The New Algorithm for the Determination of the Williams Asymptotic Expansion Coefficients for Notched Semidiscs Using the Photoelasticity Method and Finite Element Method

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Abstract. The study proposes the algorithm for the determination of the coefficients of the Williams series expansion in notched semidisks with different angles of the notch. The algorithm is based on the experimental procedure of the photoelasticity method and the finite element analysis. The large series of experiments for semidiscs was performed. Digital photoelasticity method is used to analyse experimentally the complete Williams series expansion of the stress and displacement fields in the vicinity of the crack tip in isotropic linear elastic plates under Mixed Mode loading. The distribution of the isochromatic fridge patterns is employed for obtaining the stress field near the crack tip by the use of the complete Williams asymptotic expansion for various classes of the experimental specimens (plates with two collinear cracks under tensile loading and under mixed mode loading conditions). The higher order terms of the Williams series expansion are taken into account and the coefficients of the higher order terms are experimentally obtained. The stress field equation of Williams up to fifty terms in each in mode I and mode II has been considered. The comparison of the experimental results and the calculations performed with finite element analysis has shown the importance and significant advantages of photoelastic observations for the multi-parameter description of the stress field in the neighborhood of the crack tip.

1. Introduction

Defects like cracks, sharp notches and various inclusions play a crucial role in fracture of structural components [1]. These defects can reduce the strength and the following loading can lead to the growth of cracks. The presence of cracks alters the stress and strain fields near the crack tip. In fracture mechanics, the stress intensity factor (SIF) expresses the strength of singular elastic stress field and they also characterize the near-tip stress field. SIF is a function of applied load, crack length and geometry of the cracked body. The critical SIF value will decide the propagation of crack and the resulting failure under service load. SIF can be estimated using analytical, numerical, or experimental techniques. However, analytical methods are restricted to simple configurations and boundary conditions. For complex configurations, SIF needs to be extracted using either numerical or experimental method [1]. Moreover, many works [1-6] show the importance of T-stresses in the crack-tip stress field characterization. In this work experimental evaluation of SIF and T-stresses using digital photoelasticity is carried out.

In [1] crack tip fracture parameters are estimated for SEN specimens, center crack and crack-inclusion specimen configurations using the digital photoelastic technique. Full field isochromatic and isoclinic data over the cracked specimen are estimated using the ten-step phase shifting technique in tandem with the adaptive quality guided phase unwrapping algorithm employed for phase unwrapping purpose. An over deterministic linear least square algorithm is proposed and implemented for the first time towards SIF determination involving multi-parameter stress field equation in the digital photoelasticity domain. Linearization is possible because of the availability of accurate isoclinic and isochromatic data over the model domain thus utilizing the advantage of the digital photoelasticity to its fullest extent. This linear approach has got a better convergence and attains global minimum as compared to the conventional approach of non-linear over deterministic least square minimization mentioned in the literature. The SIF values for all the three specimen configurations are found to be closely matching with the analytical or FEA estimates, thereby confirming the accuracy of the proposed methodology. The authors note that the accuracy of SIF estimate could be improved by improving the accuracy of the isoclinic parameter estimate using the white light photoelasticity thereby eliminating the isochromatic–isoclinic interaction noise.

In [7] an advanced experimental technique for determination of the stress intensity factor (SIF) and the T-stress is developed and carefully verified. The approach employs optical interferometric measurements of local deformation response to small crack length increment. Narrow notches are used for crack modeling. Initial experimental data represent inplane displacement component values measured by electronic speckle-pattern interferometry in the vicinity of the crack tip. Determination of the first four coefficients of Williams' series is the main feature of the developed technique. Relationships for transition from measured in-plane displacement components to required fracture mechanics parameters are presented. Availability of high-quality interference fringe patterns, which are free from rigid-body motion, serves as a reliable indicator of real strain state near the crack tip. Experimental verification of the proposed method is performed for non-symmetrical and symmetrical crack in thin rectangular plates subjected to uniaxial tension. The distributions of SIF and T-stress values for cracks of different length in residual stress fields near electronically welded joints of thin plates are presented as an example of practical implementing.

