The optimal aircraft gas turbine engine control in low gas mode in the conditions of external additive noise

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Abstract. Currently, mathematical and computer modeling are the basis for the design of complex technical systems, which include aircraft structures, of course. Direct tasks for calculating the strength, flight, dynamic and other aircraft structures characteristics are widely described in the literature, methods for their solution are known. In aircraft engineering the inverse problem solution is primarily connected with strength characteristic calculation of aircraft structures and heat transfer problems. The inverse problems also include aircraft engines modes controlling tasks. Due to the fuel combustion disturbance, the engine is exposed to random disturbances in the form of Gaussian white noises in real live operation. At the bench test stage, not all system feedbacks can be accurately known. Thus, direct problems describing the engine operation are represented as stochastic differential equations systems with unknown parameters and additive white noises in the right-hand side. For such mathematical models, the inverse problem solution with respect to control parameters is made difficult by the need to estimate unknown parameters that can be realized in real conditions by measuring the phase characteristics of the system.

1. Introduction

Mathematical and computer simulation is the basis for aircraft engines design [1-3]. Due to the fuel combustion disturbance, the engine is exposed to random disturbances in the form of Gaussian white noises in real live operation. At the bench test stage, not all system feedbacks can be accurately known. Thus, direct problems describing the engine operation are represented as stochastic differential equations systems with unknown parameters and additive white noises in the right-hand side.

The tasks of operation modes control aircraft engines relate to inverse problems [4-9]. It is necessary to formulate the inverse problem and solve it regarding control parameters to determine the parameters of the engine control. The engine control parameters is described by the direct stochastic model However, due to the fact that some of the parameters are unknown, we will first have to evaluate their values based on system phase characteristics measurements. This greatly complicates the solution.

2. Problem formulation

Let the original linear dynamical system be described by differential equation system

$$\dot{x} = \phi x + Ab \qquad (t_0 \le t \le t_1);$$
$$x(t_0) = x_0; \tag{1}$$

Here *t*- is the time, $[t_0, t_1]$- the measurement interval, $x = (x_1, ..., x_n)$ - state vector system (1) with components $x_i = x_i(t)$; $x_0 = (x_{0_1}, ..., x_{0_n})$- given initial values vector of state vector, $\varphi = \|\varphi_{ij}(t)\|_{n \times n}$, $A = \|a_{ij}(t)\|_{n \times s}$ - given function matrixes; b=(b1,… bs), - system parameter vector.

It is necessary to calculate such parameter values b of system (1), so that the phase coordinates $x = (x_1, ..., x_n)$ is minimally different from some given reference state, as a result of the direct problem solution $x^{\bullet} = (x_1^{\bullet}, ... x_n^{\bullet})$, $x_i^{\bullet} = x_i^{\bullet}(t)$.

By random disturbances, system (1) is transformed into a stochastic:

$$\begin{aligned} &\overset{\bullet}{X} = \varphi X + Ab + \Delta \qquad (t_0 \le t \le t_1); \\ &X(t_0) = X^0; \end{aligned} \tag{2}$$

Here $\Delta = (\Delta_1, ..., \Delta_n)$ - is random uncorrelated noise column vector $\Delta_i = \Delta_i(t)$, affecting the system with zero expectation and given autocorrelation matrix $K_\Delta(t, t') = \|K_{\Delta ij}(t, t')\|_{n \times n}$, $X^0 = (X_1^{\ 0}, ..., X_n^{\ 0})$ - random initial state vector with expected value x_0 and a given correlation matrix K_0.

So, it is necessary to create a computational procedure for solving the inverse problem for the stochastic system (2) with respect to the parameters b. We organize the solution procedure in the form of an iterative step-by-step process.

The mathematical formulation of the inverse problem can be formulated as the following unconstrained optimization problem:

$$J = \int_{t_0}^{t_1} (x^k - x^{\bullet})^T G (x^k - x^{\bullet}) dt \Rightarrow \min \tag{3}$$

where xk –phase state of the system at the k-th iteration of the solution, G-symmetric matrix of positive coefficients.

