Numerical characteristics of image geometric deformation parameters estimates convergence at stochastic gradient estimation

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Abstract. Several approaches to the numerical description of image geometric deformations parameters estimates behavior at iterations of non-identification relay stochastic gradient estimation are considered. The probability density of the Euclidean mismatch distance of deformation parameters estimates vector is chosen as an argument of the characteristics forming the numerical values. It made it possible to ensure invariance to the set of parameters of the used inter-frame geometric deformations model. The mathematical expectation, the probability of exceeding a given threshold value of the convergence rate and the confidence interval of the Euclidean mismatch distance were investigated as characteristics. Probabilistic mathematical modeling is applied to calculate the probability density of the Euclidean mismatch distance. Examples of calculation are presented.

1. Introduction

Estimation of an image sequence geometric deformations parameters is one of the main problems of image processing [1-4]. In solving this problem, non-identification stochastic estimation proved to be good [5], wherein the formation of vector estimate $\bar{\boldsymbol{\alpha}}$ of deformation parameters of reference $\left\{z_{\bar{j}}^{(1)}\right\}$ and deformed $\left\{z_{\bar{j}}^{(2)}\right\}$ images, $\left\{\bar{j}=\left(j_x, j_y\right)^T\right\}$, can be described by the procedure [6, 7]:

$$\bar{\boldsymbol{\alpha}}_t=\bar{\boldsymbol{\alpha}}_{t-1}-\boldsymbol{\Lambda}_t \boldsymbol{\beta}_t\left(Z_t, \bar{\boldsymbol{\alpha}}_{t-1}\right), \quad (1)$$

where $\boldsymbol{\beta}(\cdot)$ – stochastic gradient of an objective function, which characterizes the quality of evaluation (the mean square of the difference between the brightness of the reference and deformed images was used as the objective function to calculate examples); $\boldsymbol{\Lambda}_t$ – gain matrix, determining a value of the estimates change at the t-th iteration; Z_t – two-dimensional local sample of the reference and deformed images used to determine the stochastic gradient at the t-th iteration [8, 9].

The local sample size (LSS) largely determines the nature of estimates deformation convergence and the computational cost. The research direction is due to the fact that the problems of LSS optimization are not investigated enough. The paper discusses the possibilities of numerical description of vector estimates of geometric deformations parameters estimates behavior at iterations of non-identification relay stochastic gradient estimation.

As initial information for the numerical description of vector estimates behavior, the probability distributions of the estimates deformation parameters are chosen. The paper investigated the mathematical expectation, the probability of exceeding a given threshold value of convergence rate and the confidence interval as characteristics that form numerical values. When estimating one deformation parameter, these characteristics are directly applicable to its evaluation. If the set of parameters is estimated, then at the same iteration for each parameter different values of the optimal LSS can be obtained. Since one local sample is formed at each iteration, its value will be chosen corresponding to the maximum of the optimal volumes, which will lead to unreasonable computational costs. Therefore, in the paper, the probability distribution of the Euclidean mismatch distance (EMD) for the vector of deformation parameter estimates is chosen as the argument of the studied characteristics. This made possible to ensure the invariance of the study to the set of parameters of the deformation model used.

2. Choice of argument characteristics

For definiteness, we assume that geometric deformations of the images are estimated and described by the model of similarity, which include parallel shift parameters $\bar{h} = \left(h_x, h_y\right)^T$, angle of rotation φ, scale factor κ. In this case, we note that limitation of the parameter vector does not limit the following consideration.