The authors of [8] note that the photoelastic technique has seen some renewed interest in past few years with digital images and image processing new methods becoming readily available. However, further research is needed to improve the precision, the accuracy and the automation of photoelastic technique. The aim of this research work is to get new numerical equations for the phase-shifting method in digital photoelasticity using a plane polariscope. The model was developed to plane polariscope because of the simplicity and low cost of this equipment. To develop the phase shift and respective intensity equations only the analyzer is rotated. A ring under diametral compression is used for the experimental validation. From these intensity equations, the equations for isoclinic and isochromatic parameters are deduced by applying a new numerical technique. This approach can be used to calculate the isoclinic and isochromatic parameters using any number of images. Several analyses are performed with different number of photographic images. The results showed errors reduce when more phase-stepped images are utilized. Hence, one concludes that the uncertainties in results due to effects of errors on photoelastic images can be reduced with a larger amount of phase-stepped images.

Conventional phase shifting techniques have emerged as a powerful tool for evaluating the stress field in digital photoelasticity [9]. However, the quantity of image acquisitions they require makes the process complex and tedious. In [9] a computational hybrid method was developed for reducing the quantity of the image acquisitions in conventional phase shifting techniques. This study provides a novel approach to complete the set of acquisitions by performing some of them experimentally and simulating the remaining computationally. The accuracy of the results demonstrated that conventional phase shifting techniques could evaluate the stress field by performing fewer acquisitions and integrating computational procedures. These achievements represent a further step towards evaluating time-varying phenomena since the reduction of the acquisition time.

A novel scanning scheme for TFP (Twelve Fringe Photoelasticity) is proposed in [10] that takes into account the spatial resolution of the isochromatic fringe pattern. The proposed fringe resolution guided scanning scheme is capable of solving a variety of problems without the propagation of noise from the zones where spatial resolution of the fringes is low. Initially, a method based on the intensity gradients is proposed to generate a map that represents the fringe gradients in the model. The procedure for creating the map is discussed using theoretically simulated fringe pattern in a circular disc and later extended to experimentally recorded isochromatics using white light. The proposed scanning scheme is successfully applied to two problems – bi-axially loaded cruciform specimen with an inclined crack and thick ring subjected to internal pressure. The existing scanning schemes have limitations in solving both these problems as they do not consider the spatial resolution of the fringes. The proposed scheme eliminates the propagation of noise and restricts them within the low resolution zones since the algorithm postpones the scanning of noisy zones towards the end. The performance of the scanning scheme is independent of location of the seed point selected and hence, the user interaction is minimal. The work of J.M. Vasco-Olmo et al [11] uses DIC and photoelastic techniques to investigate the effect of single overload cycles applied during constant amplitude fatigue. Effective values of the range of stress intensity factor were calculated using the CJP model of crack tip stress and displacement fields, as this model considers both wake contact and compatibility induced influences on the applied elastic field arising from the plastic enclave generated around a fatigue crack. Values of the effective stress intensity factor are related to the observed crack growth acceleration and retardation. In addition, the paper compares the CJP results with those obtained using a compliance-based method. The present work demonstrates the utility of the CJP model in characterising fatigue crack growth rates during variable amplitude loading. It is also possible with the CJP model, through changes in the coefficient values and hence, for the first time, to shed explicit light on the contributions made by different mechanisms to the shielding effects of an overload.

In [12] closely spaced asymmetric cracks in a biaxial cruciform specimen interacting at different orientations are studied by digital photoelasticity. Isochromatic fringe patterns are captured under biaxial loading conditions with varying biaxial ratios from 0 to 1, zero indicating uniaxial loading and one indicating equi-biaxial loading. Stress Intensity Factors are determined using corrected Atluri-Kobayashi multi-parameter stress field equations in conjunction with over-deterministic least squares method. The SIF values calculated are normalized with the SIF for a single crack and compared with finite element results. Crack interaction effects at the crack tips of two interacting asymmetric cracks at different interaction angles are studied. These results may be useful in the prediction of behavior of structures with cracks when similar kinds of interaction effects are encountered.

In [13] it is shown that the limitations of the so-called over-deterministic method no longer apply when the Generalized Least Squares by Lagrange Multipliers method. The method provides not only an estimate of the value of each input quantities to be measured, but also: a fitted estimate of the value of each input quantity, the covariance matrix of all these estimates from which both the standard uncertainties and the correlation coefficients can be calculated.

