The propagation of pulses of a special shape in an inhomogeneous anisotropic medium with dispersion and torsion of the optical axis

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Abstract. An oblique incidence of polarized light pulses with different envelope shapes is considered: triangular, Gaussian, rectangular and in the form of giant laser pulses onto a flat inhomogeneous anisotropic layer with torsion of the optical axis and optical dispersion. The wavelength of an optical pulse carrier is close to the resonance wavelength of an inhomogeneous anisotropic layer. The problem of the propagation of s- and p-polarization waves through a layer, the reflection of impulses of cross-polarized components, the dependence of modulation coefficients during medium torsion and frequency detuning is considered.

1. Introduction

Currently, the interest of researchers to optical materials with anisotropy inhomogeneity and dispersion has increased significantly. The use of optical technologies in telecommunications systems is associated with effective scaling of integrated circuits and processing devices, as well as with increasing frequency and throughput. Optical properties of materials determine the direction of propagation, birefringence, polarization, phase of a light wave in a medium [1]. Matrix methods are traditionally used to calculate electromagnetic waves in planar structures [2]. They appeared in the 40s of the 20th century and were improved as new challenges and technologies developed. For an anisotropic layer, the solution has the form of a 4×4 matrix [3]. Heterogeneous structure was represented as a set of layers with homogeneous parameters. A change in the direction of the optical axis affects the transmission of light by an inhomogeneous optical medium [4]. It finds application in various optical devices, for example, polarizers, light modulators [5]. The Berreman method was developed for inhomogeneous uniaxial anisotropic media. For calculations, a model of a layered medium is used. The transfer matrix of the inhomogeneous layer is obtained by multiplying the transfer matrices of the layers. For ease of calculation, the Berreman method was later reduced to the 2 × 2 method [6] and adapted for the calculation of liquid crystal displays [7].
Most programs for modeling liquid crystal devices use simplified mathematical algorithms. The book [8] presents a rigorous approach to the optical modeling of LCD devices. The authors consider the anisotropic properties of liquid crystals, the dependence of these properties on external influences, classify theoretically rigorous and efficient methods and determine how these methods are best used to develop new applications of optics and photonics. Currently, there is a growing interest in photonic crystal waveguides, which can direct or prohibit the propagation of light at any wavelength, including the visible range. In liquid crystals, the heterogeneity of the photonic crystal structure can be provided by external fields [9].

Using external influences, you can change the direction of the optical axis of an anisotropic liquid crystal and, thus, determine the transmission spectrum of the cell or the mode of propagation of a waveguide wave. This paper is devoted to the influence of the dispersion and torsion of an anisotropic medium on the envelope shape of optical signals of various shapes: a triangular, square wave, Gaussian pulse and a signal in the form of a giant laser pulse.

2. Formulation of the problem

Consider an oblique incidence of an optical pulse on a flat anisotropic layer with torsion of the optical axis and dispersion. The dielectric constant of the medium depends on the coordinate z and the frequency ω. The drop of a wave on a layer is shown in Fig. 1. The change in the direction of the optical axis is shown with a distance from the interface z = 0. Vectors $\mathbf{k}_i$, $\mathbf{k}_r$, $\mathbf{k}_t$ - wave vectors of the incident, reflected and transmitted waves.

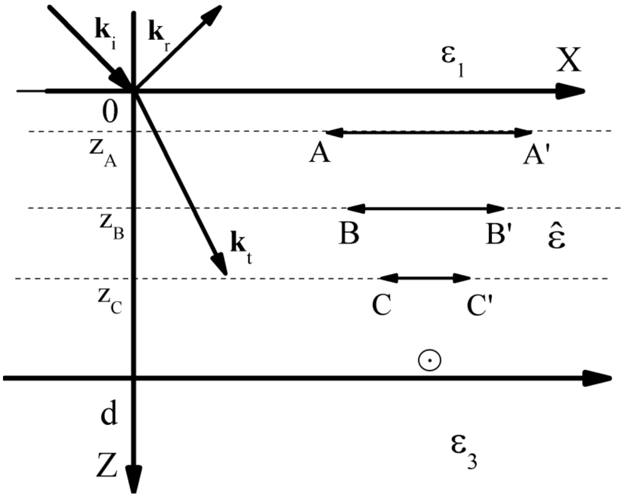

Figure 1. Oblique incidence of light on a layer with torsion of the optical axis.

