Rendezvous of two spacecraft in LEO with use tether

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Abstract. This study focuses on a dynamics of a rendezvous of a tug and a large space debris connected by a viscoelastic tether. It is assumed that the constant thrust acts on the space tug and a control is realized by changing the length of the tether. The goal is to find the control law that provides a soft docking of two bodies on an orbit. Three-body system with an additional damping device is also considered. The obtained results can be applied as applications for the tasks of implement rendezvous of two bodies using the tether.

Keywords: space debris removal, spacecraft rendezvous, space tether, control law, soft docking.

1. Introduction

In the recent years, the problems of space debris have came to the forefront of modern astronautics. According to the forecast made by Donald J. Kessler [1, 2], space debris can put an end to further space exploration. There are more than 15,000 large objects on the orbits around the Earth. Only 7% of these are active spacecraft, 17% are defunct satellites and 13% are old upper stages. All these objects are tracked. An active spacecraft or a space station can avoid collision with such objects [3-6]. Collisions of the large space debris with other debris can significantly increase numbers of the small debris on the Earth orbit. There is a large number of papers devoted to this problem [3-23]. So that many modern spacecraft are equipped with a means of de-orbiting. Different approaches have been were offered to removal the defunct satellites and the old upper stages [3-4, 7, 10, 14, 22-23]. These approaches can include the use of the tether systems [3-4, 16]. The tether can be used as a means of the soft docking of the active spacecraft (space tug) and the defunct satellites or the old upper stages (space debris) [22]. In this case the space tug and the space debris are pulled together using the tether.

This study focused on the stages of the rendezvous and the soft docking the space debris with the space tug as shown in figure 1. The goal is to study the dynamic of the maneuver of the rendezvous and to find the control law, which allow one to realize the soft docking the space debris with the space tug.

The paper is divided into two main parts. In the Section 2 a mathematical model of the two-body system is developed. In the Section 3 the damping device is considered as an additional part of the mechanical system aimed to eliminate the longitudinal oscillations of the tether. A mathematical model of the three-body system is developed. It includes the space tug, the space debris, the weightless viscoelastic tether and the damping device. The numerical analysis results show that in the end of the maneuver of the rendezvous the longitudinal oscillations of the tether occur.

2. Motion equations S_2

The two-body system includes two points S_1 (the space tug) and (the space debris), which are connected by the weightless viscoelastic tether having initial length l_0, figure 2(a). The origin of the inertial frame S coincides with the initial position of the point S_2. The unit vector $\mathbf{i}$ is in the direction of the tangent to the orbit in the point S, figure 3. The unit vector $\mathbf{j}$ is in the direction from the point S to the Earth's center. Unit vector $\mathbf{k}$ is complementary to the system to Cartesian coordinate. The positions of the material points S_1 and S_2 can be defined by the radius vectors $\boldsymbol{\rho}_1 = \{x_1, y_1, z_1\}^T$ and $\boldsymbol{\rho}_2 = \{x_2, y_2, z_2\}^T$ respectively. We suppose that the thrust tug $\mathbf{F}$ coincides with the unit vector $\mathbf{i}$. Internal interaction of the bodies points S_1 and S_2 are determined by the tether tension force

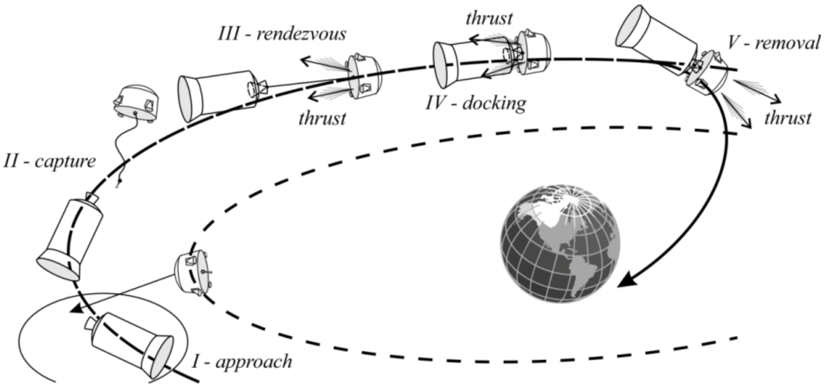

Figure 1. The stages of the active debris removal mission.