Thus, the photoelasticity techniques have been extensively used for experimentally determining the state of stress in actual mechanical components. Nowadays digital image processing and new image analysis algorithms become very important [1-15].

In this research, digital photoelasticity was employed to assess the singular and higher-order coefficients of the Williams series expansion for the stress field in the vicinity of the crack tip in semicircular bend (SCB) specimens at different crack inclination angles. To utilize the advantages of the whole – field photoelasticity and minimize the experimental errors, the overdeterministic method [4] has been used. The experimental equipment is shown in Fig. 1. The aim of this paper is to obtain the coefficients of the higher-order terms in the Williams expansion and to estimate the influence of these terms on the stress field description taking into account as many as possible terms in the asymptotic presentation of the crack tip fields.

2. Experimental procedure: photoelastic experiments using semi-circular bend specimen

The experimental setup used is shown in figure 1. All the specimens in this work were made by casting of polycarbonate. Circular and semi-circular shapes were machined from the sheet to get the test specimens. Material properties of the photoelastic material are Young's modulus $E = 3GPa$, Poisson's ration $\nu = 0.3$ and the material fringe constant is found to be $f_\sigma = 18.38 Pa\ m / fringe$.

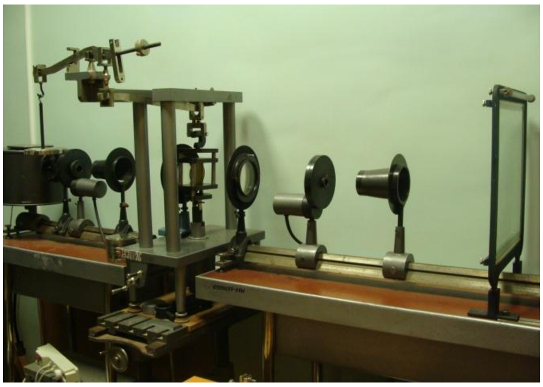

Figure 1. Actual experimental setup of transmission photoelasticity.

The schematic of the semicircular bend (SCB) specimen with an inclined crack, where P is the applied load, S is loading span in the SCB specimen, a is crack length, α is crack inclination angle is shown in figure 2. The semi-circular bend specimen subjected to three-point ben ding has received much attention in recent years for measuring the mixed mode I/II fracture resistance [16-19].

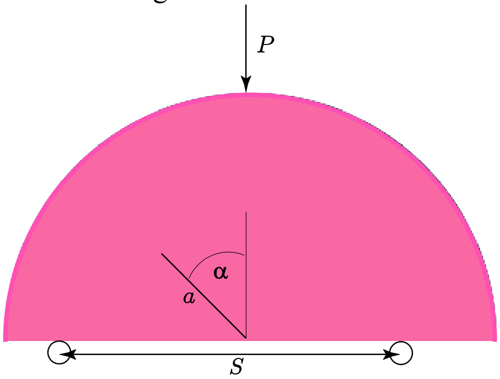

Figure 2. Loading of the SCB specimen.

The whole field fringe order and isoclinic values surrounding the crack tip are required to estimate the crack tip SIF, T-stresses and the coefficients of the higher-order terms in the Williams series expansion for the crack-tip stress field. In order to correctly label the fringe orders, color isochromatic images are grabbed in a polariscope. Isochromatic phase maps obtained for the SCB specimen with the vertical crack are shown in figure 3. The isochromatic and isoclinic data around the crack tip is estimated using the ten- step phase shifting technique [1] (figures 4). Isoclinic phase maps obtained for the SCB specimen with the vertical crack are shown in figure 4 (a-j).

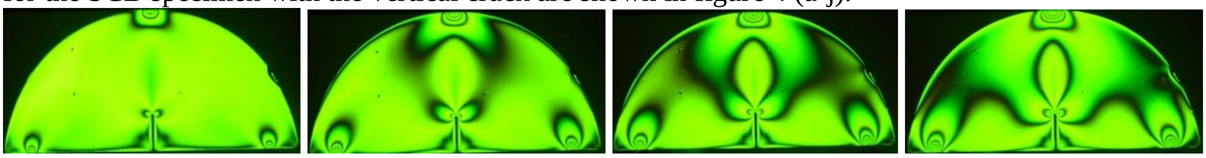

Figure 3. Colour isochromatic images for the vertical crack ($\alpha = 0^{\circ}$) in the SCB specimen.