Let, after the $(t-1)$-th iteration the vector of deformation parameters estimates has values $\hat{\bar{\alpha}}_{t-1} = \left(\hat{h}_{x(t-1)}, \hat{h}_{y(t-1)}, \hat{\varphi}_{t-1}, \hat{\kappa}_{t-1}\right)^T$. In addition, each of the estimates $\hat{h}_{x(t-1)}$, $\hat{h}_{y(t-1)}$, $\hat{\varphi}_{t-1}$ and $\hat{\kappa}_{t-1}$ correspond to its own probability distribution: $w_{t-1}\left(\hat{h}_x\right)$, $w_{t-1}\left(\hat{h}_y\right)$, $w_{t-1}(\hat{\varphi})$ and $w_{t-1}(\hat{\kappa})$. Then, on the t-th iteration in the local sample the sample from resampled reference image with coordinates (x_a, y_b) will be taken for a couple with sample from deformed image with coordinates (a,b):

$$
\begin{aligned}
x_a &= x_0 + \hat{\kappa}_{t-1}\left((a-x_0)\cos\hat{\varphi}_{t-1} - (b-y_0)\sin\hat{\varphi}_{t-1}\right) + \hat{h}_{x(t-1)}, \\
y_b &= y_0 + \hat{\kappa}_{t-1}\left((a-x_0)\sin\hat{\varphi}_{t-1} + (b-y_0)\cos\hat{\varphi}_{t-1}\right) + \hat{h}_{y(t-1)},
\end{aligned}
\qquad (2)
$$

where (x_0, y_0) – the coordinates of the center of rotation.

The method for calculating the probability distribution of estimates of the image geometric deformations parameters was proposed in [10, 11] and involves the sampling of the domain of parameter definition. Using the method it is possible to obtain discrete probability distributions (DPD) of the parameters for the selected deformation model:

$$
\begin{aligned}
&w(h_x) = \left\{p_{l_x} = P\left(\hat{h}_x = h_{l_x}\right)\right\},\ l_x = \overline{1, L_x},\quad w(h_y) = \left\{p_{l_y} = P\left(\hat{h}_y = h_{l_y}\right)\right\},\ l_y = \overline{1, L_y}, \\
&w(\varphi) = \left\{p_{l_\varphi} = P\left(\hat{\varphi} = \varphi_{l_\varphi}\right)\right\},\ l_\varphi = \overline{1, L_\varphi},\quad w(\kappa) = p_{l_\kappa} = P\left(\hat{\kappa} = \kappa_{l_\kappa}\right),\ l_\kappa = \overline{1, L_\kappa},
\end{aligned}
$$

where $P(z)$ - the probability z; L_x, L_y, L_φ и L_κ – the number of intervals for splitting the parameter space h_x, h_y, φ and κ. Then the coordinates (2) with the probability $P_{l_x l_y l_\varphi l_\kappa} = p_{l_x} p_{l_y} p_{l_\varphi} p_{l_\kappa}$ take the values:

$$
\begin{aligned}
x_{l_x l_y l_\varphi l_\kappa} &= x_0 + \kappa_{l_\kappa}\left((a-y_0)\cos\varphi_{l_\varphi} - (b-x_0)\sin\varphi_{l_\varphi}\right) + h_{l_x}, \\
y_{l_x l_y l_\varphi l_\kappa} &= y_0 + \kappa_{l_\kappa}\left((a-y_0)\sin\varphi_{l_\varphi} + (b-x_0)\cos\varphi_{l_\varphi}\right) + h_{l_y}.
\end{aligned}
$$

Thus, it is possible to calculate a probability distribution of distances between a point with coordinates (a,b) on the deformed image and possible positions of the conjugate point on the reference image for current estimates of the deformation parameters, i.e. get DPD of the euclidean mismatch distance (EMD) $w_t(r)$ at the t-th iteration.

Let consider a few examples of the results for the calculation DPD EMD. Let the images have a Gaussian autocorrelation function and a signal-to-noise ratio (ratio of the variances of image and noise) equal to 14. For parameters estimation the stochastic procedure (1) of relay type with constant elements of diagonal gain matrix: $\lambda_{hx} = \lambda_{hy} = 0{,}05$, $\lambda_\varphi = 0{,}4$ and $\lambda_\kappa = 0{,}005$ is used. The same experimental conditions are used for the examples given below.