Obviously, in the process of solving an inverse problem, the researcher may have varying degrees of awareness regarding the parameters values. Also, not all parameters should be adjusted in the process of solving the inverse problem. To reflect this circumstance, we present the parameter vector b in the form: b=(u,v,w, ϑ), and the system (1) at the k-th stage of solving the inverse problem is represented as:

$$\begin{aligned} &\dot{x}^k = \varphi x^k + Uu^k + Vv^k + Ww^k + \Theta \vartheta \qquad (t_0 \le t \le t_1); \\ &x^k(t_0) = x_0 \qquad\qquad (k = 0,1,...); \end{aligned} \tag{4}$$

Here the index k corresponds to the iteration number, $\varphi = \|\varphi_{ij}(t)\|_{nxn}$, $U = \|U_{ij}(t)\|_{nxm}$, $V = \|V_{ij}(t)\|_{nxr}$, $W = \|W_{ij}(t)\|_{nxq}$, $\Theta = \|\Theta_{ij}(t)\|_{nx\mu}$ - given function matrices $\varphi_{ij}(t)$, $U_{ij}(t)$, $V_{ij}(t)$, $W_{ij}(t)$, $\Theta_{ij}(t)$; $u^k = (u_1^{\ k}, ..., u_m^{\ k})$- adjustable parameter vector, the values u^0 of which are known at the initial (k = 0) stage of the corrections, and the final values are unknown; $v^k = (v_1^{\ k}, ..., v_r^{\ k})$ - adjustable parameter vector with unknown initial v^0 and final values; $w^k = (w_1^{\ k}, ..., w_q^{\ k})$ - adjustable parameter vector with unknown initial values w^0 and known final values $w^{\bullet}$; $\vartheta = (\vartheta_1, ..., \vartheta_\mu)$ - unknown and uncorrectable parameter vector.

Then the behavior of a real system with random disturbance affecting it at the kth stage the problem solution can be represented as:

$$\dot{X}^k = \varphi X^k + Uu^k + Vv^k + Ww^k + \Theta\vartheta + \Delta^k \quad (t_0 \le t \le t_1);$$
$$X^k(t_0) = X^{k0}, \qquad (k = 0,1,...); \tag{5}$$

For direct calculations and minimization of function (3), it is necessary to have an estimate of the initial values of those parameters which values were unknown before the calculations, so we have to have vector estimation $\theta^0 = (v^0, w^0, \vartheta)$. (5) This estimate can be obtained on the basis of system phase coordinates measurements (5). The measurement equation (meter) phase coordinates can be written as:

$$Z^k(t) = HX^k(t) + Y^k(t) \tag{6}$$

Here: $Z^k = (Z_1^k,..Z_i^k,..,Z_\rho^k)$- measurement results column vector with components $Z^k{}_i(t)$, $H = \|H_{ij}(t)\|_{\rho\times n}$ - given matrix with elements $H_{ij}(t)$, determining the measurement completeness of the state vector $X^k(t)$; $Y^k = (Y_1^k...,Y_\rho^k)$- random noise meter column vector with a given autocorrelation matrix $K_Y(t,t')$.

So, it is required to calculate (correct) the parameters of the linear dynamic system (4) on the basis of measurements (6) of the stochastic dynamic system (5) from functional minimum condition (3).
If f correction steps are required to achieve the functional minimum (3), then the state X^f will correspond to the values of the parameters $(u^f, v^f, w^\bullet, \vartheta)$.

3. Algorithm for solving the inverse problem

Extensive research was carried out to solve the problem [9-10], [12-15]. As a result, the following algorithm for solving the inverse problem was developed to select the dynamic system parameters:

1. For the initial form system (4) with measurements according to (5) - (6), the transition state matrix $\psi(t,t_0)$ is determined, which satisfies the differential equation system in the form:

$$\frac{d\psi(t,t_0)}{dt} = \varphi(t)\psi(t,t_0), \quad \psi(t_0,t_0) = E_{n\times n} \tag{7}$$

2. The functional matrix values are determined $U^*(t), V^*(t), W^*(t), \Theta^*(t)$ according to the formulas:

$$U^* = \int_{t_0}^{t}\psi(t,\tau)U(\tau)d\tau, \quad V^* = \int_{t_0}^{t}\psi(t,\tau)V(\tau)d\tau, \tag{8}$$

$$W^* = \int_{t_0}^{t}\psi(t,\tau)W(\tau)dt, \quad \Theta^* = \int_{t_0}^{t}\psi(t,\tau)\Theta(\tau)dt \tag{9}$$

3. The matrix-valued function $A(t)$ and the matrix $\overline{A}$ are calculated where is

$$A = (U^*|V^*), \quad \overline{A} = \int_{t_0}^{t_1} A^T \Gamma A dt, \tag{10}$$

G-symmetric matrix of positive coefficients.

