Numerically Efficient Kalman Filter Based Channel Estimation for OFDM Data Transmission

I.V. Semushin[a], Yu.V. Tsyganova[a], A.V. Tsyganov[b], E.F. Prokhorova[a]

[a]*Information Technologies, 432000, Ulyanovsk State University, Ulyanovsk, Russia*
[b]*Higher Mathematics, 432071, Ulyanovsk State Pedagogical University, Ulyanovsk, Russia*

Abstract

Channel estimation and prediction algorithms are developed for use in broadband OFDM data transmission over non-ideal channels. The scalar complex channel coefficients are described by Gauss–Markov AR models of a given order in state space form to model the channel fading statistics. On this basis, the conventional Kalman filtering and prediction algorithm (CKFPA) is presented as a starting point for further development.

A novel numerically stable channel estimation algorithm based on the original KFPA solution, the so-called *extended Array UD Covariance Filter* (eUD-CF) algorithm, is developed. The accuracy of the eUD-CF estimator is analyzed by the method of computational experimentation. The simulation results demonstrate that the developed algorithm can effectively restrain the CKFPAs instability problem. The aspects of a parallel implementation of the suggested algorithms are also considered.

Keywords: data transmission; Kalman filtering; channel estimation; UD-based algorithms

Introduction

In the current era of building a better-connected world, there emerged many sophisticated radio and information technologies and standards. MIMO (Multiple-Input and Multiple-Output) technology is the use of multiple antennas to improve communication performance. Now it exists in different forms: as multi-antenna MIMO (or single user MIMO), as full multi-user MIMO (MU-MIMO or network MIMO), and as partial multi-user MIMO (or multi-user and multi-antenna MIMO). MU-MIMO is considered in recent WiMAX (Worldwide Interoperability for Microwave Exchange) standard as adoptable in the specification by many communication companies. As a wireless broadband technology, WiMAX is designed to provide 30 to 40 megabit-per-second data rates, with the 2011 update providing up to 1 Gbit/s for fixed stations.

The physical layer (PHY) of WiMAX is based on the orthogonal frequency-division multiplexing (OFDM) that ensures high-speed data, video, and multimedia digital communication in applications such as digital television and audio broadcasting. OFDM has developed into a modern scheme. It is a method of encoding digital data on multiple carrier frequencies based on the idea of dividing a given data stream with high bit-rate into several parallel low bit-rate data streams. The data streams are then modulated on separate carriers often known as sub-carriers or tones.

OFDM Fundamentals [1]

The OFDM architecture (with cyclic prefix) is depicted in Fig. 1. On the transmitter (blocks 1 to 5), the input stream comes as a series of data bits in the time domain. In block 1, it is divided into N parallel sub-streams moving data with reduced data rates. In block 2, each sub-stream is converted to a required modulation format using one of the several available digital modulation techniques: PPM – pulse-position modulation also known as pulse-phase modulation, QAM – quadrature amplitude modulation, QPSK – quadrature phase shift keying, BPSK – binary phase shift keying, or others. The sub-stream converted data bits are superimposed on the orthogonal subcarriers being a set of N parallel sinusoidal oscillators tuned to N orthogonal frequencies $f_0, f_1, \ldots, f_{N-1}$. Resulting values X_k, $k = 0, 1, \ldots, (N-1)$ in the set $\{X_0, X_1, \ldots, X_{N-1}\}$ are treated in complex domain $\left(\text{with } j = \sqrt{-1}\right)$ as mapped to the IFFT inputs (to block 3). The IFFT output set of values

$$x[n] = \sum_{k=0}^{N-1} X(k)\sin\left(\frac{2\pi kn}{N}\right) - j\sum_{k=0}^{N-1} X(k)\cos\left(\frac{2\pi kn}{N}\right), \quad n = 0, 1, \ldots, (N-1)$$

is added with cyclic prefix (CP) and then converted from the parallel to serial form $s[n] = (x_0, x_1, \ldots, x_{N-1})$ treated as a single OFDM signal symbol $s[n]$ to be transmitted through the channel at time n. Adding CP in block 4 to each OFDM signal symbol assists to eradicate the ISI (Inter Symbol Interference) and ICC (Inter Carrier Interference) as long as the duration of the CP is lengthier than the channel delay spread. The delay spread of a multipath channel is defined as the root-mean-square (RMS) value of the delay of reflections in the channel which is proportionally weighted to the energy of the reflected waves initiated by the objects in the channel, the medium between the transmitting antenna and the receiving antenna.

The transmitted signal symbol $s[n]$ (at time n) is externally influenced by two phenomena: 1) multiplication by the channel impulse response $h[n]$ often refered to as "channel" and 2) addition of noise $v[n]$, thus resulting in the received signal symbol

$$y[n] = s[n]h[n] + v[n].$$

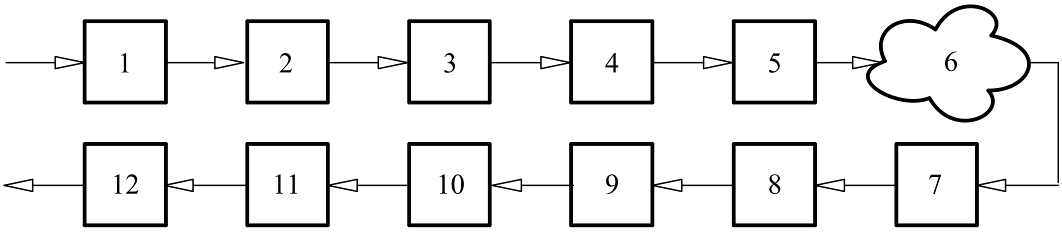

Fig. 1. OFDM Architecture with Cyclic Prefix. *Legend:* 1 – Serial to Parallel; 2 – Modulation; 3 – IFFT (Inverse FFT); 4 – Add Cyclic Prefix; 5 – Parallel to Serial; 6 – Channel; 7 – Serial to Parallel; 8 – Delete Cyclic Prefix; 9 – FFT (Fast Fourier Transform); 10 – Frequency Domain Equalizer; 11 – Demodulation; 12 – Parallel to Serial.