The equation of motion of the system can be written [24]

$$\ddot{\boldsymbol{\rho}}_i = \frac{1}{m_i}\left(\mathbf{F}_i^g + \boldsymbol{\Phi}_i^e + \boldsymbol{\Phi}_i^c + \mathbf{f}_i\right), \qquad i = 1, 2 \tag{1}$$

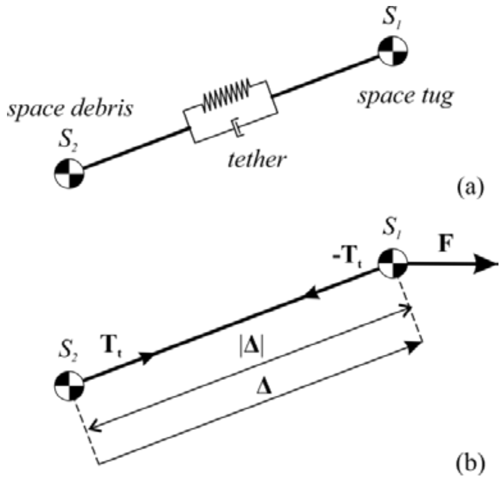

Figure 2. The two-body system: (a) - the basic scheme of the two-body system, (b) - the forces.

where the index $i = 1, 2$ represents the space tug and the space debris; m_1 and m_2 are masses of the material points S_1 and S_2; $\mathbf{f}_1$ and $\mathbf{f}_2$ are the forces of interaction acting on the space tug and the space debris; $\mathbf{r}_i = \mathbf{r} + \boldsymbol{\rho}_i$ vector defining the distance from the center of the Earth to the tug and the space debris; $\mathbf{r} = \{0, -r, 0\}^T$ is vector which defines the distance between the center of the Earth and the point S. The forces of inertia $\boldsymbol{\Phi}_i^e$, $\boldsymbol{\Phi}_i^c$ and the gravitational force $\mathbf{F}_i^g$ can be defined as

$$\mathbf{F}_i^g = -\mu \frac{m_i}{|\mathbf{r}_i|^3}\mathbf{r}_i \tag{2}$$

$$\mathbf{\Phi}_i^e = -m_i \mathbf{a}_i^e \tag{3}$$

$$\mathbf{\Phi}_i^e = -m_i \mathbf{a}_i^c = -2m_i\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_i\right) \tag{4}$$

where $\mathbf{a}_i^e = \mathbf{a}_S - \mathbf{h}_i\omega^2$ is the bulk acceleration, $\boldsymbol{\omega} = (0, 0, \omega)$ is the vector of the angular velocity vector of the coordinate system ($\omega = const$), $\mathbf{h}_i = \{x_i, y_i, 0\}$ is the vector of the defines up to an axis of rotation, $\mathbf{a}_S = -\mathbf{r}\omega$ is the acceleration of the point S.

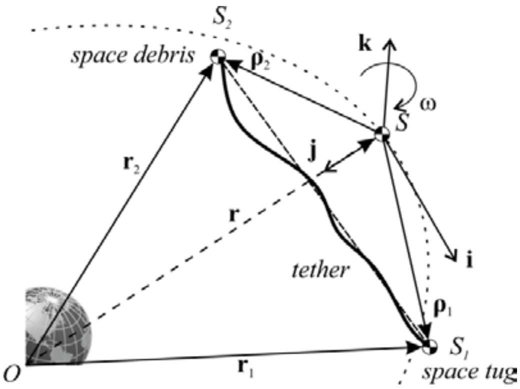

Figure 3. System coordinate.

