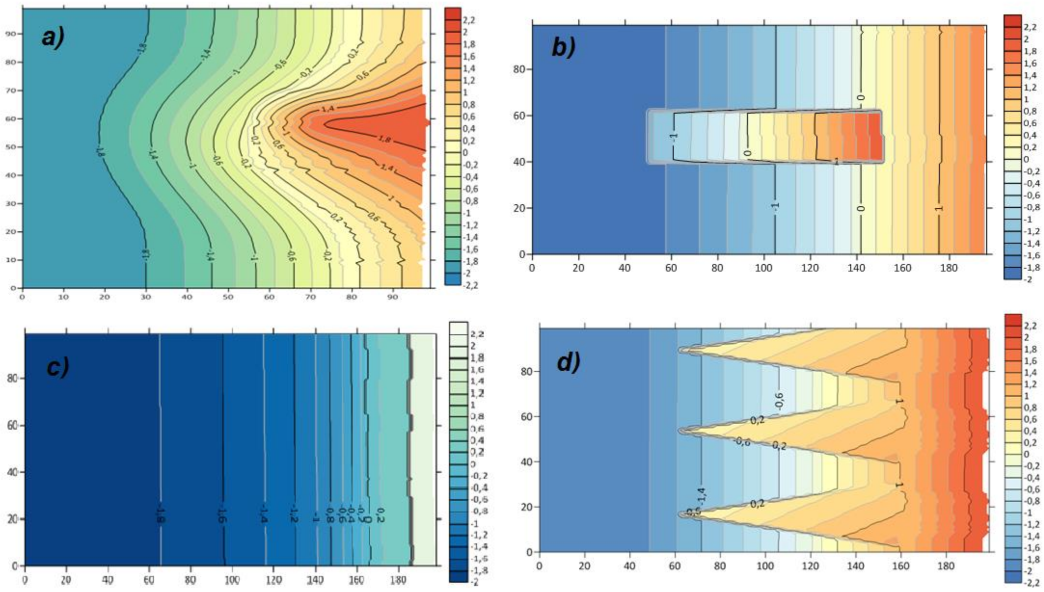

Figure 1. Isolines of the depth function of the bottom surface and the coastline.

3. The discrete model of hydrodynamics

The computational domain inscribed in a parallelepiped. For the numerical realization of the discrete mathematical model of the hydrodynamic problem posed, a uniform grid is introduced:

$$\overline{w}_h = \{t^n = n\tau, x_i = ih_x, y_j = jh_y, z_k = kh_z;\ n = \overline{0..N_t}, i = \overline{0..N_x}, j = \overline{0..N_y}, k = \overline{0..N_z};$$
$$N_t\tau = T, N_x h_x = l_x, N_y h_y = l_y, N_z h_z = l_z\},$$

where τ is the step by the time, h_x, h_y h_z are steps in space, N_t is the number of time layers, T is the upper bound on the time coordinate, N_x, N_y N_z are the number of nodes by spatial coordinates, l_x, l_y l_z are the boundaries along the parallelepiped in the direction of the axes Ox, Oy and Oz accordingly.

The method of correction to pressure was used to solve the hydrodynamic problem. The variant of this method in the case of a variable density will take the form [10-11]:

$$\frac{\tilde{u}-u}{\tau} + u\overline{u}'_x + v\overline{u}'_y + w\overline{u}'_z = \left(\mu\overline{u}'_x\right)'_x + \left(\mu\overline{u}'_y\right)'_y + \left(\nu\overline{u}'_z\right)'_z,$$
$$\frac{\tilde{v}-v}{\tau} + u\overline{v}'_x + v\overline{v}'_y + w\overline{v}'_z = \left(\mu\overline{v}'_x\right)'_x + \left(\mu\overline{v}'_y\right)'_y + \left(\nu\overline{v}'_z\right)'_z,$$
$$\frac{\tilde{w}-w}{\tau} + u\overline{w}'_x + v\overline{w}'_y + w\overline{w}'_z = \left(\mu\overline{w}'_x\right)'_x + \left(\mu\overline{w}'_y\right)'_y + \left(\nu\overline{w}'_z\right)'_z + g, \qquad (5)$$

$$P''_{xx} + P''_{yy} + P''_{zz} = \frac{\widehat{\rho}-\rho}{\tau^2} + \frac{\left(\widehat{\rho}\tilde{u}\right)'_x}{\tau} + \frac{\left(\widehat{\rho}\tilde{v}\right)'_y}{\tau} + \frac{\left(\widehat{\rho}\tilde{w}\right)'_z}{\tau},$$
$$\frac{\widehat{u}-\tilde{u}}{\tau} = -\frac{1}{\widehat{\rho}}P'_x,\ \frac{\widehat{v}-\tilde{v}}{\tau} = -\frac{1}{\widehat{\rho}}P'_y,\ \frac{\widehat{w}-\tilde{w}}{\tau} = -\frac{1}{\widehat{\rho}}P'_z,$$

