Mathematical modeling of geothermal energy from a well extraction

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Abstract. The authors of the article examine the possibility of extracting geothermal energy to the Earth's surface using the circulation of a heat carrier – water. For this purpose, an oil well is operated that is already suspended and is no longer used for its original purpose. A system of heat and mass transfer equations describing heat distribution in the ground around the well and in the coolant is presented. Various well depths, geothermal gradient, and water injection rate are considered.

1. Introduction

Energy production from petrothermal sources is possible almost anywhere in the world by deep wells drilling that reach hot dry rock layers, supplying them with a heat carrier and removing the heated transfer medium to the surface of the earth, where its heat is utilized by converting it into thermal or electrical energy. At the same time, the heat carrier that has given off heat can be re-fed into the deep well, which results in a circulation system (a closed loop) creation and up to 70% of the total cost of work is spent on wells drilling.

There exist numerous ways to energy from petrothermal sources production, as well as devices for their implementation. The disadvantages of these methods are the complexity of a circulation system creating, as well as the high cost due to the need of two or more deep well drilling. A method is known for the earth's heat utilizing and minerals mining in the zone of weakened earth's crust to produce hot steam in hot dry rocks [1]. The disadvantages of this method are an unreasonably large depth of wells being drilled, as well as its high cost, due to the need to drill two or more deep wells when their productivity is not high enough.

2. Background

Many scientists in Russia and abroad are engaged in petrothermal energy utilization. In particular, the works of such authors as S.S. Smirnov, G.A. Cheremensky, N.A. Gnatus, Yu.A. Popov, S.L. Pevsner, V.P. Pimenov, M.D. Khutorskoy, E.I. Boguslavsky, A.A. Chermoshentseva, A.N. Shulyupin, A.I. Filippov, P.N. Mikhailov, O.V. Akhmetova, N.I. Stoyanov are devoted to these issues.

There are no research projects related to the use of the suspended well stock. However, as of the beginning of 2018, the idling well stock accounted for 13.5% of the total well stock, respectively. This is exactly the stock that our research is devoted to.

2.1. Problem statement

There are two coaxial cylinders. The outer cylinder contacts with the ground, the temperature of which increases linearly with the depth. A heat carrier is pumped into the space between the cylinders, which is heated by the coming from the ground heat. A heated by petrothermal energy heat carrier is pumped

out through the inner cylinder, which is utilized on the surface to generate electricity. It is assumed that the temperature distribution has a cylindrical symmetry. It is also assumed that there is no heat exchange between the injected and the produced liquids.

Let us denote by R_2 the radius of the casing, and by R_1 – the radius of the inner pipe, which we will consider heat-insulated, by T – the temperature of the soil, by u(z) – the temperature of the liquid (heat carrier) between the pipes, and by v – the velocity of the fluid between the pipes. (Figure 1 shows the diametrical section of the well.)



Figure 1. Axial section of the well.

We assume that the temperature distribution has cylindrical symmetry about the well axis. To describe the temperature distribution *T*, u(z), we introduce a cylindrical coordinate system by directing the *z* axis down the axis of the well, and the *r* axis perpendicular to this axis. Between the pipes, we select a small motionless area (ΔV) of the height $\Delta z = z_2 - z_1$.

The temperature distribution T = T(t, r, z) in the soil around the well (i.e., at $r \ge R_2$) is set by the equation

$$\frac{dT}{dt} = a\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + a\frac{d^2T}{dz^2}$$
(1)

in which a is the soil thermal conductivity coefficient,

$$a_s = \frac{k_s}{c_s \rho_s}$$

where k_s , c_s , ρ_s are the soil thermal conductivity coefficient, heat capacity and density.