The dielectric constant of the medium depends on the z coordinate and on the frequency ω:

$$\hat{\varepsilon}(\omega,z)=\begin{pmatrix} \varepsilon_o(\omega)\cos^2(\chi(z))+\varepsilon_e\sin^2(\chi(z)) & \frac{\varepsilon_e(\omega)-\varepsilon_0(\omega)}{2}\sin(2\chi(z)) & 0 \\ \frac{\varepsilon_e(\omega)-\varepsilon_0(\omega)}{2}\sin(2\chi(z)) & \varepsilon_o(\omega)\sin^2(\chi(z))+\varepsilon_e(\omega)\cos^2(\chi(z)) & 0 \\ 0 & 0 & \varepsilon_o(\omega) \end{pmatrix}. \quad (1)$$

The dependence of the dielectric constant of the anisotropic medium on the radiation frequency is:

$$\varepsilon_{o,e}(\omega)=\varepsilon_{\perp,\parallel}+\frac{\omega_P^2 F_{\perp,\parallel}}{\omega_0^2-\omega^2-i\omega\gamma}. \quad (2)$$

For a liquid crystal MBBA: λ_0=9.07μm, $\gamma_\perp$=8.02E-3$\cdot\omega_0$, $\gamma_\parallel$=4.35E-3$\cdot\omega_0$, $F_\perp$=0.1, $F_\parallel$=0.05 [10], ε_o=5.14, ε_e=4.58 [11], $\gamma_{12}=\frac{\gamma_\perp+\gamma_\parallel}{2}$, $d=15\lambda_c$. The torsion angle of the optical axis: $\Delta\chi=90^0$.

The vector of the electric field $\vec{E}$ of the incident wave depends on the coordinates and time: $\mathbf{E}(\boldsymbol{r},t)=\mathbf{E}^{(i)}(\boldsymbol{r})A_t(t)\exp(i\omega_c t)$. The direction and length of the vector $\mathbf{E}(\boldsymbol{r})$ in the layer is determined by its properties (1). The polarization of the incident wave is generally elliptical. Let us choose as a basis for the description of the polarization of the s- and p- wave.

$$\mathbf{E}^{(i)}(\boldsymbol{r},t)=\begin{pmatrix} E_p^{(i)}(\boldsymbol{r}) \\ E_s^{(i)}(\boldsymbol{r}) \end{pmatrix} A_t(t)\exp(i\omega_c t). \quad (3)$$

An optical signal of a special form is incident on a layer with torsion of the optical axis and dispersion. We consider cases where the signal envelope has the form of a) a triangular pulse, b) a Gaussian pulse, c) a rectangular pulse, and d) a laser giant pulse. Each of the envelope functions $A_t(t)$ has its own spectrum $S(\omega)$. The medium has a dispersion, so for different values of ω the conditions for the reflection of light will be different for the same angle θ_i. The optical properties of the medium depend on ω, according to formula (1) therefore the coefficients of the reflection matrix will also be functions of frequency:

$$\mathbf{R}(\omega,\theta_i)=\begin{pmatrix} R_{pp}(\omega,\theta_i) & R_{ps}(\omega,\theta_i) \\ R_{sp}(\omega,\theta_i) & R_{ss}(\omega,\theta_i) \end{pmatrix}. \tag{4}$$

As a result, the reflected impulse will have a modified form compared to the incident one. There is a relationship between the spectra of the incident and reflected pulses:

$$\begin{pmatrix} E_p^{(r)}(\omega,\theta_i) \\ E_s^{(r)}(\omega,\theta_i) \end{pmatrix}=\mathbf{R}(\omega,\theta_i)\begin{pmatrix} E_p^{(i)}\cdot S(\omega) \\ E_s^{(i)}\cdot S(\omega) \end{pmatrix}. \tag{5}$$

The amplitude of the reflected p-polarization wave consists of two components:

$$E_p^{(r)}(\omega,\theta_i)=S(\omega)R_{pp}(\omega,\theta_i)E_p^{(i)}+S(\omega)R_{ps}(\omega,\theta_i)E_s^{(i)}. \tag{6a}$$

