The effect of climber initial velocity on orbital space elevator dynamics

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Abstract. This research is devoted to the problem of delivering a small payload to the Earth from the orbit using an orbital space elevator, which consists of two space stations connected by a massless tether and a passive climber moving along the tether. The aim of the work is to study the possibility of transferring the climber from one end of the orbital elevator to the other end due to its initial velocity. The mathematical model of the mechanical system is constructed using the Lagrange equation of the second kind. An analytical expression, which depends on the parameters of the mechanical system and determines the required initial velocity, was found using a simplified mathematical model that does not take into account the bending of the tether. The validity of the expression was verified using the original mathematical model. The analysis of the system motion was carried out, and recommendations on its mass-geometric parameters choice were given.

1. Introduction

The task of delivering payload from orbit to Earth is of great practical importance for modern astronautics. Applying space tethers can reduce the cost of this transport operation by eliminating the use of jet fuel. Various schemes of using space tether systems for solving this problem are discussed in the scientific literature: momentum exchange tethers [1, 2], variable length tethers [3-5], electrodynamic tethers [6-8], tethered towing [9, 10], space and orbital elevators[11-13]. An orbital space elevator, which consists of two space stations connected by a massless tether and a passive climber moving along the tether, is considered in this study. It is assumed that the center of mass of the system moves in a circular orbit. The payload is placed in a capsule-climber at the top station. Then the initial velocity is given to the capsule and it slides along the tether towards the lower station. Near the lower station, the capsule is decelerated and detached from the tether, after which it moves to the descent orbit. The motion of the capsule occurs in a passive mode under the action of gravitational forces, inertial forces, and reaction forces from the tether. The process physics is described in more detail in [14]. At the upper station, the centrifugal force of inertia exceeds the gravitational force. In this regard, the capsule's initial velocity can be not enough, and the climber, dropping to a certain height, will stop and then rise again to the upper end of the tether system. The aim of the work is to study the possibility of transferring the climber from one end of the orbital elevator to the other end due to its initial velocity. A mathematical model of the mechanical system will be constructed and an approximate analytical expression for determining the required initial velocity will be obtained.
The paper consists of four sections. The second section is devoted to the development of a mathematical model and obtaining an analytical expression to determine the initial climber velocity required for the transport operation. The third section presents the results of numerical simulation in

order to verify the found analytical expression. The last section gives the main conclusions of the work.

2. Mathematical model

The orbital elevator is a system, which consist of two space stations (points A and B on figure 1) connected by a weightless inextensible tether, the length of which is l. The climber (point D on figure 1) moves along the tether. Space stations and the climber are considered as material points that have masses m_A, m_B and m_D respectively. The center of mass of the orbital elevator moves in a circular orbit, whose radius is r, with an angular velocity Ω. It is assumed that the mass of the climber is many times less than the mass of the stations. The state of this mechanical system can be described by two generalized coordinates $\mathbf{q} = (\mathrm{s}, \varphi)$, where s is the distance from the upper station to the climber, φ is the angle of deviation of the tether from the local vertical.

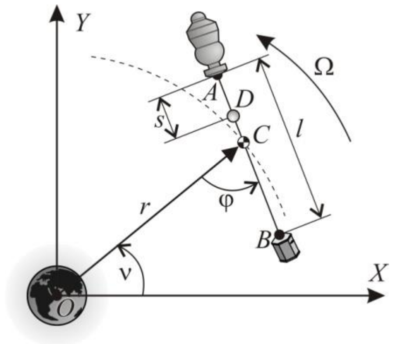

Figure 1. Orbital space elevator.

The equations of motion of the orbital elevator can be obtained using the Lagrange equations of the second kind, which have the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0, \tag{1}$$

where q_j is generalized coordinate, $L = T - W$ is the Lagrange function. In order to find the Lagrange function, we write the expressions for the kinetic T and potential W energy of the system.

$$T = \frac{m_A}{2}\left(\dot{x}_A^2 + \dot{y}_A^2\right) + \frac{m_B}{2}\left(\dot{x}_B^2 + \dot{y}_B^2\right) + \frac{m_D}{2}\left(\dot{x}_D^2 + \dot{y}_D^2\right), \tag{2}$$

$$W = -\mu\left(\frac{m_A}{\sqrt{R_A}} + \frac{m_B}{\sqrt{R_B}} + \frac{m_D}{\sqrt{R_D}}\right), \tag{3}$$

where μ is the gravitational constant of Earth, x_j, y_j are coordinates of points A, B, D in the inertial coordinate system OXY (figure 1)

$$x_A = r\cos\nu + \frac{m_B l\cos(\varphi+\nu)}{m_A + m_B}, \quad y_A = r\sin\nu + \frac{m_B l\sin(\varphi+\nu)}{m_A + m_B},$$

$$x_B = r\cos\nu - \frac{m_A l\cos(\varphi+\nu)}{m_A + m_B}, \quad y_B = r\sin\nu - \frac{m_A l\sin(\varphi+\nu)}{m_A + m_B},$$

