# **Roughness measurement by optical vortices array with nanoscale resolution**

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**Abstract.** In present report, we review the principles and applications of interference vortex method for the real time determination of polished surface roughness for transparent and reflecting materials with using of Laguerre-Gaussian probe beam array of low index as well as a reference beam. High spatial resolution caused by interference of vortices and their phase sensitivity, which is automatically analysable to retrieve the 2D and 3D shape of micro- and nanostructured surfaces with exceeding of optical diffraction limit which is applicable for non-destructive testing of roughness analysis of thin films and solid microstructures in real-time regime with longitudinal and transverse resolution down to 1.75 nm and 7 nm respectively for visible light sources. The dependence of the angle of rotation of the resulting interference pattern on the optical path difference and sample thickness for two singular beams superposition is considered in detail.

### 1. Introduction

Precise determination of surface roughness and relief is essential task for the manufacturing of optical and mechanical components with high degree of quality and smoothness in engineering, metrology research and materials science applications [1]. For this purposes a great number of instruments and measuring principles were developed last two decades. In general, they are divided into contact and non-contact [2] methods of interaction with the test sample. The first principle uses stylus probe detector and based only on mechanical interaction with an object. This method is inapplicable to the wide range of practical tasks because the scratched surface may be damaged or received parameters are not adequate. Especially it touches such cases as biological systems imaging, polymer structures analysis or metrology of thin film layers.

In contrast, optical methods of profilometry and surface analysis has a lot of advantages, among them: non-contact and non-destructible interaction with test sample, relatively high resolution, accuracy and flexible control. The principle of optical profilometry based on interferometric measurements of optical field, consisting of amplitude and phase information received from the surface being studied. Phase images may be considered as two-dimensional phase distribution or as optical path difference of interfering beams [3].

Currently the most popular methods include the measurement of light modulation envelope and phase estimation or use its combination. The phase of the electromagnetic wave is determined by using interferometer with considering of refractive index of the material in case of transparent object under investigation. Phase estimation method is the most suitable data processing method in 3D optical

microscopy and profilometers with using of both smooth plane waves and beams with phase singularities carrying helical wavefront.

A practical application of singular beams in the vortex scanning optical imaging allows to study, for example, surface geometry and optical density of the sample by analysis of singular phase transformation [4] and depends of the features of incident beam and aperture of optical systems [5]. In manuscripts [5-7] authors demonstrated a new solution for visualization and characterization of nanometer structures called Optical Vortex Scanning Microscope where the sample is scanned by moving vortex. This study demonstrates the response of the optical vortex imbedded in focused Gaussian beam to the shift from the critical plane inside the object arm of interferometer. Scanning procedure enables to cover all area of surface and plot a vortex trajectory, which has a characteristic way of movement and depends on thickness of the probe. Further research of Optical Vortex Scanning Microscope conducted with developing of analytical models and phase retrieval algorithms [7]. In the recent research [8] authors describe both theoretical and experimental results of imaging system using movable optical vortex, where the image of the probe was combined with the structured singular beam. Nevertheless, the phase distribution after the object may be recovered with quite good accuracy, thus using of optical vortices in imaging and microscopy opens new perspectives for development of relief retrieval algorithms and new optical instruments design [9-11].

By analyzing of captured interference patterns in form of vortex spiral or retrievable light structures, we can extract an information about phase shift between superposed beams and, as result, about surface roughness state and relief or sample thickness and the task for optimization of such procedures on practice is the purpose of given research.

### 2. Singular beams array in noncontact profilometry measurements

In this research we consider analytically an interference of singular beams with the wavelength  $\lambda$  carrying optical vortices of topological charge l. One of the important features of the singular beams is the screw dislocation of wavefront expressed as beam phase spatial dependence in the form:

$$\Phi(\varphi, z) = kz + l\varphi \tag{1}$$

where z is the propagation direction,  $\varphi$  is the azimuthal angle at the beam cross section and k denotes wavenumber in a free space. In simplified form, optical vortex with topological charge l may be expressed in terms of Laguerre-Gaussian mode  $LG_0^l$  with zero radial index.

Let us consider first the propagation of the paraxial  $LG_0^l$  beam along the z-axis. The transverse profile  $E_x$  of the beam have a wavenumber  $k = nk_o$ , where  $k_o = \frac{2\pi}{\lambda}$  is a wavenumber in a free space and

 $n = \sqrt{\varepsilon}$  – refractivity index of medium (for convenience it can be assumed n = 1). In the paraxial approximation, we can treat the linearly polarised electromagnetic field component as  $E = \tilde{E}_x(r, \varphi, z) \exp(-ikz)$  where  $\tilde{E}_x(r, \varphi, z)$  is the slowly varying complex amplitude which satisfies the paraxial wave equation:

$$\nabla_{\perp}^{2} E_{x}(r,\varphi,z) + 2ik \frac{\partial E_{x}(r,\varphi,z)}{\partial z} = 0$$
(2)

where transverse Laplacian  $\nabla_{\perp}^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2}$ ,  $(r, \varphi)$  – is the cylindrical coordinates and radius

is  $r = \sqrt{x^2 + y^2}$ . A particular solution to the paraxial wave equation (2) for the Laguerre-Gaussian vortex beam can be written as follows:

$$E_{x} = \frac{\omega_{0}}{w} \left(\frac{r}{w}\right)^{|l|} L_{m}^{l} \left(m, \frac{r^{2}}{w^{2}}\right) \cdot \exp\left(-\frac{r^{2}}{w^{2}}\right) \cdot \exp\left(-\frac{ikr^{2}}{2z(1+z_{R}^{2}/z^{2})}\right) \times \\ \times \exp\left(-il\varphi\right) \cdot \exp\left(i(2m+l-1)\arctan\left(\frac{z}{z_{R}}\right)\right) \cdot \exp(-ikz)$$
(3)

where  $\omega_0$  is the beam waist at the plane in z=0,  $z_R = \frac{k\omega_0^2}{2}$  denotes Rayleigh length of the beam and current beam radius is expressed as  $w = \omega_0 \sqrt{1 + (z/z_R)^2}$ . Topological charge of vortex beam is introduced as  $l = \pm 1, \pm 2,...$  The vortex position, by definition, is described by the equation  $\operatorname{Re} \tilde{E}_x(x, y, z) = \operatorname{Im} \tilde{E}_x(x, y, z) = 0$ . In equation (1) we suppose azimuthal angle is  $\varphi = \arg(x + iy)$ . Numerical calculation of intensity distribution for the array of  $3 \times 3$  vortex beams with l = 1 and typical axial minima produced by two-dimensional diffraction grating is shown on figure 1 (a) (other diffraction orders were cut by a diaphragm). Phase pattern is shown in figure 1 (b) depicts vortex phase as a helix.



**Figure 1.** Theoretically calculated intensity distribution  $(\text{Re E}(r, \varphi))^2$  (a) of array of Laguerre-Gaussian beams with dimensionless node numbers: radial index m = 0 and vortex topological charge, defined by azimuthal index l = +1 and its phase Im E $(r, \varphi)$  (b). Other beam parameters are next:  $\omega_a = 180 \text{ mcm}, \lambda = 632.8 \text{ nm}.$ 

Singular beam phase is extremely sensitive to changes of optical path between superposed waves. When the optical path of two beams travelling, for example, in arms of interferometer, are equal, it means that waves have the same waist radius and curvature of wavefront. But small variations of optical or geometrical path, which are the same in our case, provokes well-known relative phase shift whether is observable via interference spiral behaviour. Let us consider a case when singular beam propagates through the free space between mirrors of non-equable Mach-Zehnder interferometer and interfere with Gaussian beam. In the object arm we may slightly change the optical path  $\Delta = 0 \div \lambda$ , where  $\lambda$  is the wavelength.

The simplest way for rapid and computationally non intensive analysis is using of direct data from image sensors and cameras with minimal adjustment and transformations. For this purpose we propose to employ superposition of high sensitive probe beam carrying optical vortex with the reference one, but also containing singularity with opposite sign [9-11].

Making one full turn, intensity pattern coincides with itself at geometrical path difference equal to wavelength  $\lambda$ . Note, that whole picture coincides only ones for topological charges  $l = \pm 1$ . Thus, the symmetry of the pattern caused by topological charges of interfering singular beams. Further increasing of topological charge induces multiplication of high intensity spots around of image centre and are not essential for our application and we restrict ourselves only on superposition of single charged optical vortices for two reasons: at first, interference pattern of single charged vortices has well defined intensity minimum and only one symmetrical axis thus it may be automatically defined with high accuracy. Secondly, the beam spot in this case has much less radius and allow to make beam focusing more easy due to structural stability of optical vortices with topological charge  $l = \pm 1$ . Therefore this method can be used as addition to direct vortex phase analysis as quick draft regime of profile measurement. On estimated evaluation, phase rotation measurement allow to achieve vertical resolution down to 1,75 nm for He-Ne laser source with  $\lambda = 632.8$  nm and less than 1 nm for blue light laser. Rough measurements with resolution  $\approx 4$ nm can be performed by processing of interference pattern formed by two coherent optical vortices with opposite signs. The resultant intensity and phase distribution for different  $\Delta$  is depicted in figure 2.

The sharp interference curves with a characteristic "spiral", corresponding to optical vortices can be imaged by the camera and assayed. Total phase shift which is observable due to angular rotation of interference spiral can be calculated as a geometrical path difference between observable and neighboring levels of reflecting sample surface. Focused beam spot of 1 mcm on the sample surface perform the transverse resolution down to 7 nm.



**Figure 2.** Theoretically calculated intensity distribution of array of interfered Laguerre-Gaussian beams (a) with topological charge  $l = \pm 1$  and its phase (b) for different optical path after the reflecting from the sample with rough surface relief. Other beam parameters are next:  $\omega_{a} = 180 \text{ mcm}, \lambda = 632.8 \text{ nm}.$ 

# 3. Conclusion

We have analytically considered evaluation of optical phase features and sensitivity to geometrical path changes and have shown that the distinguishable spiral phase rotation occurs at  $\lambda/300$ , where  $\lambda$  – is a wavelength. Proposed technique may be applied to optically transparent and reflecting surfaces exceed optical diffraction limit. Moreover, this method applicable for non-destructive testing of live cells and biological tissues in real-time regime with simultaneous measurements of nine points on interferogram composition. Automatic processing of vortex spiral interferograms and intensity distribution with typical edge dislocation allow to achieve a vertical resolution down to 1.75 nm. The experimental evaluation of measurement accuracy and finding of optimal phase retrieval procedure is a task of future investigations.

## 4. References

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## Acknowledgments

This work was partially supported by the V.I. Vernadsky Crimean Federal University Development Program for 2015 – 2024 via grant support for young researches.