Substituting (2) - (4) in (1) one can write equations of the motion

$$\ddot{\boldsymbol{\rho}}_i = -\mu\frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} + \mathbf{r}\omega^2 + \mathbf{h}_i\omega^2 - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_i\right) + \frac{\mathbf{f}_i}{m_i}, \qquad i = 1, 2 \tag{5}$$

Next, taking into account that system moving in a circular orbit, we can rewrite (5) in form

$$\ddot{\boldsymbol{\rho}}_i = -3\omega^2\frac{y_i}{r}\mathbf{r} - \omega^2\left(\mathbf{r} - \mathbf{h}_i\right) - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_i\right) + \frac{\mathbf{f}_i}{m_i}, \qquad i = 1, 2 \tag{6}$$

This algebraic form is equivalent now to the classical Clohessy-Wiltshire-Hill equations of relative [24]. Next, we introduce a new variable

$$\boldsymbol{\Delta} = \left\{\Delta_x, \Delta_y, \Delta_z\right\} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 = \left\{x_1 - x_2, y_1 - y_2, z_1 - z_2\right\}^T \tag{7}$$

Using (6) we can write the equation for the new variable (7)

$$\ddot{\boldsymbol{\Delta}} = -3\omega^2\frac{\Delta_y}{r}\mathbf{r} - \omega^2\Delta_z - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\Delta}}\right) + \left(\frac{\mathbf{f}_1}{m_1} - \frac{\mathbf{f}_2}{m_2}\right) \tag{8}$$

2.1. The forces of interaction the space tug and the space debris

The forces $\mathbf{f}_1$ and $\mathbf{f}_2$ are determined by

$$\mathbf{f}_1 = \mathbf{F}\cdot\mathbf{i} - T_t\frac{\boldsymbol{\Delta}}{|\boldsymbol{\Delta}|}, \qquad \mathbf{f}_2 = T_t\frac{\boldsymbol{\Delta}}{|\boldsymbol{\Delta}|} \tag{9}$$

where T_t is the viscoelastic force of the tether, $\mathbf{F}$ is the thrust of the tug. The direction of the force T_t coincide with the direction of the vector $\boldsymbol{\Delta}$ as shown in figure 2 (b). The force T_t are defined as

$$T_t = \begin{cases} k_t\varepsilon_t + c_t\dot{\varepsilon}_t, & |\boldsymbol{\Delta}| \ge l \\ 0, & |\boldsymbol{\Delta}| < l \end{cases} \tag{10}$$

where k_t and c_t is a stiffness and damping ratio respectively. ε_t is the tether relative deformation, $\dot{\varepsilon}_t$ the rate-of strain is computed

$$\varepsilon_t = \frac{|\boldsymbol{\Delta}|}{l} - 1, \qquad \dot{\varepsilon}_t = \frac{\boldsymbol{\Delta}\cdot\dot{\boldsymbol{\Delta}}}{|\boldsymbol{\Delta}|\, l} - \frac{\boldsymbol{\Delta}\dot{l}}{l^2} \tag{11}$$

where l is the length control law.

2.2. Length control strategy

We derive the length control law [25, 26]

$$l = \frac{l_0}{2}\left(1 + \cos\varphi t\right) \tag{12}$$

where $\varphi = \pi / t_k$, t_k is the duration of the maneuver, l_0 is an initial length of the tether. The expression (12) satisfies the requirement that

$$\dot{l}\left(t_k\right) = 0$$

If the tether is rigid then the velocity of the space debris relative to the space tug will be equal to zero. After ending the maneuver rendezvous the final length of the tether cannot be equal to zero (because when $l = 0$ in expression (11) there will be uncertainty) so we rewriting our law in the form

$$l = l\left(t\right) = L + \frac{\left(l_0 - L\right)}{2}\left(1 + \cos\varphi t\right) \tag{13}$$

where $L = const$ is the final tether length.

3. Three-body system

To reduce the oscillations of the system we introduce a damping device as an additional element (figure 4). Now we have a three-body system which includes the space tug S_1, the space debris S_2 and the damping device S_3. S_2 and S_3 connected by the viscoelastic tether which has initial length l_0 as shown in Fig.12. S_1 and S_3 are connected by the damping device which has an initial length δ_0 as shown in Fig.5a. We modelling rigid body of the tug as two material points S_1 and S_3 where mass of the point S_1 considerably larger than mass of point S_3 ($m_1 \gg m_3$). The points S_1 and S_3 are connected by the weightless viscoelastic bar, which respond of damping device.