Similarly for s- wave:

$$E_s^{(r)}(\omega,\theta_i)=S(\omega)R_{sp}(\omega,\theta_i)E_p^{(i)}+S(\omega)R_{ss}(\omega,\theta_i)E_s^{(i)}. \tag{6b}$$

The term in (6a) describes the spectrum of the p- polarization signal, which appears when a s- wave is incident and vice versa, the term $S(\omega)R_{sp}(\omega,\theta_i)E_p^{(i)}$ in (6b) describes signal with polarization s- arising from the reflection of p- polarization. The terms $S(\omega)R_{pp}(\omega,\theta_i)E_p^{(i)}$ in (6a) and $S(\omega)R_{sp}(\omega,\theta_i)E_p^{(i)}$ in (6b) are the spectra of the signals that retain their polarization. The dependences of these four waves on time were calculated using the inverse Fourier transform.

The torsion angle χ in the layer varies depending on the z coordinate:

$$\chi(z)=\chi_0+\frac{\Delta\chi}{d}z. \tag{7}$$

The reflection matrix (4) of an inhomogeneous anisotropic layer was calculated using the Cauchy matrix found in the framework of the Wentzel- Kramers- Brillouin method.

3. References

[1] Born, M. Principles of Optics / M. Born, E. Wolf. – Cambridge: Cambridge University Press, 1999. – 952 p.

[2] Jones, R.C. A New Calculus for the Treatment of Optical Systems. VII. Properties of the N-Matrices / R.C. Jones // J. Opt. Soc. Am. – 1948. –Vol. 38(8). – P. 671-685.

[3] Berreman, D.W. Optics in Stratified and Anisotropic Media: 4×4-Matrix Formulation / D.W. Berreman // J. Opt. Soc. Am. – 1972. – Vol. 62(4). – P. 502-510.

[4] Gooch, C.H. The optical properties of twisted nematic liquid crystal structures with twist angles ≤90 degrees / C.H. Gooch, H.A. Tarry // J. Phys. D: Appl. Phys. – 1975. – Vol. 8(13). – P. 1575.

[5] Yeh, P. Optics of Liquid Crystal Displays / P. Yeh, C. Gu. – New York: John Wiley & Sons, Inc. – 2009. – 792 p.

[6] Wöhler, H. Faster 4 × 4 matrix method for uniaxial inhomogeneous media / H. Wöhler, G. Haas, M. Fritsch, D.A. Mlynski // J. Opt. Soc. Am. A. – 1988. – Vol. 5(9). – P. 1554-1557.

[7] Khoo, I.-C. Liquid Crystals / I.-C. Khoo. – New York: John Wiley & Sons, Inc., 2007. – 384 p.

[8] Yakovlev, D.A. Modeling and Optimization of LCD Optical Performance / D.A. Yakovlev, Vl.G. Chigrinov, H.-S. Kwok // Chichester: John Wiley & Sons Ltd. – 2015. – 584 p.

[9] Obayya, S. Computational Liquid Crystal Photonics: Fundamentals, Modelling and Applications / S. Obayya, M.F.O. Hameed, N.F.F. Areed. – Chichester: John Wiley & Sons Ltd. – 2016. – 280 p.
[10] Wu, S.-T. Infrared markers for determining the order parameters of uniaxial liquid crystals / S.-T. Wu // Appl. Opt. – 1987. – Vol. 26(16). – P. 3434-3440.
[11] Arakelyan, S. M. Nelinejnaya optika zhidkih kristallov / S. M. Arakelyan, Y. S. Chilingaryan. – Moscow: Nauka. – 1984. – 360 p.
[12] Wasow, W. Asymptotic Expansion for Ordinary Differential Equations / W. Wasow. – New York: Dover Publication Inc. – 1987. – 365 p.
[13] Moiseeva, N.M. The matrix solution of the 4×4 problem by the Wentzel-Kramers-Brillouin method for a planar inhomogeneous anisotropic layer / N.M. Moiseeva, A.V. Moiseev // Computer Optics. – 2018. – Vol. 42(3). – P. 354-361. DOI: 10.18287/2412-6179-2018-42-3-354-361.