Isoclinic patterns for mixed mode loading are shown in figures 7 (a-j). The skeleton of the fringe is identified first to accurately collect the data from the fringes (figures 5, 6). Global fringe thinning algorithm [12] which uses the intensity information for the location of the fringe skeleton is used to thin the fringes. A thinned image is shown in figure 8.

3. Least-squares evaluation of SIF, T-stresses and higher-order coefficients

Williams [20] had derived crack tip stress field equations in polar coordinates for a mixed mode case. The stress field equations are given as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{n}{2} a_{In} r^{(n-2)/2} \begin{Bmatrix} \left(2 + n/2 + (-1)^n\right)\cos\left(n/2-1\right)\theta - (n/2-1)\cos(n/2-3)\theta \\ \left(2 - n/2 - (-1)^n\right)\cos\left(n/2-1\right)\theta + (n/2-1)\cos(n/2-3)\theta \\ \left(n/2-1\right)\sin\left(n/2-3\right)\theta - \left[n/2 + (-1)^n\right]\sin(n/2-1)\theta \end{Bmatrix} +$$
$$+ \sum_{n=1}^{\infty} \frac{n}{2} a_{IIn} r^{(n-2)/2} \begin{Bmatrix} \left(2 + n/2 - (-1)^n\right)\sin\left(n/2-1\right)\theta - (n/2-1)\sin(n/2-3)\theta \\ \left(2 - n/2 + (-1)^n\right)\sin\left(n/2-1\right)\theta + (n/2-1)\sin(n/2-3)\theta \\ -\left(n/2-1\right)\cos\left(n/2-3\right)\theta - \left[-n/2 + (-1)^n\right]\mathrm{co}(n/2-1)\theta \end{Bmatrix} \qquad (1)$$