At the receiver (blocks 7 to 12), $y[n]$ as a series of values $(y_0, y_1, \ldots, y_{N-1})$ acquires the parallel form (in block 7) from which CP is deleted in block 8. The parallel set of values $(r_0, r_1, \ldots, r_{N-1})$ obtained by this means is put through the FFT process (in block 9) to obtain values

$$R[k] = \sum_{n=0}^{N-1} r(n) \sin\left(\frac{2\pi kn}{N}\right) + j \sum_{n=0}^{N-1} r(n) \cos\left(\frac{2\pi kn}{N}\right), \quad k = 0, 1, \ldots, (N-1)$$

in complex domain. Equalization (in block 10) is the process of adjusting the balance between these frequency components within an electronic signal which may suffer severe attenuation distortion that occurs during transmission when the transmission medium does not have a flat frequency response (or channel impulse response, CIR) across the bandwidth of the medium or the frequency spectrum of the signal. In digital subscriber line (DSL) circuits, echoes due to impedance mismatch often cause attenuation distortion so severe that some frequencies must be automatically mapped out and not used. When a channel has been "equalized," the frequency domain attributes of the signal at the OFDM system input are faithfully reproduced at the output.

The need for a good equalizer explains why the channel estimation is a crucial task.

OFDMA Basics [2]

Orthogonal Frequency-Division Multiple Access (OFDMA) is a MU version of the OFDM digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users. Each user has access to only a specific (allocated to the user) portion of the wide OFDM spectrum. This allows simultaneous low data rate transmission from several users both on the DL (downlink) and UL (uplink) direction. OFDMA, as distinct from OFDM, organizes the time (i. e. symbols) and the frequency (i. e. sub-carriers) resources into sub-channels for allocation to different users which permits for multiple access. The symbol structure of the OFDMA consists of three types of sub-carriers namely: *Data Sub-carriers*, they are used for the transmission of information data; *Pilot Sub-carriers*, they are used for channel estimation and for synchronization purposes; and *Null Sub-carriers*, these are used as guard bands and DC (Direct Current) subcarriers.

OFDMA sub-channelization involves the grouping of active data and pilot subcarriers into groups called sub-channels. The sub-channels are then dynamically allocated for subscribers based on channel condition and data requirements. Sub-channelization is supported by the WiMAX OFDMA-PHY both in the DL and UL. Using the OFDMA technique, specifically the Scalable-OFDMA (S-OFDMA), enables a flexible allocation of spectral resources for a wide range of channel bandwidths.

Research territory and a research niche

Channel estimation has attracted the attention of many researchers in the world. In [3], the authors are concerned with modeling of time-varying wireless long-term fading channels, parameter estimation, and identification from received signal strength data. Stochastic differential equations (SDEs) are used to model ultrawideband (UWB) indoor wireless channels in [4] and extended Kalman filter is employed to identify and estimate the channel parameters and states recursively. Paper [5] presents an overview of ultra-wideband (UWB) propagation channels and discusses measurement techniques and methods for extracting model parameters. Autoregressive (AR) stochastic models were considered in [6] for the computer simulation of correlated Rayleigh fading channels. The work [7] was devoted to the channel estimation of the MIMO-OFDM system based on Modified Kalman Filter (MKF). A comprehensive model for UWB propagation channels presented in [8] is based on a large number of measurement and simulation campaigns and includes the most important propagation effects in UWB channels; it was accepted as the standardized model by IEEE 802.15.4a. Authors of [9] have presented a first-order stochastic AR model for a wireless channel, which is based on SDE modeling of stationary Rayleigh fading wireless channels with "multipath reception" characterization. In [10], estimation of the channel matrix at the receiver with Extended Kalman Filter (EKF) was introduced for the blind equalization of OFDM signals that lessens a cost function which is composed of equalized and sliced symbols. An equalizer based on Kalman algorithm was developed in [11] and put into practice for the parallel OFDM modem "South Wind" (Russia production). Paper [12] investigates the estimation of time varying channels for UWB impulse radio communication, where the channel parameters considered are the attenuations and delays incurred by the signal echoes along the different propagation paths. A new form of Kalman filter with improved numerical properties is presented in [13] and its application communications signal processing is discussed.

As it can be seen from the literature, central problems in channel estimation are: (a) building a channel model and (b) using Kalman filtering technique for the model state (and parameters) estimation. Most of the recent studies have focused on a broad array of issues surrounding the AR channel models as being the best option for finding an adequate solution to the both problems.

However, the methods mentioned above as well as other existing channel estimating principles suffer from some limitations mainly concerning the treatment of the AR model as having, in this application, the lowest order (the order is unity) and the use of the simplest possible version of Kalman Filter, the so-called "conventional KF," which is known to be vulnerable to ill-conditioned data and so numerically inefficient. The steady method upon which the present channel estimation is based, eliminates these limitations by introducing a numerically efficient Kalman filter algorithm, and also it can be implemented in a parallel fashion for the OFDMA (multi-user) OFDM system.

