The lifetime of the entangled states of interacting qubits in external fields and the thermostat calculated by path integral approach

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Abstract. In this paper we study the quantum entanglement of two identical qubits which interact with external electromagnetic fields and the thermostat and each other by time-dependent dipole-dipole interaction. We develop original method in path integral approach for numerical calculation of density matrix and Peres-Horodecki parameter (the measure of qubits entanglement). The results indicate on possibility of high-entanglement states exiting and long-time non-destructive control of them.

1. Introduction

Quantum entanglement has been a subject of intense theoretical and experimental studies during the last decades because of its significance in understanding quantum mechanics and promising applications. Fundamental principles and basic methods of creating and detecting entangled states in different quantum systems can be found, e.g., in the review [1]. Thus, based on the review of modern papers, we conclude that the influence of the external electromagnetic radiation makes it possible to achieve high values of the degree of quantum entanglement of qubits state. The dipole-dipole interaction between qubits is a promising tool for management of quantum entanglement degree. However, in the proposed models, the high degree of entangled state is maintained for a limited time. There is the problem of creating a model of two identical dipole-dipole interacting qubits which interact with a coherent electromagnetic field. This theoretical model will actially predict a high lifetime of qubits entangled state, it is necessary to carry out quantum manipulation.

2. Theory

We consider interaction of two multilevel quantum systems (such as molecules, atoms or superconducting qubits) with electromagnetic field. The Hamiltonian is given as $\hat{H}_{full} = \hat{H}_{syst} + \hat{V}(\tau)$.

Our main goal is to define the density matrix $\rho(n_f\,, m_f\,, t|n_{in}, m_{in}, 0)$ of investigated quantum systems transition from eigenstate $|n_{in}, m_{in}\rangle$ at the moment $t = 0$ to the eigenstate $|n_f\,, m_f\rangle$at the moment $t > 0$. We describe the investigated system by statistical operator $\hat{\rho}(t)$. The evolution equation of $\hat{\rho}(t)$ in Dirac (interaction) picture is as follows:

$$\hat{\rho}(t) = \hat{U}_D(t)\hat{\rho}(0)\hat{U}_D^+(t), \quad \text{where} \quad \hat{U}_D(t) = \hat{T}\exp\left[-\frac{\imath}{\hbar}\int\limits_0^t \hat{V}_D(\tau)d\tau\right] \tag{1}$$

Eq. (5) in energy representation is

$$\rho_{n_f m_f n'_f m'_f}(t) = \sum_{n_{in},n'_{in}=0}^{N-1} \sum_{m_{in},m'_{in}=0}^{M-1} \langle n_f, m_f|\hat{U}_D(t)|n_{in}, m_{in}\rangle \rho_{n_{in}m_{in}n'_{in}m'_{in}}(0)\langle n'_{in}m'_{in}|\hat{U}_D^+(t)|n'_f m'_f\rangle, \tag{2}$$

We proved [2], that the density matrix Eq. (6) can be expressed as the path integral in energy eigenstates space

$$\begin{gathered}\rho_{n_f m_f n'_f m'_f}(t) = \lim_{K\to\infty} \sum_{n'_{in},n_{in},\ldots,n'_K,n_K=0}^{N-1} \sum_{m'_{in},m_{in},\ldots,m'_K,m_K=0}^{M-1} \\ \int_0^1 .. \int_0^1 d\xi_0..d\xi_K d\zeta_0..d\zeta_K d\xi'_0..d\xi'_K d\zeta'_0..d\zeta'_K \rho_{m_{in}n_{in}m'_{in}n'_{in}} \\ \exp\left[\imath\left(S[n_f, m_f, n_K, m_K, \xi_K, \zeta_K; \ldots; n_1, m_1, n_{in}, m_{in}, \xi_0, \zeta_0] - \right.\right. \\ \left.\left. S[n'_f, m'_f, n'_K, m'_K, \xi'_K, \zeta'_K; \ldots; n'_1, m'_1, n'_{in}, m'_{in}, \xi'_0, \zeta'_0]\right)\right]\end{gathered} \tag{3}$$

where $S[n_f, m_f, n_K, m_K, \xi_K, \zeta_K; \ldots; n_1, m_1, n_{in}, m_{in}, \xi_0, \zeta_0] = \sum_{k=1}^{K+1} S[l_k, n_k, l_{k-1}, n_{k-1}, \xi_{k-1}, \zeta_{k-1}]$ and dimensionless action $S[n_k, m_k, n_{k-1}, m_{k-1}; \xi_{k-1}, \zeta_{k-1}]$ in $\hbar$ units is