$$x_A = x_A - s\cos(\varphi+\nu), \qquad y_A = y_A - s\sin(\varphi+\nu),$$

$\nu = \Omega t$, R_j is the distance from the center of Earth to the j-th point

$$R_A = \frac{2lm_B r\cos\varphi}{\left(m_A + m_B\right)} + r^2 + \frac{l^2 m_B^2}{\left(m_A + m_B\right)^2},$$

$$R_B = -\frac{2l\,m_A r\cos\varphi}{\left(m_A + m_B\right)} + r^2 + \frac{l^2 m_A^2}{\left(m_A + m_B\right)^2},$$

$$R_D = \frac{2\left(\left(l-s\right)m_B - s\cdot m_A\right)r\cos\varphi}{\left(m_A + m_B\right)} + r^2 + \frac{\left(l-s\right)^2 m_B^2 - 2s\cdot m_A\left(l-s\right)m_B + m_A^2\cdot s^2}{m_B^2}.$$

Substituting expressions (2) and (3) into (1), we obtain a system of differential equations describing the motion of the considered mechanical system

$$\begin{gathered}\ddot{s}\,m_D + r\,\Omega^2 m_D\cos\varphi + \frac{\left(\left(l-s\right)m_B - s\,m_A\right)\left(\Omega+\dot{\varphi}\right)^2 m_D}{m_A+m_B} - \\ -\frac{\left(\left(m_A+m_B\right)r\cos\varphi + \left(l-s\right)m_B - s\,m_A\right)\mu\,m_D}{R_D^3\left(m_A+m_B\right)} = 0\end{gathered} \tag{4}$$

$$\begin{gathered}\frac{\ddot{\varphi}\left(\left(l^2 m_A + m_D\left(l-s\right)^2\right)m_B^2 + m_A\left(l^2 m_A - 2s\,m_D\left(l-s\right)\right)m_B + m_D m_A^2 s^2\right)}{\left(m_A+m_B\right)^2} + \\ +\frac{\left(\left(l-s\right)m_B - s\,m_A\right)m_D r\Omega^2\sin\varphi}{m_A+m_B} - \frac{2\dot{s}\left(\Omega+\dot{\varphi}\right)\left(\left(l-s\right)m_B - s\,m_A\right)m_D}{m_A+m_B} + \\ +\left(-\frac{m_A l\,m_B r\sin\varphi}{R_A^3\left(m_A+m_B\right)} + \frac{m_B l\,m_A r\sin\varphi}{R_B^3\left(m_A+m_B\right)} - \frac{m_D\left(\left(l-s\right)m_B - s\,m_A\right)r\sin\varphi}{R_D^3\left(m_A+m_B\right)}\right)\mu = 0\end{gathered} \tag{5}$$

Using the system of equations (4)-(5), let us construct the phase portrait $\dot{s}(s)$ for the initial conditions $\varphi = 0$ and $\dot{\varphi} = 0$. As can be seen from figure 2, an equilibrium point of the saddle type s^* is present on the phase portrait. A separatrix passing through it breaks the phase space into isolated areas. If the initial velocity of the climber-capsule corresponds to a point in area I on figure 2, then the capsule cannot reach the lower end of the tether. The centrifugal force of inertia will stop the capsule until it passes through the center of mass of the space tethered system, and then will return the capsule to the upper station. In order for the capsule to reach the lower station, it is necessary that its phase trajectory be in zone II. Let us find the initial velocity of the capsule corresponding to the motion along the separatrix ($\dot{s}_0$ on figure 2). For this we find the equilibrium position s^*.

In order to find an equilibrium position, let us equate to zero velocity and acceleration in the equations (4), (5): $\dot{\varphi} = \ddot{\varphi} = 0,\ \dot{s} = \ddot{s} = 0$. As result we obtain

$$s^* = \frac{l\,m_B}{m_A+m_B}, \quad \varphi^* = 0. \tag{6}$$

The value s^* corresponds to the position of the center of mass of the space tether system.

To obtain the equations of the phase trajectory, we integrate the expression (4) for $\varphi = 0,\ \dot{\varphi} = 0$

$$\frac{\dot{s}^2}{2} = \int F\left(s\right)ds + C,$$

where

$$\begin{gathered}F\left(s\right) = -r\,\Omega^2 - \frac{\left(\left(l-s\right)m_B - s\,m_A\right)\Omega^2}{m_A+m_B} + \\ +\frac{\left(\left(m_A+m_B\right)r + \left(l-s\right)m_B - s\,m_A\right)\mu}{\left(\frac{2\left(\left(l-s\right)m_B - s\cdot m_A\right)r}{\left(m_A+m_B\right)} + r^2 + \frac{\left(l-s\right)^2 m_B^2 - 2s m_A\left(l-s\right)m_B + m_A^2\cdot s^2}{m_B^2}\right)^{3/2}\left(m_A+m_B\right)}\end{gathered}$$

