Identification of dynamic errors-in-variables bilinear systems of fractional order

D.V. Ivanov[1], I.L. Sandler[2], N.V. Chertykovtseva[2]

[1]Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086
[2]Samara State University of transport, Svobody street 2 B, Samara, Russia, 443066

Abstract. An approach for the identification of dynamic single-input single-output bilinear discrete-time fractional order system models within the errors-in-variables framework for the case of white input and output noise sequences is presented. A criterion is obtained that allows obtaining highly consistent estimates of the parameters of the system.

Keywords: bilinear dynamical systems, a difference of fractional order, least square, strongly consistents.

1. Introduction

In order to describe processes of different nature, equations with derived differences of fractional order are increasingly used. Although there is no simple interpretation, which is possessed by derivatives, integrals and differences of integers, models described by equations of fractional order, used in physics and engineering [1-4], the branch of the theory of management related to the synthesis of regulators of fractional order is actively developing.

In connection with the active development and application of equations with differences and derivatives of fractional order for modeling and forecasting, methods of identification of systems described by equations and differences of fractional order are being actively developed. Most of the papers are devoted to the parametric identification of differential equations of fractional order with noise in the equation or the output signal [5-8].

The problem of identification with the presence of noises in the input and output signals is much more complicated, for example in [9] a method based on the cumulant, involving significant restrictions on signal and noise is proposed.

Articles [10-12] are devoted to identification of systems described by equations, with differences of fractional order.

In the field of physical system modelling the bilinear system (BS) models have been exploited extensively in various scientific areas, such as nuclear fission, electric networks, heat transfer, fluid flow, chemical kinetics etc., see e.g. [13-14]. Their popularity and broad applicability stems from the fact that BS models are able to satisfactorily approximate many nonlinear processes.

Furthermore, building on the theory of linear systems, the state dependent steady-state and dynamic properties of BS models are well understood. Therefore, the BS models are commonly utilised as a stepping stone when analyzing systems exhibiting nonlinear behavior. Consequently, a need to extend the EIV system description to encompass BS models is prompted. This allows a combination of both, i.e. the increased applicability offered by the BS models and the generalized system setup afforded by the EIV framework.

In this paper an approach for the identification of single-input-single-output (SISO) discrete-time fractional EIV BS models is proposed considering the case when the input and output noise sequences are white.

2. Problem statement

Consider the class of the discrete-time input-output SISO (single-input single-output) bilinear systems that can be represented by the following nonlinear auto-regressive with exogenous input process, i.e.

$$\sum_{m=1}^{r} b_0^{(m)} \Delta^{\alpha^{(m)}} z_{i-f^{(m)}} = \sum_{m=1}^{r_1} a_0^{(m)} \Delta^{\beta^{(m)}} x_{i-f_1^{(m)}} + \sum_{m=0}^{r_2} \sum_{k=1}^{r_3^{(m)}} c_0^{(mk)} \Delta^{\gamma^{(mk)}} x_{i-f_2^{(m)}} z_{i-f_3^{(k)}} \tag{1}$$

$$y_i = z_i + \xi_i,\ w_i = x_i + \zeta_i,$$

where $0 < \alpha^{(1)} \ldots < \alpha^{(r)},\ 0 < \beta^{(1)} \ldots < \beta^{(r_1)},\ 0 < \gamma^{(1k)} \ldots < \gamma^{(r_3^{(k)} k)},\ k = \overline{1, r_2}$,

$$\Delta^{\alpha^{(m)}} z_{i-f^{(m)}} = \sum_{j=0}^{i} (-1)^j \binom{\alpha^{(m)}}{j} z_{i-j-f^{(m)}},\ \Delta^{\beta^{(m)}} x_{i-f_1^{(m)}} = \sum_{j=0}^{i} (-1)^j \binom{\beta^{(m)}}{j} x_{i-j-f_1^{(m)}},$$

$$\Delta^{\gamma^{(mk)}} x_{i-f_2^{(m)}} z_{i-f_3^{(k)}} = \sum_{j=0}^{i} (-1)^j \binom{\gamma^{(mk)}}{j} \left(x_{i-j-f_2^{(m)}} y_{i-j-f_3^{(k)}} \right),$$

$$\binom{\alpha^{(m)}}{j} = \frac{\Gamma(\alpha^{(m)}+1)}{\Gamma(j+1)\Gamma(\alpha^{(m)}-j+1)},\ \binom{\beta^{(m)}}{j} = \frac{\Gamma(\beta^{(m)}+1)}{\Gamma(j+1)\Gamma(\beta^{(m)}-j+1)},\ \binom{\gamma^{(mk)}}{j} = \frac{\Gamma(\gamma^{(mk)}+1)}{\Gamma(j+1)\Gamma(\gamma^{(mk)}-j+1)},$$

$$\Gamma(\alpha) = \int_0^{\infty} e^{-t} t^{\alpha-1} dt,$$

$f^{(m)}, f_1^{(m)}, f_2^{(m)}, f_3^{(k)}$ are non-negative values of delays,

z_i, y_i are noise-free output and input,

x_i, w_i are the measured output and input,

ξ_i, ζ_i are noise of observation in the output and the input signals,

The following assumptions are introduced **A1.**The discrete-time bilinear system is asymptotically stable, observable and controllable.