The paper is organized as follows. Section 1 creates the mathematical framework for the problem considered by describing the Gauss–Markov AR model written in the state space concerning channel states and measurements (the receiver inputs). The mathematical notation used here or hereafter is as is customary in stochastic models and estimation theory, not as is the convention in radio engineering. In Section 2, the concept of Conventional Kalman filter is briefly introduced intending to the task of channel estimation. Development results of a novel numerically stable channel estimation algorithm based on the original solution, the so-called *Extended Array UD Covariance Filter* (eUD-CF) algorithm, is presented in Section 3. In Section 4, a parallel architecture implementing the eUD-CF algorithm for channel estimation most efficiently, is given, followed by conclusions in the paper ending.

1. Channel Model for OFDM Links

Consider the Gauss–Markov AR model written in the state space:

$$\begin{aligned} x_{t+1} &= \Phi_t x_t + \Gamma w_t, \ t = 0, 1, \ldots; \\ y_t &= H_t x_t + v_t, \ t = 1, 2, \ldots \end{aligned} \tag{1}$$

where combined state vector $x_t \triangleq \left[x_t^{(1)T} \mid \cdots \mid x_t^{(m)T}\right]^T$ consists of m sub-vectors $x_t^{(k)T} \triangleq \left[x_{t-n_k+1}^{(k)} \mid \cdots \mid x_{t-1}^{(k)} \mid x_t^{(k)}\right]$, $k = 1, \ldots, m$, corresponding to m different sub-channels; y_t is the measurement vector; the process noise w_t and the measurement noise v_t are mutually independent random (Gaussian) sequences, i. e., $w_t \sim \mathcal{N}(0, Q_t)$ and $v_t \sim \mathcal{N}(0, R_t)$. Let these noises be independent of some random initial state $x_0 \sim \mathcal{N}(\bar{x}_0, P_0)$.

Matrices that determine the system (1) have the following block (array) forms:

$$\begin{aligned} \Phi_t &= \operatorname{diag}\left[\Phi_t^{(1)}, \Phi_t^{(2)}, \cdots, \Phi_t^{(m)}\right], \quad \Gamma = \operatorname{diag}\left[\Gamma^{(1)}, \Gamma^{(2)}, \cdots, \Gamma^{(m)}\right], \quad Q_t = \operatorname{diag}\left[q_t^{(1)}, q_t^{(2)}, \cdots, q_t^{(m)}\right]; \\ H_t &= \operatorname{diag}\left[H_t^{(1)}, H_t^{(2)}, \cdots, H_t^{(m)}\right], \quad R_t = \operatorname{diag}\left[r_t^{(1)}, r_t^{(2)}, \cdots, r_t^{(m)}\right] \end{aligned} \tag{2}$$

where

$$\left.\begin{aligned} \Phi_t^{(k)} &= \left[\begin{array}{c|ccc} \emptyset & & I & \\ \hline a_{n_k-1,t}^{(k)} & \cdots & a_{1,t}^{(k)} & a_{0,t}^{(k)} \end{array}\right] \\ \Gamma^{(k)} &= [0, \cdots, 0, 1]^T, \ n_k\text{-dimensional} \\ H_t^{(k)} &= [0, \cdots, 0, s_t^{(k)}], \ n_k\text{-dimensional} \end{aligned}\right\} \quad \begin{aligned} k &= 1, \ldots, m \\ m &= \text{the number of sub-channels} \\ n_k &= 1, \ldots, N \\ n_k &= \text{the } k^{\text{th}} \text{ sub-channel dimension} \\ N &= \text{the greatest possible } n_k \end{aligned} \tag{3}$$

The following nomenclature (Table 1) gives an insight into variables and parameters used in the channel model:

Table 1. Model nomenclature

Name	Meaning	Range	Dimension
x_t	channel (state of)	$\mathbb{C}^n$	$n = \sum_{k=1}^m n^{(k)}$
Φ_t	channel transition	$\mathbb{C}^{n\times n}$	$n \times n$
Γ	channel noise input matrix	$\mathbb{R}^{n\times m}$	$n \times m$
w_t	channel noise	$\mathbb{C}^m$	m
y_t	observed signal	$\mathbb{C}^m$	m
H_t	channel pilot subcarriers	$\mathbb{C}^{m\times n}$	$m \times n$
v_t	channel observation noise	$\mathbb{C}^m$	m
Q_t	channel noise covariance	$\mathbb{C}^{m\times m}$	$m \times m$
R_t	channel observation noise covariance	$\mathbb{C}^{m\times m}$	$m \times m$

Our goal is to estimate the unknown state vector x_t, i. e., to calculate, at each discrete time instant t, the one-step predicted estimate $\hat{x}_{t|t-1}$ minimizing the MSE criterion $\mathrm{E}\left[(x_t - \hat{x}_{t|t-1})^H (x_t - \hat{x}_{t|t-1})\right]$, given the available mesurements $Y_1^{t-1} = [y_1^T \mid \ldots \mid y_{t-1}^T]^T$. The well-known Kalman filter algorithm [14, 15] is an ideal theoretical tool for solving the linear estimation problem.

2. Channel Estimation Algorithm Based on Conventional Kalman Filter

Let us formulate the Conventional Kalman Filter (CKF) algorithm for UWB OFDM channel estimation with the above model. Taking into account the block diagonal structure of system matrices (3), we design the estimation algorithm in the form of m concurrent CKFs as it is described in the below.