where $V = \{u, v, w\}$ are the components of the velocity vector, $\{\widehat{u},\widehat{v},\widehat{w}\},\{\tilde{u},\tilde{v},\tilde{w}\}$ are the components of the velocity vector fields on the «new» and intermediate time layers, respectively, $\overline{u} = \left(\tilde{u}+u\right)/2$,

$\hat{\rho}$ and ρ are the distribution of the density of the aqueous medium on the new and previous time layers, respectively.

In the construction of discrete mathematical models of hydrodynamics, the fullness of the control cells was taken into account, which makes it possible to increase the real accuracy of the solution in the case of a complex geometry of the investigated region by improving the approximation of the boundary.

Through $o_{i,j,k}$ marked the volume of fluid (VOF) of the cell (i,j,k) [10]. VOF is determined by the pressure of the liquid column inside this cell. If the average pressure at the nodes that belong to the vertices of the cell in question is greater than the pressure of the liquid column inside the cell, then the cell is considered to be full $\left(o_{i,j,k}=1\right)$. In the general case, VOF can be calculated by the following formula:

$$o_{i,j,k}=\frac{P_{i,j,k}+P_{i-1,j,k}+P_{i,j-1,k}+P_{i-1,j-1,k}}{4\rho g h_z}. \qquad (6)$$

We introduce the coefficients q_0, q_1, q_2, q_3, q_4, q_5, q_6, describing VOF of regions located in the vicinity of the cell (control areas). In the case of boundary conditions of the third kind $c'_n(x,y,t)=\alpha_n c+\beta_n$, the discrete analogues of the convective uc'_x and diffusion $\left(\mu c'_x\right)'_x$ transfer operators, obtained with the help of the integro-interpolation method, taking into account the VOF, can be written in the following form [12]:

$$\left(q_0\right)_i uc'_x \simeq \left(q_1\right)_i u_{i+1/2}\frac{c_{i+1}-c_i}{2h_x}+\left(q_2\right)_i u_{i-1/2}\frac{c_i-c_{i-1}}{2h_x},$$

$$\left(q_0\right)_i \left(\mu c'_x\right)'_x \simeq \left(q_1\right)_i \mu_{i+1/2}\frac{c_{i+1}-c_i}{h_x^2}-\left(q_2\right)_i \mu_{i-1/2}\frac{c_i-c_{i-1}}{h_x^2}-\left|\left(q_1\right)_i-\left(q_2\right)_i\right|\mu_i\frac{\alpha_x c_i+\beta_x}{h_x}.$$

Similarly, approximations for the remaining coordinate directions will be recorded. The error in approximating the mathematical model is equal to $\mathrm{O}\left(\tau+\|h\|^2\right)$, where $\|h\|=\sqrt{h_x^2+h_y^2+h_z^2}$. The conservation of the flow at the discrete level of the developed hydrodynamic model is proved, as well as the absence of non-conservative dissipative terms obtained as a result of discretization of the system of equations. A sufficient condition for the stability [13, 14] and monotony of the developed model is determined on the basis of the maximum principle [15-16], with constraints on the step with respect to the spatial coordinates: $h_x<\left|2\mu/u\right|$, $h_y<\left|2\mu/v\right|$, $h_z<\left|2\nu/w\right|$ or $\mathrm{Re}\le 2N$, where $\mathrm{Re}=\left|V\right|\cdot l/\mu$ is the Reynolds number [10], l is the characteristic size of the region $N=\max\left\{N_x,N_y,N_z\right\}$. Discrete analogs of the system of equations (5) are solved by an adaptive modified alternating-triangular method of variational type [17-19].

4. Description of the software package

A software package has been developed for implementing model hydrodynamic problems implemented in C ++. Parallel algorithms implemented in the software package for solving model problems of the systems of grid equations arising in the process of discretization by a modified adaptive alternating triangular method of variational type were developed using MPI technology. Fiure 2 shows the scheme of the algorithm of the program that numerically implements the developed 3D models of wave hydrodynamics.