A2. The system structure, i.e. $r, r_1, r_2, r_3^{(k)}\ f^{(m)}, f_1^{(m)}, f_2^{(m)}, f_3^{(k)}, \alpha^{(m)}, \beta^{(m)}, \gamma^{(mk)}$ is known a priori.

A3. The true input signal x_i is a random process persistently exciting of sufficiently high order with $\mathrm{E}(x_i) = 0,\ \mathrm{E}(x_i^2) = \sigma_x^2 \leq \infty$:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \varphi_x^{(i)} \left(\varphi_x^{(i)}\right)^T = H, \text{ a.s.},$$

$$\operatorname{liim}_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \varphi_x^{(i)} \left(\varphi_x^{(i)}\right)^T = H,\ \varphi_x^{(i)} = \left(\Delta^{\beta^{(1)}} x_{i-f_1^{(1)}}, \ldots, \Delta^{\beta^{(r_1)}} x_{i-f_1^{(r_1)}} \right)^T,$$

matrix H is restricted positive definite.

A4. Input/output noise sequences $\{\xi_i\}$ and $\{\zeta_i\}$ are zero mean, ergodic, white signals with known variances, denoted σ_x^2 and σ_y^2.

A5. Input/output noise sequences $\{\xi_i\}$ and $\{\zeta_i\}$ mutually uncorrelated and uncorrelated with the noise-free signals $\{z_i\}, \{x_i\}$.

The system parameter vector is defined as

$$\theta_0 = \left(b_0^T \mid a_0^T \mid c_0^T\right)^T,\ b_0 = \left(1 b_0^{(2)} ... b_0^{(r)}\right)^T,\ a_0 = \left(a_0^{(1)} ... a_0^{(r_1)}\right)^T,$$

$$c_0 = \left(c_0^{(11)} \dots c_0^{(1r_3^{(1)})} \mid c_0^{(21)} \dots c_0^{(2r_3^{(1)})} \mid \dots \mid c_0^{(r_2 1)} \dots c_0^{(r_2 r_3^{(r_2)})}\right)^T.$$

The regressor vector for the measured signals is defined as

$$\varphi_i = \left(\left(\varphi_y^{(i)}\right)^T \mid \left(\varphi_w^{(i)}\right)^T) \mid \left(\varphi_{wy}^{(i)}\right)^T\right)^T,\ \varphi_y^{(i)} = \left(\Delta^{\alpha^{(1)}} y_{i-f^{(1)}} \dots \Delta^{\alpha^{(r)}} y_{i-f^{(r)}}\right)^T,$$

$$\varphi_w^{(i)} = \left(\Delta^{\beta^{(1)}} w_{i-f_1^{(1)}}, \dots, \Delta^{\beta^{(r_1)}} w_{i-f_1^{(r_1)}}\right)^T,\ \varphi_{wy}^{(i)} = \left(\varphi_{wy}^{(i1)} \mid \varphi_{wy}^{(i2)} \mid \quad \mid \varphi_{wy}^{(ir_3^{(m)})}\right)^T,$$

$$\varphi_{wy}^{(ik)} = \left(\Delta^{\gamma^{(1k)}} w_{i-f_2^{(1)}} y_{i-f_3^{(k)}} \quad \dots \quad \Delta^{\gamma^{(r_2 k)}} w_{i-f_2^{(r_2)}} y_{i-f_3^{(k)}}\right).$$

Given N samples of the measured signals, i.e. $\{y_i\}, \{w_i\}$, determine the vector $\hat{\theta}$.