For $k = 1, \ldots, m$ implement the discrete Kalman Filter $\mathcal{F}_k$ to estimate the unobservable state vector $x_t^{(k)}$ from the noise-corrupted measurements $\left(Y^{(k)}\right)_1^{t-1}$ as follows ($t = 0, 1, \ldots$):

$$\hat{x}^{(k)}_{t+1|t} = \Phi^{(k)}_t \hat{x}^{(k)}_{t|t-1} + K^{(k)}_{p,t} e^{(k)}_t, \qquad \hat{x}^{(k)}_{0|-1} = \bar{x}^{(k)}_0, \tag{4}$$

$$K^{(k)}_{p,t} = \Phi^{(k)}_t P^{(k)}_{t|t-1} H^{(k)H}_t \left(R^{(k)}_{e,t}\right)^{-1}, \qquad e^{(k)}_t = y^{(k)}_t - H^{(k)}_t \hat{x}^{(k)}_{t|t-1}, \qquad R^{(k)}_{e,t} = H^{(k)}_t P^{(k)}_{t|t-1} H^{(k)H}_t + R^{(k)}_t. \tag{5}$$

Matrix $P^{(k)}_{t|t-1}$ appearing in the above formulas is the error covariance matrix, i. e.,

$$P^{(k)}_{t|t-1} = \mathbf{E}\left\{\left(x^{(k)}_t - \hat{x}^{(k)}_{t|t-1}\right)\left(x^{(k)}_t - \hat{x}^{(k)}_{t|t-1}\right)^H\right\},$$

and it satisfies the difference Riccati equation

$$P^{(k)}_{t+1|t} = \Phi^{(k)}_t P_{t|t-1} \Phi^{(k)H}_t + \Gamma^{(k)} Q^{(k)}_t \Gamma^{(k)T} - K^{(k)}_{p,t} R^{(k)}_{e,t} K^{(k)H}_{p,t}, \qquad P^{(k)}_{0|-1} = \Pi^{(k)}_0 > 0. \tag{6}$$

Remark 1. At any discrete time instant t, one can easily obtain the predicted estimate $\hat{x}_{t|t-1} = \left[\hat{x}^{(1)T}_{t|t-1} \,\middle|\, \cdots \,\middle|\, \hat{x}^{(m)T}_{t|t-1}\right]^T$.

Remark 2. It is obvious that the bank of KFs $\{\mathcal{F}_k \,|\, k = 1, \ldots, m\}$ is ideal for organization of a parallel computing scheme that can significantly accelerate the process of channel estimation.

Many recent papers were devoted to the solution of various channel estimation problems with the use of Kalman filtering technique [16, 17, 18, 19]. However, it is well-known that the KF algorithm in its conventional form is numerically unstable due to the Riccati computational procedure described by equation (6) (see, for instance, the discussion in [20]). Matrix $P^{(k)}_{t|t-1}$ has the physical meaning of variance of the state prediction error, $x^{(k)}_t - \hat{x}^{(k)}_{t|t-1}$, and therefore has to be nonnegative-definite. As mentioned in [21], round-off errors may destroy this property thus resulting in a failure of filter $\mathcal{F}_k$. The problem of machine round-off errors is unavoidable due to the limited machine precision of real floating-point numbers. Unfortunately, it is impossible to fully remedy the problem. However, one can significantly reduce the effect of round-off errors by designing some algebraically equivalent Kalman filter implementations which may become the desired numerically efficient algorithms. Such solutions are based on variety of matrix factorization methods as applied to the prediction error covariance matrices involved in the filter equations.

Since the invention of the KF in the 1960s, there has been a keen interest in the development of numerically stable and efficient KF implementation methods [14]. In this paper we propose, instead of conventional KF, a new numerically efficient *extended array UD covariance filter* (eUD-CF) for solving the channel estimation problem.

3. Numerically Stable Channel Estimation Algorithm Based on Extended Array UD Covariance Filter

The main idea of extended array algorithm is that all the quantities of the discrete filter are updated in an array form using the orthogonal matrix transforms. It means that numerically stable orthogonal transforms are used for updating the corresponding factors of the state error covariance and state estimate at each iteration step. More precisely, orthogonal operators are applied to the pre-array (which accommodates the filter quantities available at the current step) to get the post-array in a special form. Then, the required updated filter quantities are simply read out from the post-array. This feature makes the array algorithms better suited to the parallel computations and to the very large scale integration (VLSI) implementations [22, 23]. The first information-type extended array filter was built in [24]. The covariance-type extended array square-root algorithms were constructed in [23].

The special feature of the UD KF methods is that they are based on the modified Cholesky factorization of the state prediction error covariance matrix $P = UDU^T$, where U is a unit upper triangular matrix, D is a diagonal matrix and $D > 0$. The first UD implementation of the KF was Bierman's sequential algorithm [25].

Recently, a new extended array UD covariance filter (eUD-CF) was proposed in [26]. Its benefits are as follows:

1) robustness of computations against round-off errors,
2) lack of square-root operation,
3) avoiding the matrix inverse operation on each iteration of algorithm,
4) a compact and convinient extended array form of the UD filter.

Let us reformulate the algorithm eUD-CF adjusting it to the specific model structure (1) and (2).