On the analogy with the two body system (8), the motion equations of the three-body system can b written as

$$\ddot{\boldsymbol{\rho}}_i = -3\omega^2 \frac{y_i}{r}\mathbf{r} - \omega^2 z_i - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_i\right) + \frac{\mathbf{f}_i}{m_i}, \quad i = 1,2,3 \tag{14}$$

Next, we make the change of variables

$$\begin{aligned} \boldsymbol{\Delta} &= \left\{\Delta_x, \Delta_y, \Delta_z\right\} = \boldsymbol{\rho}_3 - \boldsymbol{\rho}_2 = \left\{x_3 - x_2,\, y_3 - y_2,\, z_3 - z_2\right\}^T \\ \boldsymbol{\delta} &= \left\{\delta_x, \delta_y, \delta_z\right\} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_3 = \left\{x_1 - x_3,\, y_1 - y_3,\, z_1 - z_3\right\}^T \end{aligned} \tag{15}$$

Using the equation (14) one can write the equation for the new variable (15)

$$\begin{aligned} \ddot{\boldsymbol{\Delta}} &= -3\omega^2 \frac{\Delta_y}{r}\mathbf{r} - \omega^2\Delta_z - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\Delta}}\right) + \left(\frac{\mathbf{f}_3}{m_3} - \frac{\mathbf{f}_2}{m_2}\right) \\ \ddot{\boldsymbol{\delta}} &= -3\omega^2 \frac{\delta_y}{r}\mathbf{r} - \omega^2\delta_z - 2\left(\boldsymbol{\omega}\times\dot{\boldsymbol{\delta}}\right) + \left(\frac{\mathbf{f}_1}{m_1} - \frac{\mathbf{f}_3}{m_3}\right) \end{aligned} \tag{16}$$

For the three-body system the forces of the interaction are determined as

$$\mathbf{f}_1 = \mathbf{F}\cdot\mathbf{i} - T_d\frac{\boldsymbol{\delta}}{|\boldsymbol{\delta}|}, \qquad \mathbf{f}_2 = T_t\frac{\boldsymbol{\Delta}}{|\boldsymbol{\Delta}|}, \qquad \mathbf{f}_3 = T_d\frac{\boldsymbol{\delta}}{|\boldsymbol{\delta}|} - T_t\frac{\boldsymbol{\Delta}}{|\boldsymbol{\Delta}|}$$

where T_d is a viscoelastic force of the damping device

$$T_d = \begin{cases} k_d\varepsilon_d + c_d\dot{\varepsilon}_d, & |\boldsymbol{\delta}| \ge \delta_0 \\ 0, & |\boldsymbol{\delta}| < \delta_0 \end{cases} \tag{17}$$

where ε_d and $\dot{\varepsilon}_d$ is the relative deformation and rate-of-strain the damping device respectively

$$\varepsilon_d = \frac{|\boldsymbol{\delta}|}{\delta_0} - 1, \quad \dot{\varepsilon}_d = \frac{\boldsymbol{\delta} \cdot \dot{\boldsymbol{\delta}}}{\delta_0 |\boldsymbol{\delta}|} \tag{18}$$

The force T_t is given by (10) when $\boldsymbol{\Delta}$ is calculated by the expression (15).

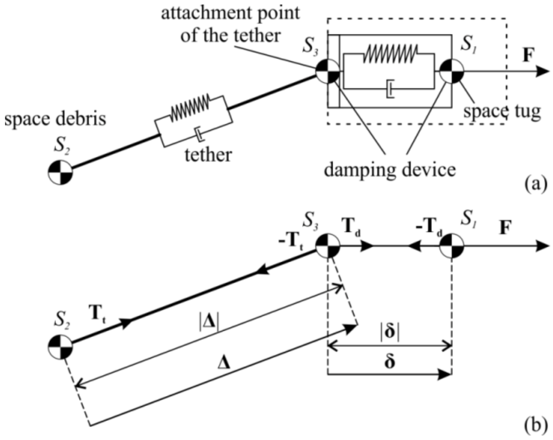

Figure 4. Three-body system.