$$\begin{gathered}S[n_k, m_k, n_{k-1}, m_{k-1}; \xi_{k-1}, \zeta_{k-1}] = 2\pi(n_k - n_{k-1})\xi_{k-1} + 2\pi(m_k - m_{k-1})\zeta_{k-1} - \\ -\frac{1}{\hbar}V_{n_k m_k n_{k-1} m_{k-1}}(\tilde{t}_k) 2\cos[2\pi(n_k - n_{k-1})\xi_{k-1} + 2\pi(m_k - m_{k-1})\zeta_{k-1} - \\ - \left(\omega_{n_k n_{k-1}} + \omega_{m_k m_{k-1}}\right)\tilde{t}_k]\Delta t_k,\end{gathered} \tag{4}$$

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where $S[n_f, m_f, n_K, m_K, \xi_K, \zeta_K; \ldots; n_1, m_1, n_{in}, m_{in}, \xi_0, \zeta_0] = \sum_{k=1}^{K+1} S[l_k, n_k, l_{k-1}, n_{k-1}, \xi_{k-1}, \zeta_{k-1}]$ and dimensionless action $S[n_k, m_k, n_{k-1}, m_{k-1}; \xi_{k-1}, \zeta_{k-1}]$ in $\hbar$ units is

$$S[n_k, m_k, n_{k-1}, m_{k-1}; \xi_{k-1}, \zeta_{k-1}] = 2\pi(n_k - n_{k-1})\xi_{k-1} + 2\pi(m_k - m_{k-1})\zeta_{k-1} - \frac{1}{\hbar} V_{n_k m_k n_{k-1} m_{k-1}}(\tilde{t}_k) 2\cos[2\pi(n_k - n_{k-1})\xi_{k-1} + 2\pi(m_k - m_{k-1})\zeta_{k-1} - \left(\omega_{n_k n_{k-1}} + \omega_{m_k m_{k-1}}\right)\tilde{t}_k]\Delta t_k, \quad (8)$$

3. The heat bath influence model

We consider two identical qubits which interact with electromagnetic field and each other by dipole-dipole interaction. Hamiltonian of this model has the following form:

$$\hat{H}_{full} = \hat{H}_Q + \hat{V}(\tau) \qquad \text{and} \qquad \hat{V}(\tau) = \hat{V}_{QF}(\tau) + \hat{V}_{QQ}(\tau) + \hat{V}_{QH}(\tau) \quad (9)$$

where $\hat{H}_Q$ is hamiltonian of two non-interacting qubits with eigenenergies $E_{nm} = E_n + E_m$ and eigenvectors $|n, m\rangle$. The numbers of qubits states are $N = 0, 1$ and $M = 0, 1$. $\hat{V}(\tau)$ is interaction operator.

Here $\hat{V}_{QF}(\tau)$ is operator which describe interaction between qubits and one mode electromagnetic field:

$$\hat{V}_{QF}(\tau) = \hbar\Omega_R \left[(\hat{\sigma}_1^+ + \hat{\sigma}_1^-)\cos(\omega\tau + \phi_1) + (\hat{\sigma}_2^+ + \hat{\sigma}_2^-)\cos(\omega\tau + \phi_2)\right], \quad (10)$$

where Rabi frequency $\Omega_R = dE_0/\hbar$, d is qubits dipole moment, E_0 is electric field amplitude, $\sigma_1^+, \sigma_1^-, \sigma_2^+, \sigma_2^-$ are ladder operators of first and second qubit, ω is electromagnetic field frequency, $\phi_1 = kx_1, \phi_2 = kx_2$ are phases of electric field harmonic oscillations in coordinates x_1 and x_2 of first and second qubits, $k = \omega/c$ is wavenumber of electromagnetic field. $\hat{V}_{QQ}(\tau)$ is operator wich descibe dipole-dipole interaction between qubits:

$$\hat{V}_{QQ}(\tau) = \hbar\tilde{\Omega}_{QQ}(\tau)\left(\hat{\sigma}_1^-\hat{\sigma}_2^+ + \hat{\sigma}_1^+\hat{\sigma}_2^-\right), \quad \text{where} \quad \tilde{\Omega}_{QQ}(\tau) = \Omega_{QQ}\cos\left(\omega_{QQ}\tau - \phi_1\right)\cos\left(\omega_{QQ}\tau - \phi_2\right) \quad (11)$$

where Ω_{QQ} and ω_{QQ} are dipole-dipole interaction amplitude and frequency respectively.