3. Criteria for identification

This system can be reformulated in equation error form as

$$a_0^T \varphi_y^{(i)} - b_0^T \varphi_w^{(i)} - c_0^T \varphi_{wy}^{(i)} = \varepsilon_i,$$

where the equation error or residual is given by

$$\varepsilon_i = b_0^T \varphi_\xi^{(i)} - a_0^T \varphi_\zeta^{(i)} - c_0^T \left(\varphi_{x\xi}^{(i)} + \varphi_{\zeta z}^{(i)} + \varphi_{\zeta\xi}^{(i)}\right),\ \varphi_\xi^{(i)} = \left(\Delta^{\alpha^{(1)}} \xi_{i-f^{(1)}}, \dots, \Delta^{\alpha^{(r)}} \xi_{i-f^{(r)}}\right)^T,$$

$$\varphi_\zeta^{(i)} = \left(\Delta^{\beta^{(1)}} \zeta_{i-f_1^{(1)}}, \dots, \Delta^{\beta^{(r_1)}} \zeta_{i-f_1^{(r_1)}}\right)^T,\ \varphi_{x\xi}^{(i)} = \left(\varphi_{x\xi}^{(i1)} \mid \varphi_{x\xi}^{(i2)} \mid \dots \mid \varphi_{x\xi}^{(ir_3^{(m)})}\right)^T,$$

$$\varphi_{x\xi}^{(ik)} = \left(\Delta^{\gamma^{(1k)}} x_{i-f_2^{(1)}} \xi_{i-f_3^{(k)}} \quad \dots \quad \Delta^{\gamma^{(r_2 k)}} x_{i-f_2^{(r_2)}} \xi_{i-f_3^{(k)}}\right),\ \varphi_{\zeta z}^{(i)} = \left(\varphi_{\zeta z}^{(i1)} \mid \varphi_{\zeta z}^{(i2)} \mid \dots \mid \varphi_{\zeta z}^{(ir_3^{(m)})}\right)^T,$$

$$\varphi_{\zeta z}^{(ik)} = \left(\Delta^{\gamma^{(1k)}} \zeta_{i-f_2^{(1)}} z_{i-f_3^{(k)}} \quad \dots \quad \Delta^{\gamma^{(r_2 k)}} \zeta_{i-f_2^{(r_2)}} z_{i-f_3^{(k)}}\right),\ \varphi_{\zeta\xi}^{(i)} = \left(\varphi_{\zeta\xi}^{(i1)} \mid \varphi_{\zeta\xi}^{(i2)} \mid \quad \mid \varphi_{\zeta\xi}^{(ir_3^{(m)})}\right)^T.$$

From requirement **A4, A5** it follows that generalized error ε_i has zero mean. We obtain that variance of generalized error equal to

$$\sigma_\varepsilon^2 = \sigma_\varsigma^2 + \sigma_\xi^2 b_0^T H_\alpha b_0 + \sigma_\zeta^2 a_0^T H_\beta a_0 + c_0^T H_\gamma c_0$$

$$H_\alpha = E\left[\sum_{i=1}^{N} \varphi_\xi^{(i)} \left(\varphi_\xi^{(i)}\right)^T\right] = \sigma_\xi^2 \begin{pmatrix} h_\alpha^{(11)} & h_\alpha^{(21)} & \dots & h_\alpha^{(1r)} \\ h_\alpha^{(21)} & h_\alpha^{(22)} & \dots & h_\alpha^{(2r)} \\ \vdots & \vdots & \ddots & \vdots \\ h_\alpha^{(r1)} & h_\alpha^{(r2)} & \dots & h_\alpha^{(rr)} \end{pmatrix},$$

$$h_\alpha^{(mk)} = E\left(\Delta^{\alpha^{(m)}} \xi_{i-f^{(m)}} \cdot \Delta^{\alpha^{(k)}} \xi_{i-f^{(k)}}\right) = \lim_{N\to\infty} \frac{1}{N} \left(\sum_{j=0}^{N-1} \binom{\alpha^{(m)}}{j - f^{(m)}} \binom{\alpha^{(k)}}{j - f^{(k)}} \cdot \frac{N-j}{N}\right),\ m = \overline{1,r},\ k = \overline{1,r},$$

$$H_\beta = E\left[\sum_{i=1}^{N} \varphi_\zeta^{(i)} \left(\varphi_\zeta^{(i)}\right)^T\right] = \sigma_\zeta^2 \begin{pmatrix} h_\beta^{(11)} & h_\beta^{(12)} & \dots & h_\beta^{(1r_1)} \\ h_\beta^{(21)} & h_\beta^{(22)} & \dots & h_\beta^{(1r_1)} \\ \vdots & \vdots & \ddots & \vdots \\ h_\beta^{(r_1 1)} & h_\beta^{(r_1 2)} & \dots & h_\beta^{(r_1 r_1)} \end{pmatrix},$$

$$h_\beta^{(mk)} = E\left(\Delta^{\beta^{(m)}} \zeta_{i-f_1^{(m)}} \cdot \Delta^{\beta^{(k)}} \zeta_{i-f_1^{(k)}}\right) = \lim_{N\to\infty} \frac{1}{N} \left(\sum_{j=0}^{N-1} \binom{\beta^{(m)}}{j - f_1^{(m)}} \binom{\beta^{(k)}}{j - f_1^{(k)}} \cdot \frac{N-j}{N}\right),\ m = \overline{1,r_1},\ k = \overline{1,r_1},$$