Figure 1 shows an example of the calculated DPD estimates $\hat{\varphi}$ and $\hat{\kappa}$ with $\mu = 1$ and the parameter mismatch $h_x = h_y = 4$, $\varphi = 15^o$ and $\kappa = 1{,}2$, after 60 iterations. Note that the probability distribution of estimates of all parameters are close to the Gaussian. Figure 2a shows DPD $w_{60}(r)$ of absolute value EMD: $r_{l_x l_y l_\varphi l_\kappa} = \sqrt{(a - x_{l_x l_y l_\varphi l_\kappa})^2 + (b - y_{l_x l_y l_\varphi l_\kappa})^2}$. In this case, the expectation value of the EMD is 1.9, and the variance is 4.0.

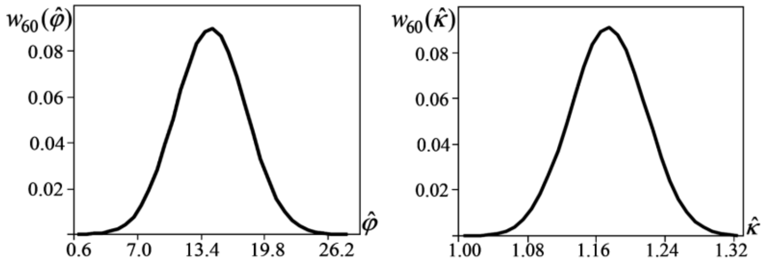
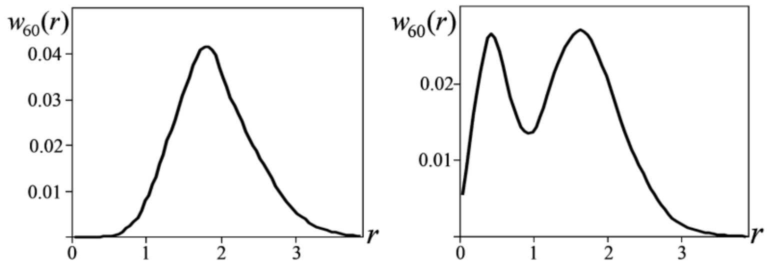


Figure 1. Examples of DPD deformation parameter estimates.

Figure 2b shows an example calculation for $\mu = 3$. At the same time, each point of the local sample plan corresponds to its own distribution of EMD, and the totality of all points corresponds to the total distribution. The plan was formed as follows. The coordinates (j_{x1}, j_{y1}) of the first count on the deformed image were chosen randomly within a certain domain, and the other two according to the rule: $(j_{xi}, j_{yi}) = (\text{int}\,[R_i \sin \phi_i], \text{int}[R_i \sin \phi_i])$, where $i = 1, 2$; $\text{int}\,[z]$ - integer part of z; $\phi_{1,2} = arctg((j_{y1} - y_0)/(j_{x1} - x_0)) \pm 120^o$; R_i - random numbers.



a) b)

Figure 2. Examples DPD of EMD.

The result shows that with increasing LSS the distribution of EMD is not normalized. This is due to that EDM has non-linear dependence on deformation parameters, with the result that different points of the local sample plan give statistically significantly different mathematical expectations and variances of EDM.

3. Characteristics of changes in the vector estimations

Using the probability distribution of EMD we find the expression for the numerical description of image geometric deformations parameters estimates behavior at iterations of non-identification relay stochastic gradient estimation. As the characteristics that form the numerical values, we consider mathematical expectation, the probability of exceeding a given threshold value of the convergence rate and the confidence interval EMD.

3.1 Mathematical expectation of change EMD

The mathematical expectation determines its convergence rate to zero at a particular iteration. At LSS $\mu = m$, this characteristic can be found through a change in the distribution of EMD on adjacent iterations:

$$\mathrm{M}[\Delta r]\Big|_{\mu=m} = \int_0^{\infty} r\left(w_{t-1}(r) - w_t(r)\right)dr . \quad (3)$$

A positive value (3) corresponds to the improvement of the parameter estimates vector $\hat{\bar{\alpha}}$, a negative value corresponds to the deterioration.