First, we use the modified Cholesky decomposition of covariance matrix $P_{t|t-1} = U_{P_{t|t-1}} D_{P_{t|t-1}} U^H_{P_{t|t-1}}$ where U is a unit upper triangular matrix and D is a diagonal matrix. Second, we introduce the following notations:

$$\hat{z}_t = \left(U_{P_{t|t-1}} D_{P_{t|t-1}}\right)^{-1} \hat{x}_{t|t-1} \quad \text{and} \quad b_t = -\left(U_{R_{e,t}} D_{R_{e,t}}\right)^{-1} e_t.$$

Finally, given the measurements $\left(Y^{(k)}\right)_1^{t-1}$ for $k = 1, \ldots, m$, then the new eUD-CF algorithm intended to estimate the unobservable state vector $x^{(k)}_t$ takes the following form.

Algorithm 1. *Extended array UD Covariance Filter* (eUD-CF)

0. Initial data: $\hat{z}_0 = 0$ $P_{0|-1} = \Pi_0 > 0$. Calculate the modified Cholesky factors $\{U_{\Pi_{0|-1}}, D_{\Pi_{0|-1}}\}$.
   Set $U_{Q_0} = I$, $D_{Q_0} = Q_0$, $U_{R_0} = I$, $D_{R_0} = R_0$.
1. Recursively update $\{U_{P_{t+1|t}}, D_{P_{t+1|t}}\}$ and $\hat{z}_{t+1}$ as follows ($t \geq 0$):
   1.1. For $\{U_{P_{t|t-1}}, D_{P_{t|t-1}}\}$, $\{U_{R_t} = I, D_{R_t} = R_t\}$, $\{U_{Q_t} = I, D_{Q_t} = Q_t\}$, $\hat{z}_t$ construct a pair of the pre-arrays $\{\mathcal{A}_t, \mathcal{D}_t\}$:

$$\mathcal{D}_t = \text{diag}\{D_{Q_t}, D_{P_{t|t-1}}, D_{R_t}\}, \quad \mathcal{A}_t^H = \begin{bmatrix} 0 & \hat{z}_t^H & -y_t^H \left(U_{R_t} D_{R_t}\right)^{-H} \\ \Gamma_t U_{Q_t} & \Phi_t U_{P_{t|t-1}} & 0 \\ 0 & H_t U_{P_{t|t-1}} & U_{R_t} \end{bmatrix}. \tag{7}$$

   1.2. Apply the modified weighted Gram-Schmidt (MWGS) orthogonalization of the columns of $\mathcal{A}_t$ with respect to the weighting matrix $\mathcal{D}_t$ to obtain a pair of the post-arrays $\{\mathcal{A}_t^\dagger, \mathcal{D}_t^\dagger\}$:

$$\mathcal{D}_t^\dagger = \text{diag}\{(*), D_{P_{t+1|t}}, D_{R_{e,t}}\}, \quad \mathcal{A}_t^\dagger = \begin{bmatrix} 1 & \hat{z}_{t+1}^H & b_t^H \\ 0 & U_{P_{t+1|t}} & K_{p,t} U_{R_{e,t}} \\ 0 & 0 & U_{R_{e,t}} \end{bmatrix}$$

   such that $\mathcal{A}_t^H = \mathcal{A}_t{}^\dagger B_t^H$ and $\mathcal{A}_t{}^H \mathcal{D}_t \mathcal{A}_t = \mathcal{A}_t{}^\dagger \mathcal{D}_t{}^\dagger \left(\mathcal{A}_t{}^\dagger\right)^H$, where B_t is the MWGS-UD transformation that produces the block upper triangular matrix $\mathcal{A}_t{}^\dagger$ and diagonal matrix $\mathcal{D}_t{}^\dagger$; $(*)$ is a value of no interest.

Remark 3. In our algoritm description, we omit the superscript $^{(k)}$ for simplicity of notation.
Remark 4. At any discrete time instant t, one can easily obtain the predicted estimate

$$\hat{x}_{t|t-1} = \left(U_{P_{t|t-1}} D_{P_{t|t-1}}\right) \hat{z}_t. \tag{8}$$

The practical implementation scheme of eUD-CF algorithm is very simple. It consists of only 3 steps:

① Fill in the block pre-arrays with the input data.

② Perform the MWGS-UD transformation.

③ Extract the required output data from the block post-arrays.

The eUD-CF implementation scheme is shown in Fig. 2.

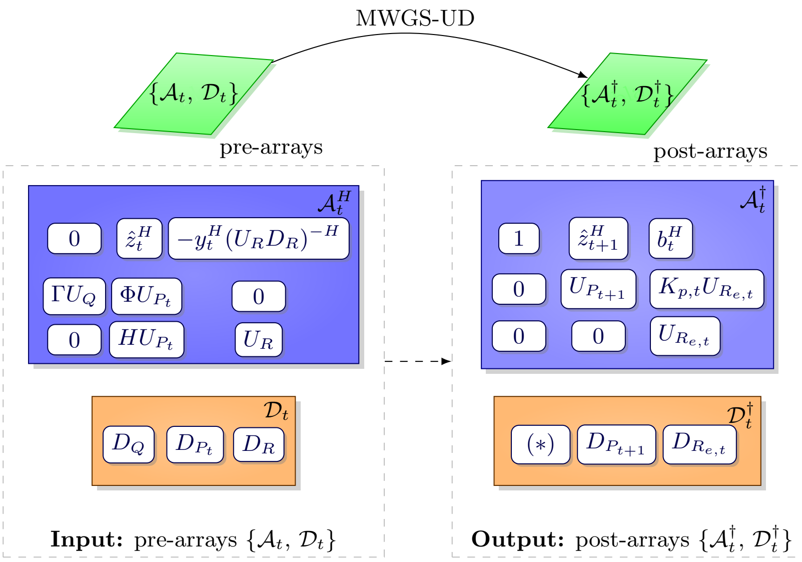

Fig. 2. The eUD-CF implementation scheme.

