The contribution of one-meson interaction to fine and hyperfine structure of muonic hydrogen

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Abstract. We study the interaction between the muon and proton in muonic hydrogen induced by the effective exchange of axial vector, scalar, pseudoscalar and tensor mesons in the light-by-light scattering. The transition form factor $\gamma + \gamma \to meson$ is taken in the monopole form over each photon momentum squared which is usually used in the analysis of corresponding experimental date. Numerical contributions of meson interaction to fine and hyperfine structure are obtained.

1. Introduction

The processes of one meson production and decay play an essential role among different interactions of mesons. They have been studied experimentally for quite a long time, and a rich experimental data have been accumulated [1]. With the development of the quark model and nonperturbative methods of quantum chromodynamics, such reactions were constantly in the field of intensive theoretical studies.

A new round of interest in $\gamma + \gamma \to meson$ processes is connected with their possible role as a new source of interactions between leptons and nucleons. Since in atomic physics there are precise experiments to measure the fine and hyperfine structure of the spectrum, any new contributions to the particle interaction operator are of interest and can be studied experimentally. Our first estimates of the contribution of effective meson exchanges in muonic hydrogen show that this contribution can be significant [2, 3]. In this work we study the role of effective exchanges of different mesons for the position of the energy levels in muonic hydrogen.

2. General formalism

The amplitude of the one-meson exchange between the muon and the proton arises as a result of the transition of two virtual photons into exchanged meson. The vertex function describing this process plays a central role in the study of meson exchange, since a good prediction of the magnitude of the shift in energy levels depends primarily on it. The general parameterization of $meson \to \gamma + \gamma$ vertex function is different for the mesons of different spin. In the case of scalar meson it takes the form:

$$T_S^{\mu\nu}(t, k_1, k_2) = 4\pi\alpha \Big\{ A(t^2, k_1^2, k_2^2)(g^{\mu\nu}(k_1 \cdot k_2) - k_1^\nu k_2^\mu) + \tag{1}$$

$$B(t^2,k_1^2,k_2^2)(k_2^\mu k_1^2-k_1^\mu(k_1\cdot k_2))(k_1^\nu k_2^2-k_2^\nu(k_1\cdot k_2))\Big\},$$

where $A(t^2,k_1^2,k_2^2)$, $B(t^2,k_1^2,k_2^2)$ are two scalar functions on three variables, $k_{1,2}$ are four momenta of virtual photons, t is the four momentum of scalar meson.

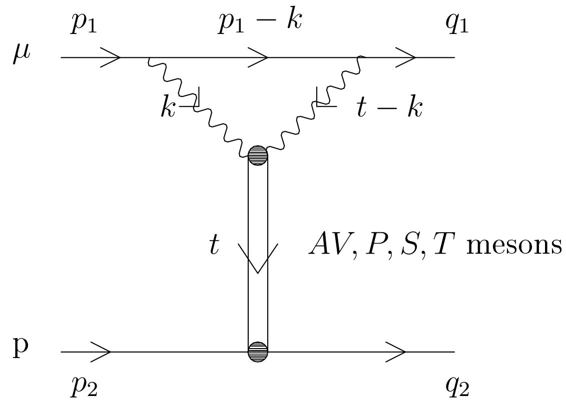

Figure 1. Muon-proton interaction induced by meson exchange.

Then the muon-proton interaction amplitude via the scalar meson exchange can be written as follows:

$$i\mathcal{M}=\frac{\alpha^2 g_s}{\pi^2}\int\frac{d^4kA(t^2,k^2,k^2)(g^{\mu\nu}(k_1\cdot k_2)-k_1^\nu k_2^\mu)[\bar u(q_1)\gamma_\mu(\hat p_1-\hat k+m_1)\gamma_\nu u(p_1)][\bar v(p_2)v(q_2)]}{k^4(k^2-2m_1k_0)(\mathbf{t}^2+M_s^2)},\tag{2}$$