4. Numerical analysis

To illustrate the performance of the developed system, numerical simulations are used. Let us consider the two-body system and the three-body system with parameters presented in Table 1. We compare the motion of the system without damping device (8) and with it (16). The simulations are run for 3000 seconds for the initial altitude is 800 km and the initial velocities of the space tug and space debris to each other and correspond to a circular orbit. The rendezvous maneuver is performed within the first 2500 seconds, during which the tether length is reduced to the value L. After the rendezvous maneuver is terminated, the whole system continues to orbital motion. Simulations are based on the numerical integration of the motion equations (8) and (16) by means of a Runge–Kutta algorithm of fourth order with variable step.

Table 1. System parameters.

Parameters	Value
Initial tether length l_0, m	1000
Final tether length L, m	0.1
Initial length o the damping device δ_0, m	0.3
Space tug mass m_1, kg	800
Space debris mass m_2, kg	2000
Mass of the damping device m_3, kg	8
Duration of the maneuver t_k, s	2500
Stiffness of the tether k_t, N	6000
Damping ratio of the tether c_t, Ns	4000
Stiffness of the damping device k_d, N	10
Damping ratio of the damping device c_d, Ns	100

Table 2 presents initial conditions for the our systems which is describes by the equations (8) and (16) respectively. Figures 5-7 presents the result of numerical simulations for two cases: without damping device and with it.

Table 2. Initial conditions.

$\mathbf{\Delta} = \left\{\Delta_x, \Delta_y, \Delta_z\right\}^T$, m	$\left\{1000, 0, 0\right\}^T$
$\dot{\mathbf{\Delta}} = \left\{\dot{\Delta}_x, \dot{\Delta}_y, \dot{\Delta}_z\right\}^T$, m/s	$\left\{0, 0, 0\right\}^T$
$\boldsymbol{\delta} = \left\{\delta_x, \delta_y, \delta_z\right\}^T$, m	$\left\{0.3, 0, 0\right\}^T$
$\dot{\boldsymbol{\delta}} = \left\{\dot{\delta}_x, \dot{\delta}_y, \dot{\delta}_z\right\}^T$, m/s	$\left\{0, 0, 0\right\}^T$

Figure 5 shows the changing of the angle α which define as

$$\alpha = \mathrm{ArcTan}\left(\frac{\Delta_y}{\Delta_x}\right).$$

Figure 5 (a) shows the case without the damping device (the first case), figure 5 (b) shows case with it (the second case). Figure 5 shows that the amplitude of oscillations of the angle α in the second case almost two times less than in the first case. The value of angle α begins to grow in the end of the maneuver rendezvous. After that his value oscillates around zero.

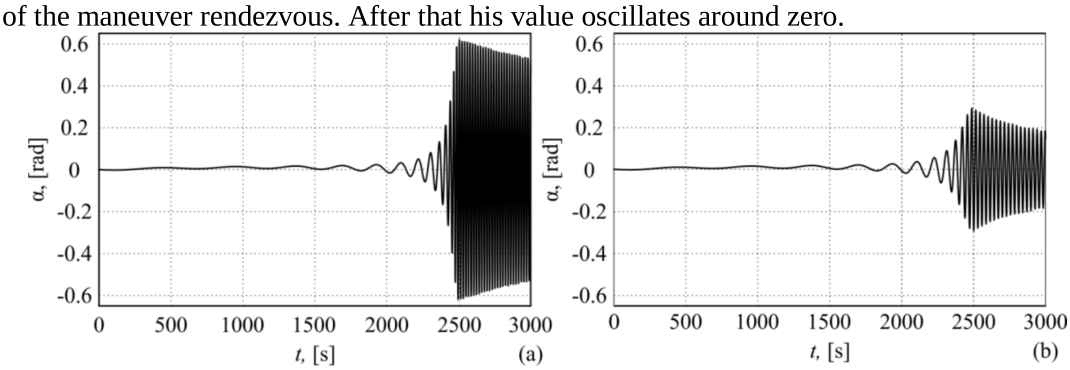

Figure 5. Change of the angle α : a - for the two-body system; b for the three-body system.