In what follows, we discuss the accuracy of the designed eUD-CF estimator presented in Algorithm 1. It has the main property to improve accuracy and robustness of the computations for a finite-precision computer arithmetics. To check this fact, we consider the set of ill-conditioned test problems from [14].

Example1. (Set of ill-conditioned test problems)

Consider the state-space model (1) with $\{F, G, H, Q, R\}$ given by

$$\Phi = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, G = \begin{bmatrix}0\\0\\0\end{bmatrix}, Q = \begin{bmatrix}1\end{bmatrix}, \quad R = \begin{bmatrix}\delta^2 & 0\\ 0 & \delta^2\end{bmatrix}, H = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1+\delta\end{bmatrix}$$
$$\text{with } x_0 \sim \mathcal{N}(0, I_3)$$

where I_3 denotes the 3×3-identity matrix. To simulate round-off we assume that $\delta^2 < \varepsilon_{\text{round-off}}$, but $\delta > \varepsilon_{\text{round-off}}$ where $\varepsilon_{\text{round-off}}$ denotes the unit round-off error[1], i.e., the machine precision limit.

The above set of ill-conditioned problems demonstrates how a problem that is well conditioned, as posed, can be made ill-conditioned by the filter. It is often used in the Kalman filtering community for observing the influence of round-off errors on various KF implementations. The difficulty is in matrix inversion $R_{e,t}$. After processing only the first measurement y_1, the matrix $R_{e,0} = H\Pi_0 H^T + R$ becomes singular in machine precision, i.e., as $\delta \to \varepsilon_{\text{round-off}}$. This yields the failure of the conventional Kalman filter.

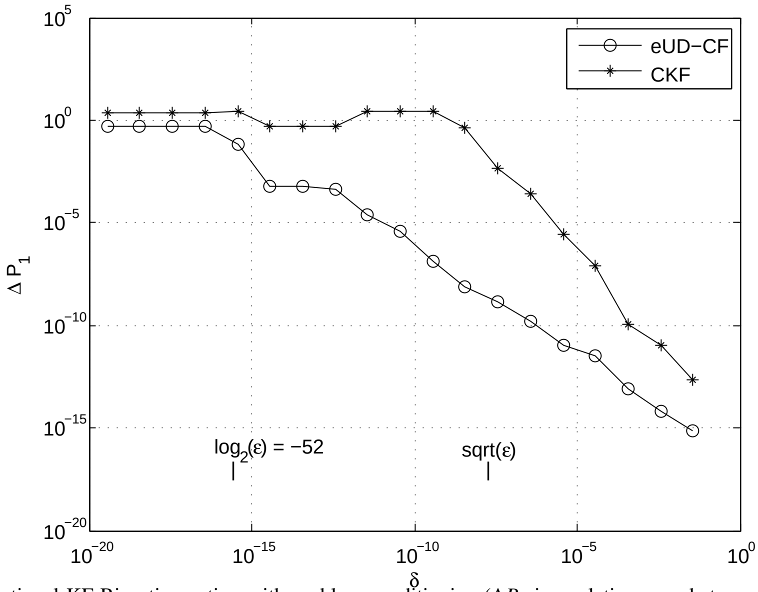

Fig. 3. Degradation of the conventional KF Riccati equation with problem conditioning (ΔP_1 is a relative error between calculated (numerical) $P_{1|0}$ and exact (symbolic) $P_{1|0}$).

We perform the only one iteration of conventional Kalman filter (CKF) and eUD-CF algorithm 1 for various values of δ while $\delta \to \varepsilon_{\text{round-off}}$. Figure 3 in the below illustrates how the conventional Kalman filter (CKF) and the new eUD-CF method perform on the variably ill-conditioned problem of Example 1 as the conditioning parameter $\delta \to 0$. Both solution methods were implemented as MATLAB m-files and in the same double precision setting.

For this simple example, the accuracy of the eUD-CF method appears to degrade more gracefully than the conventional Kalman Filter as $\delta \to \varepsilon_{\text{round-off}}$, the machine precision limit. The eUD-CF based solution still maintains about nine digits ($\approx$ 30 bits) of accuracy at $\delta \approx \sqrt{\varepsilon_{\text{round-off}}}$, when the CKF has essentially no bits of accuracy in the computed solution.

4. The New eUD-CF based Channel Estimation Framework

In this section, we propose a new channel estimation framework with m concurrent eUD-CF algorithms. The general scheme, which is diagrammatically shown in Fig. 4, is composed of a bank of blocks with MWGS-UD transforms, each filled in with its own subsystem matrices $\Phi_t^{(k)}$, $\Gamma^{(k)}$, $Q_t^{(k)}$, $H_t^{(k)}$, $R_t^{(k)}$. Each k-th eUD-CF algorithm, $k = 1, \ldots, m$ calculates its own estimate $\hat{z}_t^{(k)}$ independently of the other filters. Thus, the system (1) state estimate $\hat{x}_t$ is constructed as follows:

$$\hat{x}_t = \left[\left(U_{P_{t|t-1}}^{(1)} D_{P_{t|t-1}}^{(1)}\right)\hat{z}_t^{(1)} \mid \ldots, \left(U_{P_{t|t-1}}^{(m)} D_{P_{t|t-1}}^{(m)}\right)\hat{z}_t^{(m)}\right]^T.$$

This framework can be naturally implemented on a set of parallel processors because of its inherent parallel structure.

