Studies of 3D-Ising model with long-range interaction

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Abstract

Monte-Carlo simulations were used to study thermodynamic properties of Ising models on cubic lattices with long-rang interactions. We took into account interactions with the next-nearest neighbors and varied their values J_4 from 0 to 1. The results of simulations are in good agreement with an analytic formula that defines the critical inverse temperature as a function of the parameter J_4. The behavior of the internal energy and the heat capacity indicates that at the critical point the heat capacity has an infinite singularity.

Keywords

Ising model, cubic lattice, long-range interaction, critical temperature

1. Introduction

When examining an Ising model on cubic lattices we account for interactions between the nearest neighbors only, in the system there is a phase transition of the second kind and it is characterized by an infinite singularity of the heat capacity at the critical point. We do not know for sure if it is a logarithmic singularity or the heat capacity is a power function with a very small exponent, but it is important that when the temperature tends to the critical value the heat capacity increases unlimitedly. Theoretical estimates [1], [4], and [6] as well as results of a lot of computer simulations [2], [5], and [7] argue in favor of this behavior.

2. Main question

We were interested in what happened if we also accounted for interactions with the next-nearest neighbors too. How such a change in the model affects the character of the singularity at the critical point? Let us explain what is the reason for our interest.

We can also include interactions with the next-next-nearest neighbors and the next-next-next neighbors and so on. In this way sooner or later it will be possible to describe the system with the aid of the mean field model. In the framework of the mean field approximation the phase transition in the Ising model is of the second kind too, but the heat capacity has a finite jump. Consequently, if we take into account more and more remote neighbors, somewhere the singularity type of the heat capacity will change. In other words, we will obtain a finite jump in place of the infinite singularity.

3. Conclusion

The goal of this paper was to determine if the account for the next-nearest neighbors changed the singularity type. Our analysis did not allow us to obtain a decisive answer; however, we found that when the number of the interacting neighbors increases the behavior of the heat capacity in the vicinity of the critical point changed significantly.

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