Operator $\hat{V}_{QH}(\tau)$ descibe the interaction of qubits with a thermostat:

$$\hat{V}_{QH}(\tau) = \hbar\Omega_H(T)\left[(\hat{\sigma}_1^+ + \hat{\sigma}_1^-)\cos(\omega\tau + \alpha_1) + (\hat{\sigma}_2^+ + \hat{\sigma}_2^-)\cos(\omega\tau + \alpha_2)\right] \quad (12)$$

where

$$\Omega_H(T) = \frac{d}{\hbar}\left(\frac{\hbar\omega}{\varepsilon_0 V\left(e^{\hbar\omega/kT} - 1\right)}\right)^{1/2} \quad (13)$$

where T is temperature, α_1 and α_2 are random phases of heat field. Qubits are influenced by one mode of the thermostat field. Its frequency ω coincides with the frequency of the quantum transition of qubits. Other modes of the electromagnetic field of a thermostat have little effect on quantum transition of qubits, so they are not taken into account. The parameter $\Omega_H(T)$ characterized the interaction of qubits with a thermostat.

4. Results of numerical simulation

We will make numerical calculations of the Peres-Horodecki parameter in the proposed model depending on the time and temperature of the thermostat. The interaction parameters of the qubits, the external electromagnetic field are taken the same as in the work [3]. We will make out numerical calculations of the Peres-Horodecki parameter for cases when the temperature of the thermostat has values $T_1 < T_2 < T_3$ and, accordingly, $\Omega_H(T_1) = \Omega_R$, $\Omega_H(T_2) = 5\Omega_R$, $\Omega_H(T_3) = 10\Omega_R$.
The results of the calculations are presented in three graphs in Fig.1.

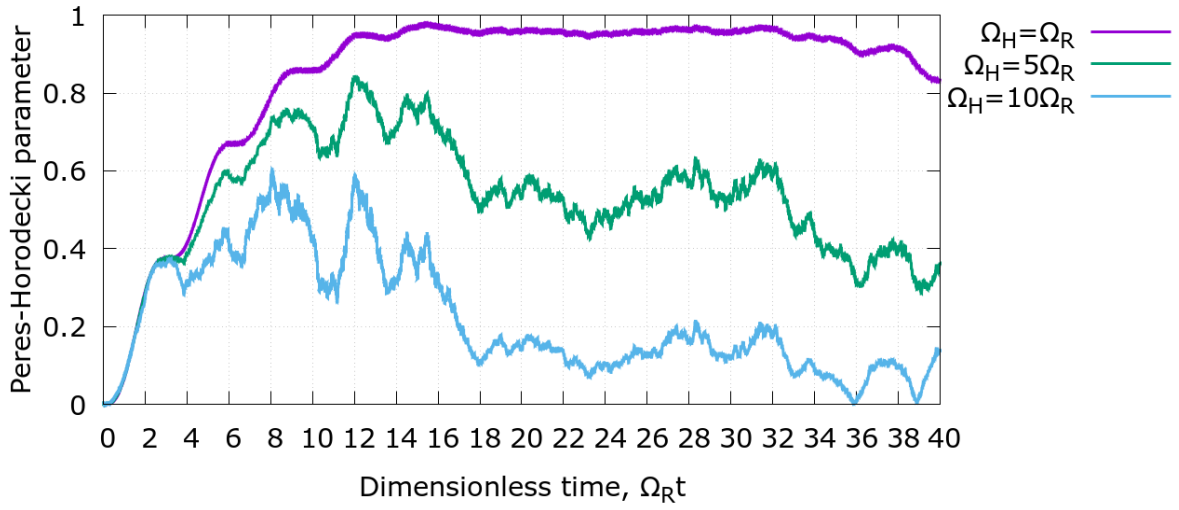

Figure 1. Graphs of the dependence of the Perese-Horodecki parameter of the qubits on the time of their interaction with an external electromagnetic field and a thermostat. The graphs correspond to different values of the thermostat temperature.

The first graph represents the dependence of the Perese-Horodecki parameter on time in the interaction of qubits with a thermostat at a temperature T_1. It shows that the value of the parameter deviates from the unit very little.
The second and third plots plotted for temperatures T_2 and T_3 show a significant decrease in the parameter. A thermostat with a high temperature destroys the entanglement of qubits. This confirms the experiment.

5. Conclusion

In the path integral approach we describe the entanglement of two identical qubits model by the use of Peres-Horodecki parameter. This model include interacion with laser driven field and heat bath. We also simulate a variable dipole-dipole interaction between qubits. The results of the numerical experiments indicate that the interaction of qubits with the thermal field has a significant impact on the dynamics of entanglement. However, when the intensity of interaction

(Rabi frequency) with the thermal field does not exceed the intensity of interaction with the driven laser field, the effect of thermal noise can be neglected.

6. References

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