[1] Computer round-off for the floating-point arithmetic is often characterized by a single parameter $\varepsilon_{\text{round-off}}$, the latter defined in different sources as the largest number such that either $1 + \varepsilon_{\text{round-off}} = 1$ or $1 + \varepsilon_{\text{round-off}}/2 = 1$ in machine precision.

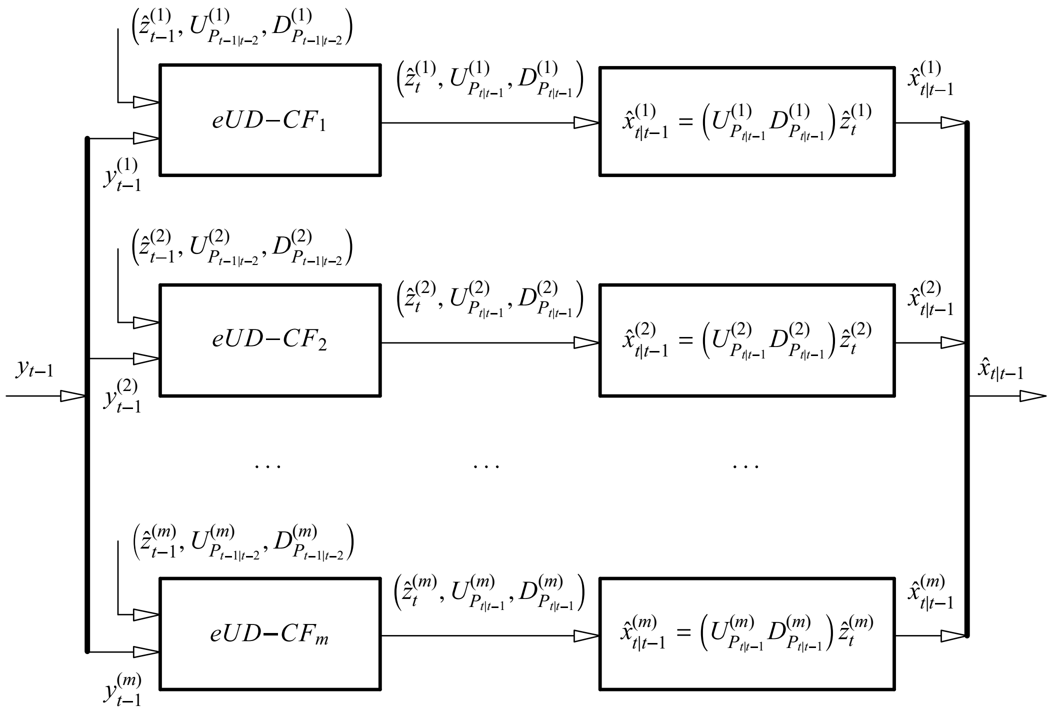

Fig. 4. The $eUD-CF$ based multi-sub-channel estimation framework.

Conclusions

The Kalman filter computations are intrinsically recursive thereby disembarrassing itself of any necessity to keep the past measurements in memory as time goes on. This feature is a well known one of great virtue. It makes using KF in practical implementations the most feasible option as compared with some other filtering methods. On the flip side, this useful feature may be reduced to zero if one tries to implement the Kalman filter in its originally developed form, the so-called "conventional KF algorithm." The divergence of KF estimates occurs while performing the covariance Riccati iterations embedded into the KF algorithm.

This paper demonstrates how one can apply the newest—divergency free—implementations of the Kalman filtering process for dealing with the OFDM channel impulse response estimation problem. Out of many candidates in the science of Kalman filter numerics developed in more recent times, it singles one of such implementations called the *extended array UD Covariance Filter (eUD-CF)*. Its powerful features are as follows:

1. The channel estimating results are robust against the round-off errors.
2. The computations do not contain the most time-consuming square-root operation.
3. The Riccati iterations are free of matrix inverse operations.
4. The compact and regular orthogonal array form of algorithm poses the best option for parallel computations in the OFDMA multi-sub-channel organization.

References

[1] Viswanathan, M. Simulation of Digital Communication Systems Using Matlab [eBook] [Text] / M. Viswanathan. — Second edition. — Los Gatos, CA : Smashwords, Inc., 2013. —Sep. 25. —ISBN : 9781301525089.

[2] Yin, H. OFDMA: A Broadband Wireless Access Technology [Text] / H. Yin, S. Alamouti // IEEE Sarnoff Symposium. —2006. —P. 1–4.

[3] Recursive Estimation and Identification of Time-Varying Long-Term Fading Channels [Text] / M. M. Olama, K. K. Jaladhi, S. M. Djouadi, Ch. D. Charalambous // Research Letters in Signal Processing. —2007. —Vol. 2007. —P. 1–6.

[4] Modeling, Real-Time Estimation, and Identification of UWB Indoor Wireless Channels [Text] / M. M. Olama, S. M. Djouadi, Ya. Li, A. Fathy // International Journal of Antennas and Propagation. —2013. —Vol. 2013. —P. 1–8.

[5] Molisch, A. F. Ultrawideband Propagation Channels— Theory, Measurement, and Modeling [Text] / A. F Molisch // IEEE Transactions on Vehicular Technology. —2005. —Vol. 54, no. 5. —P. 1528–1545.