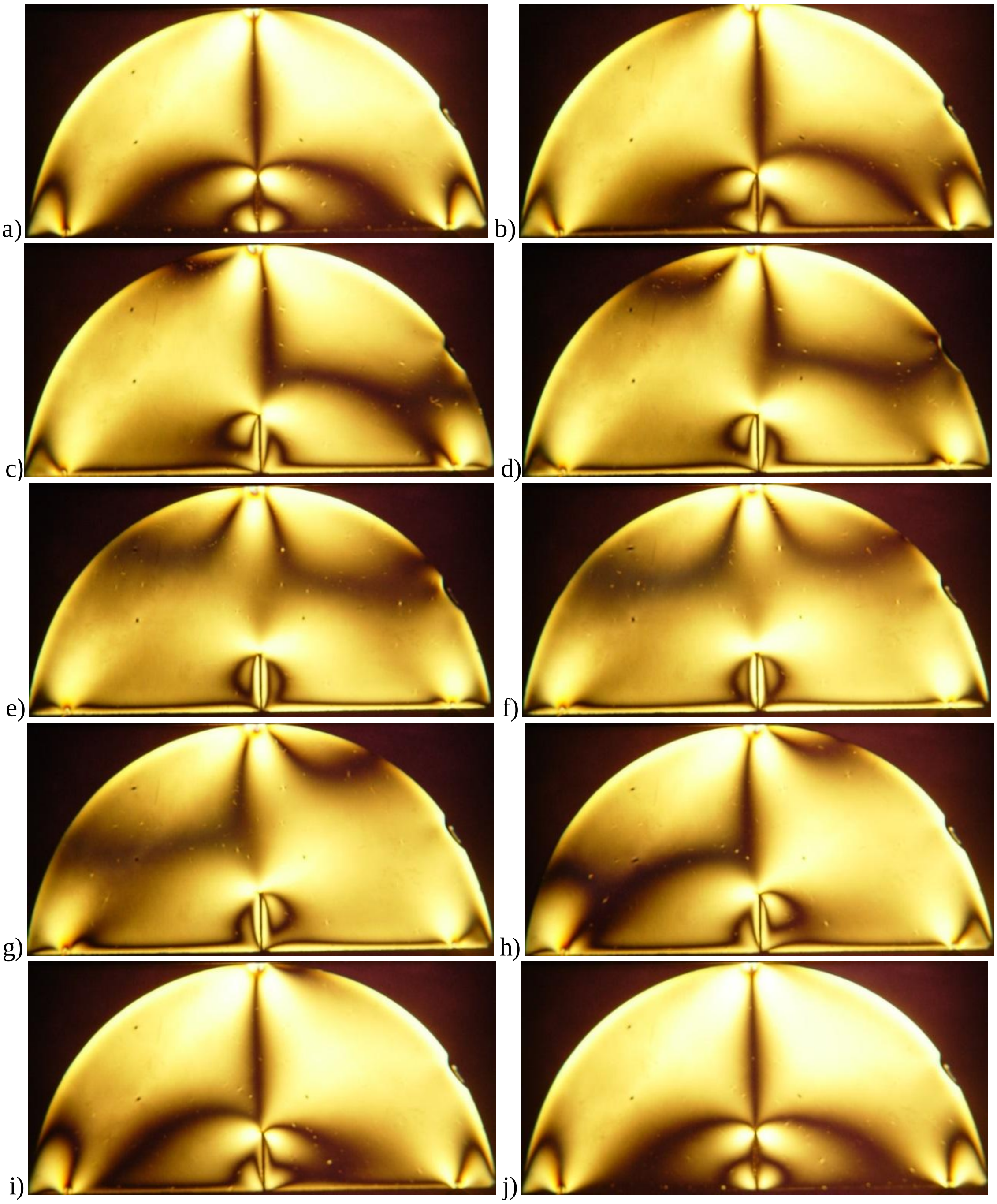

Figure 4. Isoclinic phase maps for mode I crack in the SCB specimen.

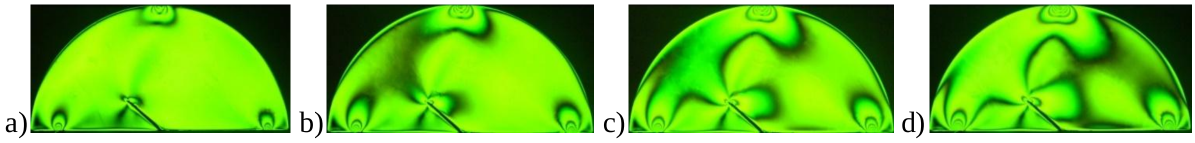

Figure 5. Color isochromatic images for the inclined crack at $\alpha = 45^\circ$ in the SCB specimen.

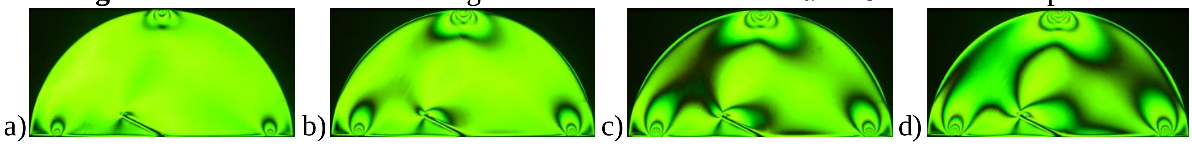

Figure 6. Color isochromatic images for the inclined crack at $\alpha = 60^\circ$ in the SCB specimen.