The developed software package includes: a control unit (contains a cycle for a time variable, and the functions are called: calculating the velocity field without taking into account the elevation function, calculating the elevation function of the velocity, calculating the two-dimensional velocity field, checking the presence of the structure on the surface of the aquatic environment and data output); the input unit of the initial distributions for calculating the velocity of currents and the level elevation function (the initial distributions of the velocity field and the elevation functions of the level and the initial values of the degree of filling of the calculated cells are set); block of construction of grid equations for 3D fields of water flow rates without pressure; block of construction of grid equations for calculating the pressure field and the function of elevation of the level; a unit for checking the presence on the surface of the aquatic environment of the structure; the unit for calculating the velocity field with regard to pressure; a block for solving grid equations with a modified adaptive alternating triangular method of variational type; block output values of the velocity field and the pressure function (field elevation function level).

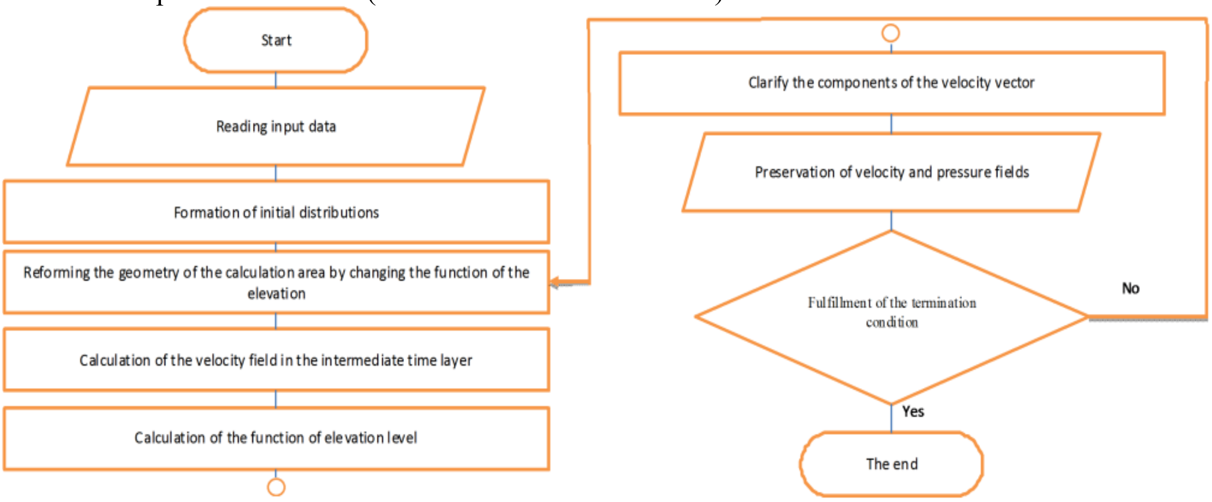

Figure 2. Algorithm of the software complex.

5. Results of numerical experiments based on wave-hydrodynamic model

The developed software package allows to set a complex configuration of a surface object, as well as the type and characteristics of the source of oscillations. The developed software package allows to set a complex configuration of the surface object, as well as the type and characteristics of the source of oscillations. Fiure 3 the results of forecasting changes in wave hydrodynamic processes during the flow of an underwater body, taking into account the complex geometry of the object in the liquid. The figure describes the level and bottom elevation function for the bottom without structures, single breakwater (buna), breakwater wall and set of breakwaters.

The developed numerical algorithms for solving model problems and implementing a set of programs can be used to study hydrophysical processes in coastal water systems [16], as well as to find the velocity field and pressure of the water environment, to assess the possible impact on the coastline in the presence of various coastal protective and shore fortifications.

The figure shows that because of walls of the wave is reflected from it and with force rolls back, washing away the coast. The disappearance of beaches leads to more intense erosion of the underwater slope, and this, in turn, accelerates the deformation or destruction of the walls. There is also erosion and decrease in the width of the beach, that is, they contribute to the rapid disappearance of beaches.

Dam is a transverse to the shoreline structure that helps to hold sediments and preserve a natural or artificial beach. Dam is an active coastal protection structure, which delays the beach-forming sediments moving along the shore. The presence of dam in itself makes a change in the dynamics of the coastal zone. First of all, they violate the longitudinal structure of the movement of mud and sand sediments, and therefore the leeward coast will almost always experience a shortage of material. The protective properties of dams have local character.