[6] Baddour, K. E. Autoregressive Modeling for Fading Channel Simulation [Text] / K. E. Baddour, N. C. Beaulieu // IEEE Transactions on Wireless Communications. —2005. —Vol. 4, no. 4. —P. 1650–1662.

[7] Dalwadi, D. C. A Novel Channel Estimation Technique of MIMO-OFDM System based on Modified Kalman Filter [Text] / D. C. Dalwadi, H. B. Soni // Indian Journal of Science and Technology. —2016. —Vol. 9, no. 36. —P. 1–6.

[8] A Comprehensive Standardized Model for Ultrawideband Propagation Channels [Text] / A. Molisch, D. Cassioli, C-C. Chong [et al.] // IEEE Transactions on Antennas and Propagation. —2006. —Vol. 54, no. 11. —P. 3151–3166.

[9] Feng, T. S. Stochastic Differential Equation Theory Applied to Wireless Channels [Text] / T. (S). Feng, T. R. Field, S. Haykin // IEEE Transactions On Communications. —2007. —Vol. 55, no. 8. —P. 1478–1483.

[10] Kumar, S. Channel Estimation Using Extended Kalman Filter with Sliced Multi Modulus Blind Equalization Algorithm (SMMA) [Text] / S. Kumar, B. L. Malleswari // Journal of Theoretical and Applied Information Technology. —2016. —Vol. 90, no. 2. —P. 228–235.

[11] Zabirov, D. P. Equalizer on Basis of Kalman Algorithm [Text] / D. P. Zabirov // ONV (Omskiy Nauchnyi Vestnik): Information Technology, Radio Engineering and Communications. — 2008. — Vol. 1, no. 64. — P. 155–158. — [In Russian]. URL: http://cyberleninka.ru/article/n/ekvalayzer-na-osnove-algoritma-kalmana.
[12] Sethi, S. Channel Estimation Using Kalman Filter for UWB Communication Systems [Text] / S. Sethi, R. Pasand, Nielsen J. // Proceedings of MILCOM 2006, IEEE Military Communications Conference. (23-25 Oct. 2006). — [S. l.] : IEEE, 2006.
[13] Grewal, M. S. Kalman Filter Implementation With Improved Numerical Properties [Text] / M. S. Grewal, J. Kain // IEEE Transactions on Automatic Control. — 2010. — Vol. 55, no. 9. — P. 2058–2068.
[14] Grewal, M. S. Kalman Filtering: Theory and Practice Using MATLAB [Text] / M. S. Grewal, A. P. Andrews. — New Jersey : Prentice Hall, 2001.
[15] Kailath, T. Linear Estimation [Text] / T. Kailath, A. H. Sayed, B. Hassibi. — New Jersey : Prentice Hall, 2000. — 856 p.
[16] Zhang, S. A Kalman Filtering Channel Estimation Method Based on State Transfer Coefficient Using Threshold Correction for UWB Systems [Text] / Sh. Zhang, D. Wang, J. Zhao // International Journal of Future Generation Communication and Networking. — 2014. — Vol. 7, no. 1. — P. 117–124.
[17] Zhou, J. OFDM System Channel Estimation Algorithm Research Based on Kalman Filter Compressed Sensing [Text] / J. Zhou, G. Xia, J. Wang // Journal of Theoretical and Applied Information Technology. — 2013. — Vol. 49, no. 1. — P. 119–125.
[18] Shi, L. Ultra Wideband Channel Estimation Based on Kalman Filter Compressed Sensing [Text] / L. Shi, Z. Zhou, L. Tang // Transaction of Beijing Institute of Technology. — 2012. — Vol. 32, no. 2. — P. 170–173.
[19] Sternad, M. Channel Estimation and Prediction for Adaptive OFDM Downlinks [Text] / M. Sternad, D. Aronsson // Proceedings of VTC 2003, Vehicular Technology Conference. (6–9 Oct. 2003). — [S. l.] : IEEE, 2003.
[20] Verhaegen, M. Numerical Aspects of Different Kalman Filter Implementations [Text] / M. Verhaegen, P. Van Dooren // IEEE Trans. on Automat. Contr. — 1986. — Vol. AC-31, no. 10. — P. 907–917.
[21] Hassibi, B. Array Algorithms for H^∞ Estimation [Text] / B. Hassibi, T. Kailath, A. H. Sayed // IEEE Trans. Automat. Contr. — 2000. — Vol. 45, no. 4. — P. 702–706.
[22] Hotop, H.-J. New Kalman Filter Algorithms Based on Orthogonal Transformations for Serial and Vector Computers [Text] / H.-J. Hotop // Parallel Computing. — 1989. — Vol. 40, no. 12. — P. 233–247.
[23] Park, P. New Square-Root Algorithms for Kalman Filtering [Text] / P. Park, T. Kailath // IEEE Trans. on Automatic Control. — 1995. — Vol. 40, no. 5. — P. 895–899.
[24] Dyer, P. Extension of Square-Root Filtering to Include Process Noise [Text] / P. Dyer, S. McReynolds // J. Optimiz. Theory Appl. — 1969. — no. 3. — P. 444–459.
[25] Bierman, G. J. Factorization Methods For Discrete Sequential Estimation [Text] / G. J. Bierman. — New York : Academic Press, 1977. — 256 p.
[26] Tsyganova, Y. V. On the UD Filter Implementation Methods [Text] / Yu. V. Tsyganova // University Proceedings. Volga Region (Physical and Mathematical Sciences). — 2013. — no. 3. — P. 84–104. — [In Russian].