We obtain the temperature distribution equation u = u(t, r, z), where $R_1 \le r \le R_2$. Due to the smallness of the $R_2 - R_1$ distance we assume that u does not depend on the r variable. Between the pipes, we select a small stationary cylindrical area (ΔV) with a height of $\Delta z = z_2 - z_1$. An infinitesimal layer of liquid with a thickness of dz moving at the speed of V (figure 2) will pass the Δz distance in the time $\Delta t = \Delta z/v$ and absorb a certain amount of heat equal to

$$Q_{adsorb} = cdm\Delta u \tag{2}$$

where c is a specific heat capacity of the liquid ; dm = pSdz is the mobile layer mass;

Consequently, we obtain the thermal conductivity equation:

$$\frac{du}{dt} + v \frac{du}{dz} = A \left(T \bigg|_{r=R_2} - u \right) + a_{eff} \frac{d^2 u}{dz^2}$$
(3)

where $A = \frac{2R_2\alpha}{c\rho(R_2^2 - R_1^2)}$, $a_{eff} = \frac{k_{eff}}{c\rho}$, α is heat transfer from the ground to the heat carrier

coefficient, c and p are specific heat capacity and water density.

We calculate the depth z from the level where the ground temperature does not depend on temperature fluctuations on the ground surface and is equal to the T_0 value. We assume that before the well operation the ground temperature increased linearly with the depth. Therefore, at the initial moment, the temperature in the ground is set by the equation

$$T\Big|_{t=0} = T_0 + G_Z \tag{4}$$

where G is geotemperature gradient. At a distance R from the well axis this temperature remains stable over time:

$$T\Big|_{r=R} = T_0 + G_Z \tag{5}$$

We assume that at the initial depth, the temperature does not change over time and is equal to:

$$T\Big|_{z=0} = T_0 \tag{6}$$

We will also assume that at the initial moment the temperature of the injected heat carrier in the annular space is the same throughout the borehole and is equal to T_l .

$$u\Big|_{t=0} = T_l \tag{7}$$

The temperature of the injected liquid at the initial depth is constant and also equal to T_L

$$u\Big|_{z=0} = T_l \,. \tag{8}$$

Finally, we assume that heat exchange occurs between the soil and the liquid in the annular space, described by the equation of the form:

$$k\frac{dT}{dr} = \alpha \left(T - u\right) \bigg|_{r=R_2} \quad (t > 0).$$
⁽⁹⁾

We apply the grid method for the numerical solution of the problem (1) - (9).

2.2. The mathematical model reliability results

The mathematical model reliability was confirmed experimentally. Calculations of the liquid temperature at the well bottom at depths of 2000; 2500 m with a geothermal gradient of 0.05 $^{\circ}$ C/m were made. Figures 2-3 show the curves of the liquid temperature dynamics for 3 months (90 days) from the beginning of injection. A month after the pumping starting the temperature of the liquid tapers off to a steady-state regime.

Different injection rates were considered: 3.6; 7.2; 10.8; 14.4 m³ per hour. The graphs show that as the pumping speed increases, the temperature of the liquid decreases. For example, increasing the injection rate from 3.6 to 7.2 m³ per hour reduces the water temperature from 40°C to 30°C.

Figure 4 shows the curves of the liquid temperature dependence on the injection rate at different geothermal gradients. At low geothermal gradients (less than 0.02°C/m), the injection rate has a smaller effect on the water temperature.



Figure 2. Change in the temperature of the liquid at the bottomhole at different injection rates (m³ per h), the geothermal gradient is 0.05 ^oC/m, the depth is 2000 m.



Figure 3. Change in the temperature of the liquid at the bottomhole at different injection rates (m³ per h), the geothermal gradient is 0.05 ^oC/m, the depth is 2500 m.



Figure 4. The temperature of the liquid at time t=90 days at the well bottom, depending on the injection rate at a different geothermal gradient, at a depth of 2000 m.

3. Key findings

- 1. A mathematical model describing the heat exchange processes in a petrothermal well and in the surrounding it soil mass has been developed.
 - 2. Basing on the mathematical model and the experiment, it is proposed to select wells so that geothermal energy production is cost-effective, for example, at a depth of 2000 m, with a temperature gradient of 5°C per 100 m, the injection rate of heat exchanger should not be more than 3.6 m3 per hour.
 - 3. The work can be useful for oil companies in order to save energy and make the use of suspended wells profitable.

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5. Литература

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