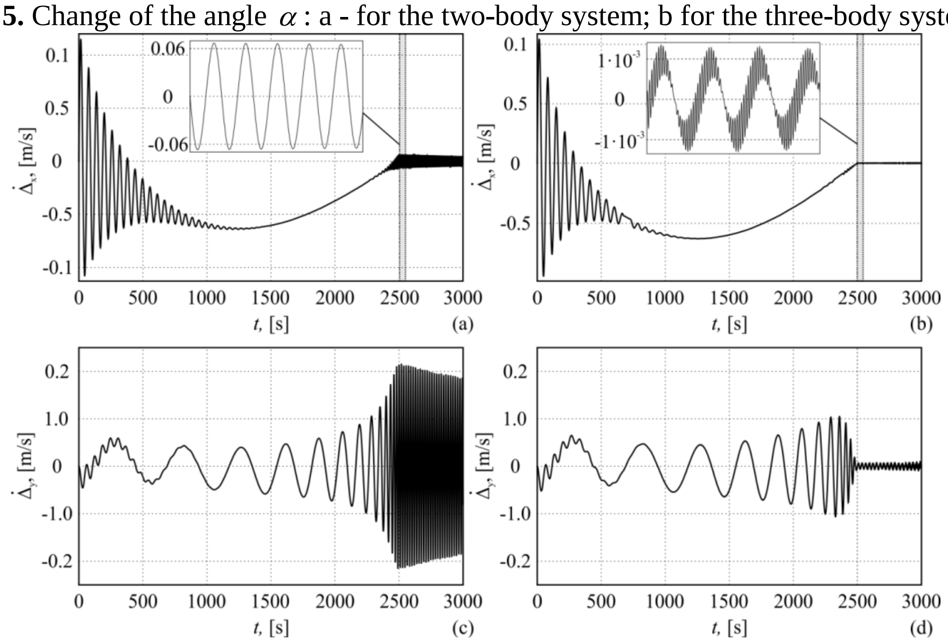

Figure 6. Change of the velocities $\dot{\Delta}_x, \dot{\Delta}_y$: a, c - for the two-body system; b, d for the three-body system.

Figures 6-7 depict respectively the graphics of change of the velocities $\dot{\Delta}_x, \dot{\Delta}_y$ and coordinate Δ_x, Δ_y in the first and the second cases. Note that in this work the motion of the system occurs in plane Sxy, this is due to the choice of the initial conditions when $\Delta_z = \delta_z = \dot{\Delta}_z = \dot{\delta}_z = 0$.

Figure 7 shows that after the end of the maneuver rendezvous, values of velocities $\dot{\Delta}_x, \dot{\Delta}_y$ oscillates around zero in both cases. Figure 8 depict the oscillation of the coordinates Δ_x, Δ_y in the end maneuver in the both case. Note that the amplitude of oscillations for $\dot{\Delta}_x, \dot{\Delta}_y, \Delta_x, \Delta_y$ after the end of the maneuver in the second case more less than in the first case. As we can see from figures 6 (b, d) and 7 (b, d) the oscillations of the tether have decreased.

Figures 6-7 depict that law (13) ensure the performance of the rendezvous and can be used for tasks of implement of the rendezvous of spacecraft particular for tasks the rendezvous of the space tug and the space debris.

5. Conclusion

We consider here a dynamics of the active debris removal system which consist of the tug and the space debris connected by the viscoelastic tether. The control law for the tether was proposed. The high frequency oscillations of the viscoelastic tether appeared on the final docking phase. Therefore, the mechanical damper has been added to the system for damping oscillations of the tether.

We developed mathematical models: the two-body system, without the damping device, and the three-body system, with damping device, to study of dynamics during the rendezvous maneuver. It was shown that, the maneuver rendezvous leads to oscillations of the tether in the case without damping device. Adding the damping device allowed to eliminate or significantly reduce the oscillations of the tether.

In general, we believe that the above approach to reduce of oscillation of tether can provide good results for a large variety of applications for missions rendezvous spacecrafts. Further research on the subject should verify the dynamics with regard to the consideration of the tug and the space debris as solids bodies.