The development of a 3D model of wave hydrodynamic processes based on field data made it possible to describe the movement of the aquatic environment of a shallow water body taking into account wave propagation towards the shore. A modern software package has been created that is adapted for modeling hydrodynamic wave processes, is used in a wide range of parameters to calculate velocity fields and pressures of the aquatic environment, and to estimate the hydrodynamic impact on the coast in the presence of surface waves. The source of perturbations is set at a given distance from the shoreline. The modeling area has dimensions of 50 by 50 m and a depth of 2 m, the peak point rises above sea level by 2 m. Suppose that the liquid is at rest at the initial moment of time. When solving the posed model problem, a grid of 100 by 100 by 40 sizes of calculated nodes was used; the time step was 0.01 s.

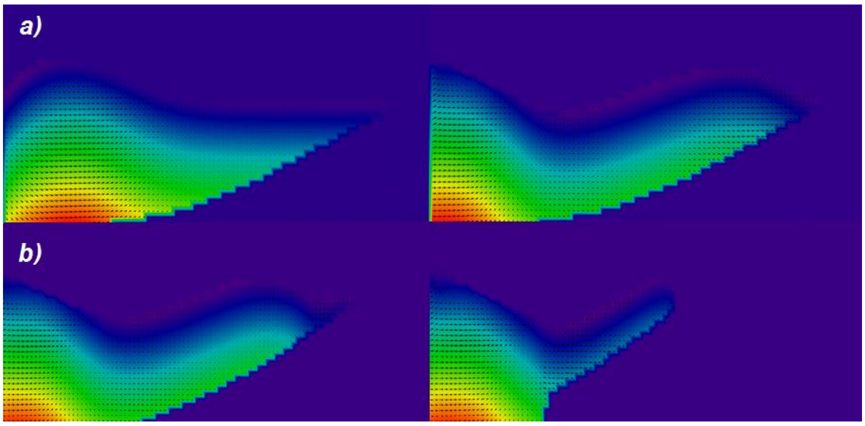

Figure 3. The field of the vector of the velocity of movement of the aquatic environment (cut of the XOZ plane).

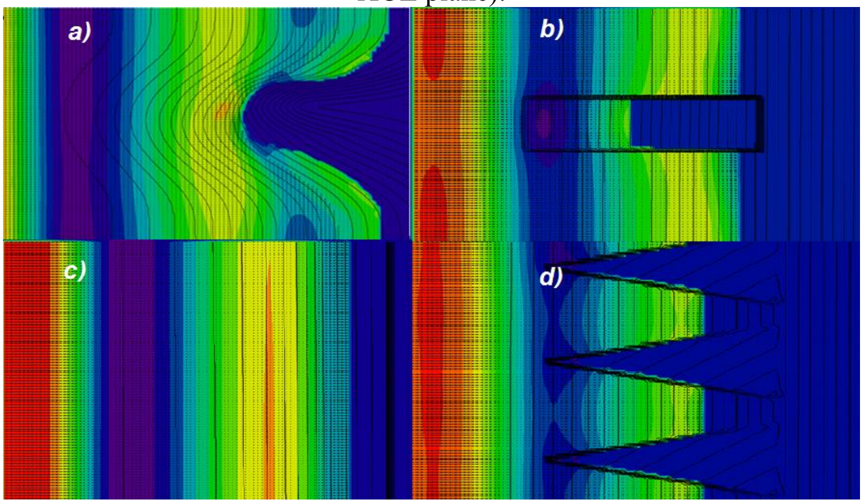

Figure 4. Graphs of the bottom relief.

Figure 4 demonstrates the field of the vector of the velocity of movement of the aquatic environment in case the geometry of the bottom, single breakwater (dam), breakwater wall and set of breakwaters, the level elevation function dynamically changes, and flooding and shallowing zones are formed. Accounting for flooding and drainage of coastal areas was carried out at the expense of recalculation VOF of the cells. The proposed approach allows solving problems in regions with a complex and dynamically tunable boundary geometry. The developed software complex has a distinctive feature, when modeling the propagation of surface oscillations, the wave output to the coastline is taken into account.

6. Conclusion

The paper considers the construction and study of three-dimensional wave models in coastal marine systems, taking into account the dynamically changing geometry of the bottom and the presence of shore protection structures. 3D models are used to study the hydrophysical processes in coastal water systems, as well as to assess the hydrodynamic impact on shore protection structures and coastal structures in the presence of gravitational surface waves. The 3D model gives the most realistic description of physical wave processes near the coastline. The description of the developed software is given, which allows changing the characteristics of the location and oscillations of the source, as well as taking into account the shape of zones and the degree of intensity of drainage and flooding of coastal areas. The paper considers the following shore protection structures: single breakwater (dam), breakwater wall, set of breakwaters. The developed software package can be widely used for practical studies of the calculation of the force effect of waves on the geometry of the bottom surface, as well as surface and coastal infrastructure.

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