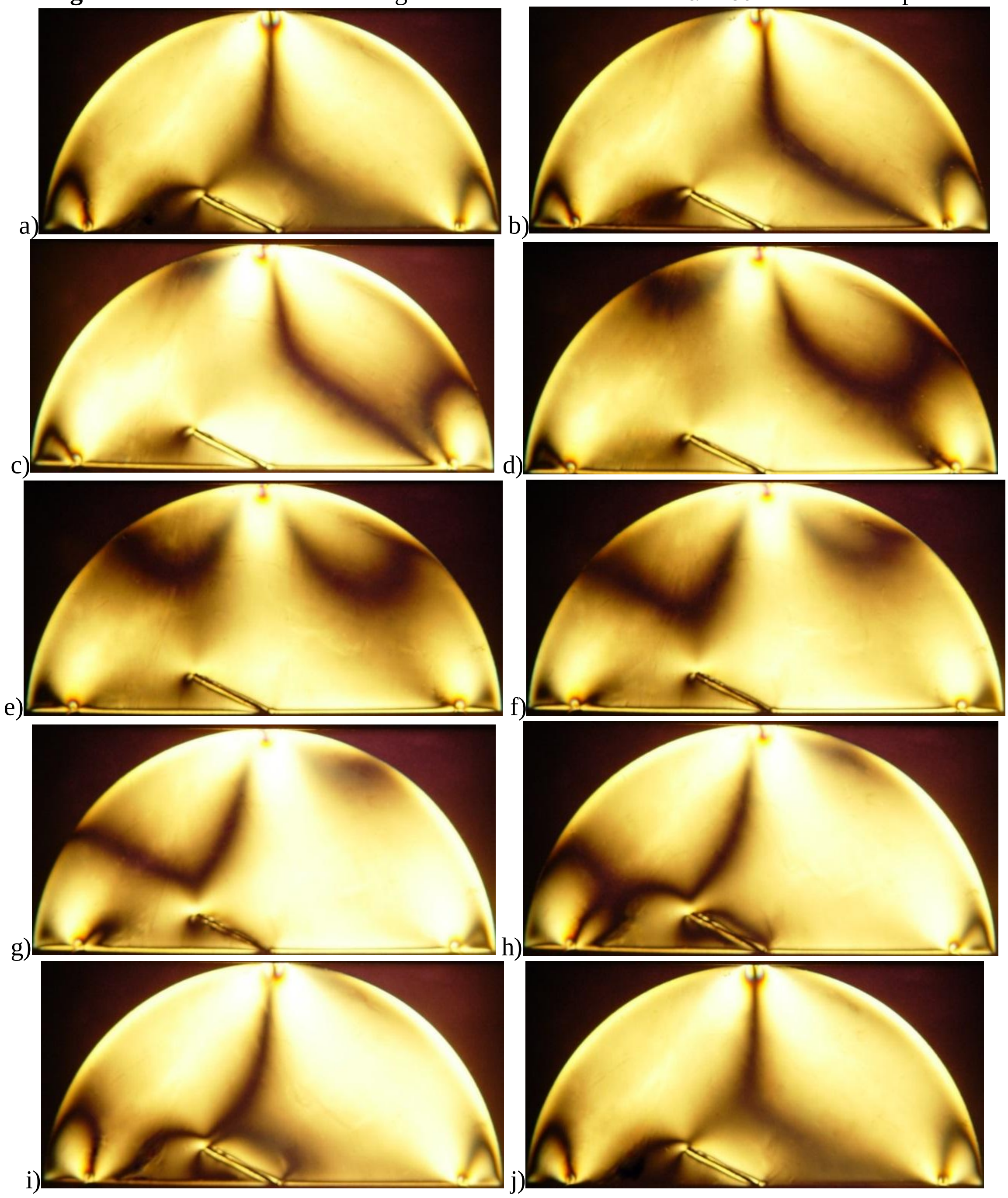

Figure 7. Isoclinic phase maps for the mixed-mode loading of the crack inclined at $\alpha = 60^\circ$ in the SCB specimen.

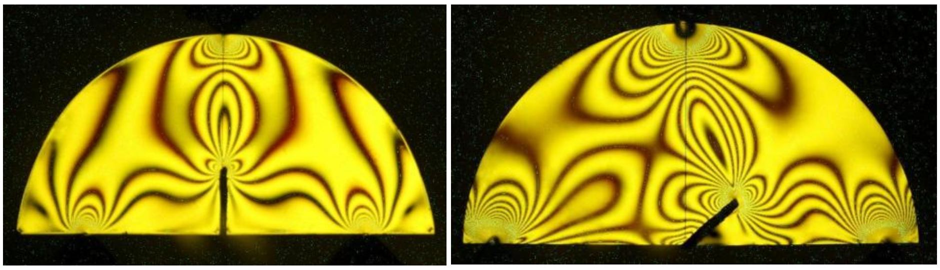

Figure 8. Fringe thinned images for the vertical crack and for the mixed-mode loading of the crack inclined at $\alpha = 45^{\circ}$ in the SCB specimen.