where p_1, p_2 are four momenta of particles in initial state, q_1, q_2 are four momenta of particles in final state. We set $t=q_1-p_1=0$ because this momentum is small and we consider further the leading order in fine structure constant α contribution to the interaction operator. This leads to the cancellation of the term with the function $B(p^2,k_1^2,k_2^2)$. Using projection operator on muon-proton states with spin S=0, S=1 [4, 5, 6] we can construct the interaction operator for these states. For example, in the case of triplet state we find:

$$i\mathcal{M}(^3S_1)=\frac{\alpha^2 g_s}{16m_1^2m_2^2\pi^2}\int\frac{d^4}{k^4(k^2-2k_0m_1)}A(t^2,k^2,k^2)(g_{\mu\nu}k^2-k_\mu k_\nu)\tag{3}$$

$$Tr\Big[\frac{1+\gamma_0}{2\sqrt2}\hat\varepsilon(\hat p_2-m_2)(\hat q_2-m_2)\hat\varepsilon^*\frac{1+\gamma_0}{2\sqrt2}(\hat q_1+m_1)\gamma_\mu(\hat p_1-\hat k+m_1)\gamma_\nu(\hat p_1+m_1)\Big]\frac{1}{\mathbf{t}^2+M_s^2},$$

where m_1, m_2 are the masses of a muon and proton, M_S is the mass of scalar meson. After the trace calculation using the package Form we obtain in the leading order:

$$\mathcal{T}_{2S}=k^2(3k_0+2m_1)-2m_1k_0^2.\tag{4}$$

The same contribution occurs for the singlet state $^1S_0$, so in the leading order the scalar meson exchange doesn't contribute to hyperfine structure. At the same time there is a shift of the level 2S as whole which is determined by (4). The typical momentum integral contributing to the shift has the following form:

$$\mathcal{I}_1=\int\frac{d^4(k^2+2k_0^2)}{k^2(k^4+a_1^2k_0^2)}\frac{1}{(k^2+1)^2}=\frac{\pi}{24}\left[-9+36\ln2+2a_1^2(-7+12\ln2)-12(3+2a_1^2)\ln a_1\right],\tag{5}$$

where $a_1 = 2m_1/\Lambda$. An analytical value $\mathcal{I}_1$ is presented after an expansion over $2m_1/\Lambda$ up to terms of the second order. Such integral appears if we suppose that the parameterization of function $A(t^2, k^2, k^2)$ for scalar mesons has the same form as for pseudoscalar and axial-vector mesons (monopole form for variables k_1^2 and k_2^2):

$$A(t^2, k^2, k^2) = A_S \frac{\Lambda^4}{(\Lambda^2 + k^2)^2}, \tag{6}$$

where $A_S = A(0, 0, 0)$. The parameterization of the form (6) is widely used in the analysis of different experimental data. The contribution to interaction potential takes the form:

$$\Delta V_{2S}^{Ls}(t) = \frac{\alpha^2 g_s m_1 A_S}{6} \Big[-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1 \Big] \frac{1}{\mathbf{t}^2 + M_s^2}. \tag{7}$$

Averaging (7) over wave function of 2S-state we obtain the shift of 2S-level in the form:

$$\Delta E^{Ls}(2S) = \frac{\alpha^5 \mu^3 g_s m_1 A_S}{96\pi M_s^2} \frac{(2 + \frac{W^2}{M_S^2})}{(1 + \frac{W}{M_S})^4} \Big[-9 + 36 \ln 2 + 2a_1^2(-7 + 12 \ln 2) - 12(3 + 2a_1^2) \ln a_1 \Big]. \tag{8}$$

An analytical expression (8) is used below for numerical estimate of the contribution. To do this, you need to have reliable parameter values, including A_S.

3. Conclusion

Numerical estimate of contributions of mesons with different spins to the muon-proton interaction potential are obtained in the case of S- and P-states. After averaging these potentials over the wave functions we find corresponding contributions to the Lamb shift (2P-2S) and hyperfine structure. They are obtained on the basis of equations (6)-(8) taking the values of parameters for corresponding exchanged mesons. The obtained contributions to the Lamb shift (2P-2S) in muonic hydrogen are large and should be used for precise comparison with experimental data of CREMA collaboration.

4. References

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