When using DPD value $\mathrm{M}[\Delta r]$ is determined by the ratio:

$$\mathrm{M}[\Delta r]\Big|_{\mu=m} = \sum_{i=1}^{L_r} r_i\left(p_{i,t-1} - p_{i,t}\right),$$

where L_r - the number of intervals splitting the domain of definition EMD. Clearly, that the expectation value $\mathrm{M}[\Delta r(+k)]$ of improving the vector of parameter estimates with increasing LSS by k can be found as: $\mathrm{M}[\Delta r(+k)] = \sum_{i=1}^{L_r} r_i\left(p_{i,t}\big|_{\mu=m} - p_{i,t}\big|_{\mu=m+k}\right)$.

For example, Figure 3a shows the dependence $w_{60}(r) - w_{61}(r)$ on EMD, which calculated at $\mu = 1$ for 61 iterations. Obviously, that with small EMD the probability differences are negative, for large ones they are positive. At the same time, the average value describes the improvement of the vector of estimates and equal to 0.017. Figure 3b shows an example of dependence $w_{60}(r)\big|_{\mu=1} - w_{60}(r)\big|_{\mu=2}$ on EMD. As in Figure 3a, for small EMD, the probability differences are negative, for large ones, they are positive. The average value is 0.02.

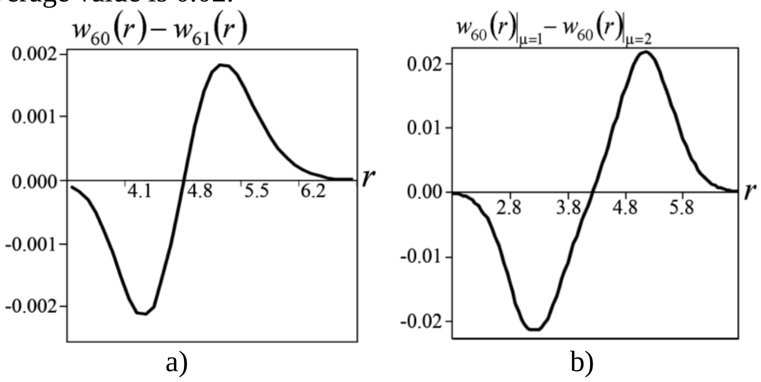


Figure 3. Examples of differences in the DPD of EMD on adjacent iterations (a) and different LSS (b).

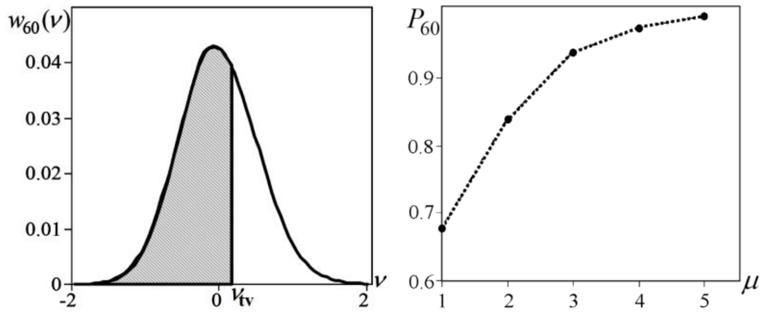
3.2. Probability of exceeding a given threshold value of EMD convergence rate

It is necessary to determine the distribution $w_t(\nu)$ of the convergence rate ν at the iterations of estimation and then determine the probability. The probability that ν at the t–th iteration will exceed a given threshold ν_{tv} is:

$$P_t = 1 - \int_{-\infty}^{\nu_{\text{tv}}} w_t(\nu)d\nu . \quad (4)$$

The distribution $w_t(\nu)$ can be found as the difference DPD of EDM on adjacent iterations, also for a unit of time we take a dimensionless value between iterations: $w_t(\nu) = r\left(w_{t-1}(r) - w_t(r)\right)$. In this case, the convergence rate can be estimated either at each iteration, or after a certain number of iterations.