The coefficients are the unknown mode I and mode II parameters respectively. The SIFs can be computed from the coefficients as $K_I = a_{I1}\sqrt{2\pi}$, $K_{II} = -a_{II1}\sqrt{2\pi}$. The coefficient a_{I2} is related to T-stress as $\sigma_{0x} = T = -4a_{I2}$. The stress optic law relates the fringe order N and the in-plane principal stresses σ_I and σ_{II} as

$$Nf_{\sigma} / h = \sigma_1 - \sigma_2 \qquad (2)$$

where f_{σ} is the material stress fringe value and h is the thickness of the specimen. For a plane stress problem, the stress components are related to the principal stresses as

$$\sigma_1, \sigma_2 = (\sigma_{11} + \sigma_{22}) / 2 \pm \sqrt{(\sigma_{11} - \sigma_{22})^2 / 4 + \sigma_{12}^2}. \qquad (3)$$

Substituting Eq. (3) in (2) one can define an error function for the data point:

$$g_m = ((\sigma_{11} - \sigma_{22}) / 2)_m^2 + (\sigma_{12})_m^2 - ((N_m f_{\sigma}) / (2h))^2. \qquad (4)$$

Eq. (4) is a non-linear equation in terms of the unknown parameters a_{Ij}, a_{IIj}. Initial estimates are made for these unknown parameters and possibly the error will not be zero since the estimates are not accurate [4, 12]. The estimates are corrected using an iterative process based on Taylor series expansion of g_m. One can arrive at the solution of the incremental change by solving a simple matrix problem. The results are given in Table 1.

Table 1. Crack tip fracture parameters for SCB specimen.

Fracture parameter	$n = 2$	$n = 4$	$n = 8$
K_I (MPa $\sqrt{mm}$)	23.90	23.99	24.00
K_{II} (MPa $\sqrt{mm}$)	0.45	0.41	0.40
a_{I1} (MPa $\sqrt{mm}$)	9.53	9.58	9.59
a_{I2} (MPa)	-0.44	-0.456	-0.457
a_{I3} (MPa $(mm)^{-1/2}$)		0.145	0.146
a_{I4} (MPa $(mm)^{-1}$)		0.001	0.000
a_{I5} (MPa $(mm)^{-3/2}$)			0.021
a_{I6} (MPa $(mm)^{-2}$)			0.0006
a_{I7} (MPa $(mm)^{-3/2}$)			-0.01
a_{I8} (MPa $(mm)^{-3}$)			0.0001

4. Conclusions

In this study, digital photoelasticity technique is used to estimate the crack tip fracture parameters for different crack configurations. The paper introduces a Java application programmed for the advanced determination of the fracture characteristics: coefficients of the Williams series expansion (WE) for the stress field in the vicinity of the crack tip. The tool allows us to collect experimental points from the photoelasticity tests on the cracked specimens. An automatic routine implemented as a Java application permits to determine the values of coefficients of higher order terms of the WE that describe crack-tip fields. These values are calculated using the over-deterministic method which is also applied to the results of the finite element analysis of some mode I and mixed mode test geometry. Thus, the Java application provides an analytical reconstruction of the crack-tip stress field by means of the truncated WE and enables detailed analysis of the crack-tip stress field approximation. The developed procedures simplify the analysis of the description of mechanical fields at a greater distance from the crack tip considerably. The presented study is focused on the optimization of the selection of FE nodal results entering the over-deterministic technique used to determine the values of coefficients of the higher order terms of the WE. The results of FEM modelling are compared with the experimental results obtained by the photoelasticity method. The experimental approach based on the photoelasticity method allows us to observe the von Mises equivalent stress distribution in the whole specimen and confirm the FEM studies performed. The comparison shows that in the specimen the pure mode II loading can't be realized. Analysis of numerical studies and the isochromatic fringe patterns allows us to conclude that in the semicircular disc the mixed mode loadings are realized for all angles from 0° to 80°. The experimental SIF and T-stress values estimated using the proposed method are compared with finite element analysis (FEA) results, and are found to be in good agreement.

5. Acknowledgments

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