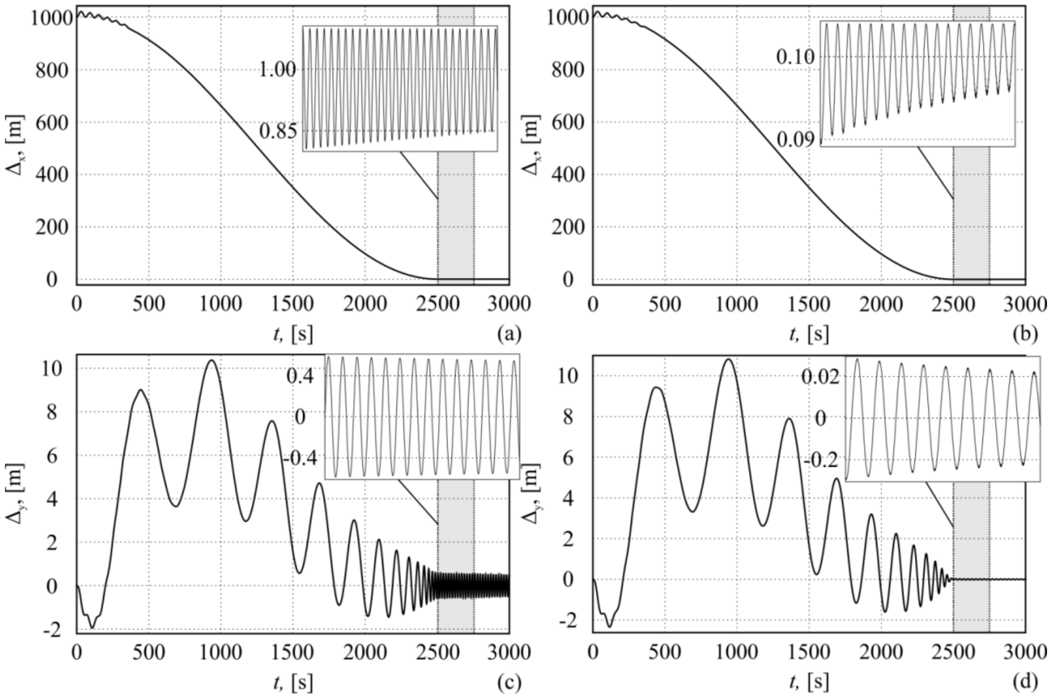

Figure 7. Change of the coordinate Δ_x, Δ_y: a, c - for the two-body system; b, d for the three-body system.

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7. References

[1] Kessler, D.J. Collision frequency of artificial satellites: the creation of a debris belt / D.J. Kessler, B.G. Cour-Palais // Journal of geophysical research. – 1978. – Vol. 83. – P. 2637-2646.

[2] Kessler, D.J. The kessler syndrome: implications to future space operations / D.J. Kessler, N.L. Johnson, J.C. Liou, M. Matney // Advances in the Astronautical Sciences. – 2010. – Vol. 137. – P.1-15.

[3] Pelton, J.N. New solutions for the space debris problem / J.N. Pelton. – Springer Cham Heidelberg New York Dordrecht London, 2015.

[4] Bonnal, C. Active debris removal: Recent progress and current trends / C. Bonnal, J.M. Ruault, M.C. Desjean // Acta Astronautica. – 2013. – Vol. 85. – P. 51-60.

[5] Anselmo, L. Ranking upper stages in low Earth orbit for active removal / L. Anselmo, C. Pardini // Acta Astronautica. – 2016. – Vol. 122. – P. 19-27.

[6] Aslanov, V.S. Rigid Body Dynamics for Space Applications / V.S. Aslanov. – Elsevier, 2017. – 420 p.

[7] DeLuca, L.T. Active debris removal by a hybrid propulsion module / L.T. DeLuca, F. Bernelli, F. Maggi, P. Tadini, C. Pardini // Acta Astronautica. – 2013. – Vol. 91. – P. 20-33.