Figure 4a shows an example of the DPD convergence rate of the EDM at 60 iterations at $\mu = 1$. At the same time, threshold value ν_{tv} is equal to 0.21 and the probability (4) of exceeding (shaded domain) is equal to 0.68. Figure 4b shows the dependence of probability exceeding the convergence rate of the EDM of the selected threshold value on the volume of the local sample. Clearly, if you increase μ, then the probability increases, and reaches at $\mu = 5$ value 0.99.



a) b)

Figure 4. DPD convergence rate and probability of exceeding the threshold value.

3.2. Confidence interval of the EMD

The change in the boundaries of the confidence interval at adjacent iterations is used as the numerical value of the deformation estimates of the confidence interval EMD for a given confidence probability:

$$\Delta r_{\text{ci}} = r_{\text{l}t} - r_{\text{l}(t+1)} + r_{\text{r}t} - r_{\text{r}(t+1)}\,, \tag{5}$$

where the indices “l” and “r” mean left and right limits of the confidence intervals, respectively.

Similarly, through the boundaries of the confidence intervals, one can describe the change in the vector of estimates with increasing LSS from $\mu = m$ to $\mu = m + k$:

$$\Delta r_{\text{ci}}(+k) = r_{\text{lt}}\big|_{\mu=m} - r_{\text{lt}}\big|_{\mu=m+k} + r_{\text{rt}}\big|_{\mu=m} - r_{\text{rt}}\big|_{\mu=m+k}\,. \tag{6}$$

Note that expressions (5) and (6) cannot be non-negative. It is also necessary to take into account the mismatch signs of the current estimate $\hat{\alpha}_t$ and the parameter values α, when analyzing the measurement of boundaries of the confidence intervals of individual deformation parameters (such as the angle of rotation, parallel shift, etc.), which can be positive and negative:

$$\Delta r_{\text{ci}} = \left(r_{\text{lt}} - r_{\text{l}(t+1)} + r_{\text{rt}} - r_{\text{r}(t+1)}\right) sign\ (\hat{\alpha}_t - \alpha),$$

$$\Delta r_{\text{ci}}(+k) = \left(r_{\text{lt}}\big|_{\mu=m} - r_{\text{lt}}\big|_{\mu=m+k} + r_{\text{rt}}\big|_{\mu=m} - r_{\text{rt}}\big|_{\mu=m+k}\right) sign\,(\hat{\alpha}_t - \alpha)_t\,.$$

4. Conclusion

The paper presented numerical description of image geometric deformations parameters estimates behavior at iterations of non-identification relay stochastic gradient estimation. As initial information for solving the problem, we consider the probability distribution of parameter estimates.

We chose the probability distribution EDM estimates of deformation parameters as an argument of characteristics. They form numerical values, which made it possible to ensure invariance to the set of parameters of the used inter-frame geometric deformations model.

The mathematical expectation, the probability of exceeding a given threshold value of the convergence rate and the confidence interval of the EMD were investigated as characteristics. The investigated characteristics can be used to optimize the volume and plan of the local sample according to various criteria. In particular, when using the probability of exceeding a given threshold value of the convergence rate to optimize the LSS, and after calculating the DPD of EMD, the task is reduced to finding the LSS.

Results showed that the confidence interval is a less informative parameter in comparison with mathematical expectation of change and with probability of exceeding a given threshold value of the convergence rate EMD. This is due to the fact that the probability distribution of EMD significantly changes from iteration to iteration. Therefore, on adjacent iterations, the change in the limits of the confidence interval does not always characterize the improvement of the estimates vector.

5. References

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