4. The functional matrix elements are determined $V^{**}(t), W^{**}(t), \Theta^{**}(t)$ according to formulas:

$$V^{**} = HV^*, \; W^{**} = HW^*, \; \Theta^{**} = H\Theta^* \tag{11}$$

5. The matrix-valued function is formed $B(t)$:

$$B = (V^{**}, W^{**}, \Theta^{**}) \tag{12}$$

6. Disturbance process $\Delta_k^* = \Delta_k^*(t)$ correlation matrix $K^*{}_\Delta(t,\tau)$ is calculated by using the known correlation matrix K_0 и $K_\Delta(t',t'')$ and defined by the expression:

$$K_\Delta^*(t,\tau) = \psi(t,t_0)K_0\psi^T(\tau,t_0) + \int_{t_0}^{t_1}\int_{t_0}^{t_1}\psi(t,t')K_\Delta(t',t'')\psi^T(\tau,t'')dt'dt'' \tag{13}$$

7. The correlation matrix $K_{t\tau}$ of disturbance meter process is calculated $Y_k^*(t)$:

$$K_{t\tau} = H(t)K_\Delta^*(t,\tau)H^T(\tau) + K_Y(t,\tau) \quad (14)$$

8. The integral equations system is solved

$$\int_{t_0}^{t_1} f^*(t)\mathrm{B}dt = E_{g\times g}, \quad f^*(t) - \Lambda^* \int_{t_0}^{t_1} f^*(\tau)K_{t\tau}d\tau = \mathrm{B}^T \quad (15)$$

relatively to unknown functions $f^*(t)$ for effective, unbiased and stable unknown parameters values vector estimation $\theta^0 = (v^0, w^0, \vartheta)$.

9. Based on the obtained measurements $Z^k(t)$, the unknown parameters vector $\theta^0 = (v^0, w^0, \vartheta)$ is calculated in the 0-th approximation:

$$(\tilde{v}^{0k-1}, \tilde{w}^{0k-1}, \tilde{\vartheta}^{k-1}) = \int_{t_0}^{t_1} f^* \bar{Z}^{k-1^*} dt \quad (16)$$

$$\bar{Z}^{k-1^*} = \frac{1}{k}\sum_{j=0}^{k-1} Z^{j^*}. \quad (17)$$

As a result, when $k = 1$, we obtain the estimates $(\tilde{v}^{00}, \tilde{w}^{00}, \tilde{\vartheta}^0)$.

10. Estimates of $(\tilde{u}^{fk-1}, \tilde{v}^{fk-1})$ adjusted parameters u^f, v^f are figured out in accordance with the formula:

$$(\tilde{u}^{fk-1}, \tilde{v}^{fk-1}) = \bar{\mathrm{A}}^{-1}\int_{t_0}^{t_1} \mathrm{A}^T\Gamma(x^\bullet - \psi(t,t_0)x_0 - W^*w^\bullet - \Theta^*\tilde{\vartheta}^{k-1})dt \quad (18)$$

If $k = 1$, estimates $(\tilde{u}^{f0}, \tilde{v}^{f0})$ are determined in the zero approximation.

11. Corrective additives are calculated $\delta u^0, \delta v^0, \delta w^0$:

$$\delta u^{k-1} = \tilde{u}^{fk-1} - u^0, \delta v^{k-1} = \tilde{v}^{fk-1} - \tilde{v}^{0k-1}, \delta w^{k-1} = w^\bullet - \tilde{w}^{0k-1}, \ (k=1) \quad (19)$$

12. The system is corrected when $k = 1$, due to the addition increment correction δu^0, δv^0 и δw^0, and to the parameter u^0 and unknown parameter estimations v^0 и w^0:

$$u^k = u^0 + \delta u^{k-1}, v^k = v^0 + \delta v^{k-1}, w^k = w^0 + \delta w^{k-1} \quad (20)$$

13. Based on previous process state estimation x^k, corresponding to parameter values u^k, v^k, w^k paragraphs 9-13 are repeated when $k = k+1$ and subsequent approximation to the adjusted state is determined x^f, corresponding to the adjusted parameters $u^f, v^f, w^\bullet, \vartheta$.

This process is repeated until the desired corrected state is obtained with a given accuracy.

4. Conclusions

The developed algorithm allows us to effectively solve inverse problems for dynamic stochastic systems with unknown parameters by measuring their phase characteristics. The performed computational experiments have proved the applicability of the developed algorithm for the control of complex technical systems in a perturbed environment.

5. References

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