[8] Sabatini, M. Elastic issues and vibration reduction in a tethered deorbiting mission / M. Sabatini, P. Gasbarri, G.B. Palmerini // Advances in Space Research. – 2016. – Vol. 57. – P. 1951-1964.

[9] Wen, H. Constrained tension control of a tethered space-tug system with only length measurement / H. Wen, Z.H. Zhu, D. Jin, H. Hu // Acta Astronautica. – 2016. – Vol. 119. – P. 110-117.

[10] Benvenuto, R. Dynamics analysis and GNC design of flexible systems for space debris active removal / R. Benvenuto, S. Salvi, M. Lavagna // Acta Astronautica. – 2015. – Vol. 110. – P. 247-265.

[11] Pang, Z. Chaotic motion analysis of a rigid spacecraft dragging a satellite by an elastic tether / Z. Pang, B. Yu, D. Jin // Acta Mechanica. – 2015. – Vol. 226. – P. 2761-2771.

[12] Lee, J. Input shaped large thrust maneuver with a tethered debris object / J. Lee, H. Schaub, H. // Acta Astronautica. – 2014. – Vol. 96. – P. 128-137.

[13] Lee, J. Tethered towing using open-loop input-shaping and discrete thrust levels / J. Lee, H. Schaub // Acta Astronautica. – 2014. – Vol. 105. – P. 373-384.

[14] Nishida, S. Strategy for capturing of a tumbling space debris / S. Nishida, S. Kawamoto // Acta Astronautica. – 2011. – Vol. 68. – P. 113-120.

[15] Misra, A.K. Dynamics and control of tether connected two-body systems a brief review / A.K. Misra // Acta Astronautica. – 2008. – Vol. 63. – P. 1169-1177.

[16] Aslanov, V.S. 2012. Dynamics of the tethered satellite system / V.S. Aslanov, A.S. Ledkov. – Elsevier, 2012. – 356 p.

[17] Aslanov, V.S. Chaos Behavior of Space Debris During Tethered Tow / V.S. Aslanov // Journal of Guidance, Control, and Dynamics. – 2016. – Vol. 39. – P. 2399-2405.

[18] Aslanov, V.S. Dynamics, analytical solutions and choice of parameters for towed space debris with flexible appendages / V.S. Aslanov, V.V. Yudintsev // Advances in Space Research. – 2015. – Vol. 12. – P. 660-667.

[19] Aslanov, V.S. Behavior of Tethered Debris With Flexible Appendage / V.S. Aslanov, V.V. Yudintsev // Acta Astronautica. – 2014. – Vol. 104. – P. 91-98.

[20] Aslanov, V.S. Dynamics of towed large space debris taking into account atmospheric disturbance / V.S. Aslanov, A.S. Ledkov // Acta Mechanica. – 2014. – Vol. 225. – P. 2685-2697.

[21] Aslanov, V.S. Dynamics of large debris connected to space tug by a tether / V.S. Aslanov, V.V. Yudintsev // Journal of Guidance, Control, and Dynamics. – 2013. – Vol. 36. – P. 1654-1660.

[22] Lee, J. Tethered tug for large low earth orbit debris removal / J. Lee, C.R. Seubert, H. Schaub, V. Trushkyakov, E. Yutkin // AAS/AIAA Asrodynamics Specialists Conference, 2012.
[23] Park, H. Experiments on Autonomous Spacecraft Rendezvous and Docking Using an Adaptive Artificial Potential Field Approach / H. Park, M. Romano, J. Virgili-Llop, R.I. Zappulla // AAS, 2016. – P. 4461-4478.
[24] Schaub, H. Analytical Mechanics of Aerospace Systems / H. Schaub, J.L. Junkins. – AIAA, 2003. – P. 578.
[25] Aslanov, V.S. Rendezvous of non-cooperative spacecraft and tug using a tether system / V.S. Aslanov, R.S. Pikalov // Engineering Letters. – 2017. – Vol. 25. – P. 142-146.
[26] Aslanov, V.S. The control of the rendezvous of two spacecraft using a tether system / V.S. Aslanov, R.S. Pikalov, E.R. Gunchin // Russian Aeronautics (in print).