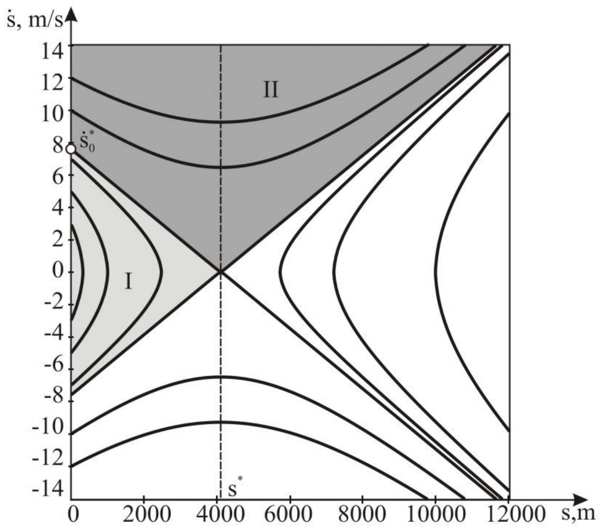

Figure 2. Phase portrait.

After integration, we obtain the equation of the phase trajectory in the form

$$\dot{s}=\sqrt{2\left(-r\,\Omega^2 s-\frac{1}{2}\frac{\Omega^2 s\left(2l\,m_B-m_A s-m_B s\right)}{m_A+m_B}+\frac{\mu\left(m_A+m_B\right)}{\left(l\,m_B+m_A r-m_A s+m_B r-m_B s\right)}+C\right)}, \tag{7}$$

where C is the integration constant. In order to determine the initial velocity corresponding to the stationary value s^*, we find constant C , substituting $s=s^*,\dot{s}=0$ in the equation (7). After substituting this value into equation (7) and the taking $s=0$, $\Omega=\sqrt{\mu r^{-3}}$, we obtain the desired value of the initial velocity:

$$\dot{s}_0^*=\frac{m_B l}{\left(m_A+m_B\right)}\sqrt{\frac{\mu\left(lm_B+3r\left(m_A+m_B\right)\right)}{r^3\left(lm_B+r\left(m_A+m_B\right)\right)}} \tag{8}$$

Analysis of equation (8) shows that the initial velocity depends on the length of the tether, the masses of the stations and the radius of the orbit of the orbital elevator. The most obvious recommendation for choosing system parameters is to use a heavy upper station and a light lower one. In this case, the required velocity will decrease.

Thus, in order for the capsule to reach the lower end of the tether system at a fixed position $\varphi=0$, its initial velocity must satisfy the condition

$$\dot{s}_0>\dot{s}_0^*. \tag{9}$$

3. Results of numerical simulation

To verify the found condition (9), we perform the numerical integration of a system of equations (4), (5) with various initial conditions. The considered mechanical system has the following parameters: $r=6731000{+}300000$ m, $l=30000$ m, $m_A=6300$ kg, $m_B=1000$ kg, $m_D=100$ kg. For this parameters expression (8) gives $\dot{s}_0^*=7.622\text{m/s}$. Figures 3, 4 shows results of numerical integration for various initial velocities. It is seen that at low velocities the capsule cannot overcome the position of the center of mass and returns to the upper station.

Figure 4 shows that the movement of the capsule causes a buildup of the space cable system, but the amplitude of oscillations is insignificant. For example, in the case of $\dot{s}_0=10\text{m/s}$ the angle φ amplitude does not exceed 0.01 rad.

Thus, the results of numerical integration showed that the analytical expression (8) found is in good agreement with the results of numerical calculations.

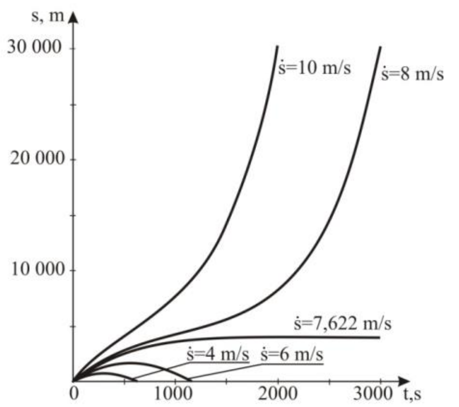

Figure 3. Dependence of the climber position on time.

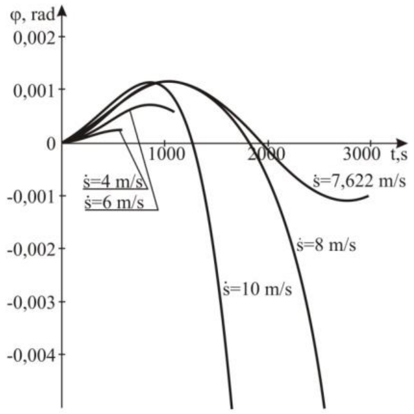

Figure 4. Dependence of the tether deflection angle on time.

4. Conclusion

The problem of the descent of payload from orbit using an orbital space elevator was considered in this study. The mathematical model of the orbital space elevator was constructed using the Lagrange equation of the second kind. An analytical expression, which depends on the parameters of the mechanical system and determines the required initial velocity, was found using a simplified mathematical model. The validity of the expression was confirmed by the results of the numerical integration.

5. References

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