$$H_\gamma = E\left[\sum_{i=1}^{N}\left(\varphi_{\zeta y}^{(i)}\left(\varphi_{\zeta y}^{(i)}\right)^T + \varphi_{w\xi}^{(i)}\left(\varphi_{w\xi}^{(i)}\right)^T - \varphi_{\zeta\xi}^{(i)}\left(\varphi_{\zeta\xi}^{(i)}\right)^T\right)\right] = \left(\begin{array}{c:c:c:c} H_\gamma^{(11)} & H_\gamma^{(12)} & \dots & H^{(1r_2)} \\ \hdashline H_\gamma^{(21)} & H_\gamma^{(22)} & \dots & H_\gamma^{(2r_2)} \\ \hdashline \vdots & \vdots & \ddots & \vdots \\ \hdashline H_\gamma^{(r_2 1)} & H_\gamma^{(r_c 2)} & \dots & H_\gamma^{(r_2 r_2)} \end{array}\right),$$

$$H_\gamma^{(\mathrm{mm'kk'})} = \sigma_\xi^2 \begin{pmatrix} h_\gamma^{(11\mathrm{kk'})} & h_\gamma^{(12\mathrm{kk'})} & \dots & h_\gamma^{(1\mathrm{r}_3^{(k')}\mathrm{kk'})} \\ h_\gamma^{(21\mathrm{kk'})} & h_\gamma^{(22\mathrm{kk'})} & \dots & h_\gamma^{(2\mathrm{r}_3^{(k')}\mathrm{kk'})} \\ \vdots & \vdots & \ddots & \vdots \\ h_\gamma^{(\mathrm{r}_3^{(k)}1\mathrm{kk'})} & h_\gamma^{(\mathrm{r}_3^{(k)}2\mathrm{kk'})} & \dots & h_\gamma^{(\mathrm{r}_3^{(k)}\mathrm{r}_3^{(k')}\mathrm{kk'})} \end{pmatrix},$$

$$h_{\zeta y}^{(mm'kk')} = E\left(\Delta^{\gamma^{(mk)}}\zeta_{i-f_2^{(m)}}y_{i-f_3^{(k)}}\cdot\Delta^{\gamma^{(m'k')}}\zeta_{i-f_2^{(m')}}y_{i-f_3^{(k')}}\right) =$$

$$= \sigma_\zeta^2 \lim_{N\to\infty}\frac{1}{N}\left(\sum_{j=0}^{N-1}\begin{pmatrix}\gamma^{(\mathrm{mk})}\\ j-f_2^{(k)}\end{pmatrix}\begin{pmatrix}\gamma^{(\mathrm{m'k'})}\\ j-f_2^{(\mathrm{k'})}\end{pmatrix}\tilde{h}_y^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(k)}-f_2^{(\mathrm{k'})}\right|}\right),$$

$$\tilde{h}_y^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(k)}+f_2^{(\mathrm{k'})}\right|} = h_y^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(m)}+f_2^{(\mathrm{m'})}\right|}\frac{N-j}{N},$$

$$h_y^{|m|} = \lim_{i\to\infty}\frac{1}{N}\sum_{i=0}^{N}y_i y_{i+m},$$

$$h_{w\xi}^{(mm'kk')} = E\left(\Delta^{\gamma^{(mk)}}w_{i-f_2^{(m)}}\xi_{i-f_3^{(k)}}\cdot\Delta^{\gamma^{(m'k')}}w_{i-f_2^{(m')}}\xi_{i-f_3^{(k')}}\right) =$$

$$= \sigma_\xi^2 \lim_{N\to\infty}\frac{1}{N}\left(\sum_{j=0}^{N-1}\begin{pmatrix}\gamma^{(\mathrm{mk})}\\ g-f_3^{(k)}\end{pmatrix}\begin{pmatrix}\gamma^{(\mathrm{m'k'})}\\ j-f_3^{(\mathrm{k'})}\end{pmatrix}\tilde{h}_w^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(k)}-f_2^{(\mathrm{k'})}\right|}\right),$$

$$\tilde{h}_w^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(k)}+f_2^{(\mathrm{k'})}\right|} = h_w^{\left|f_3^{(k)}-f_3^{(\mathrm{k'})}-f_2^{(m)}+f_2^{(\mathrm{m'})}\right|}\frac{N-j}{N},$$

$$h_w^{|m|} = \lim_{i\to\infty}\frac{1}{N}\sum_{i=0}^{N}w_i w_{i+m},$$

$$h_{\zeta\xi}^{(mm'kk')} = E\left(\Delta^{\gamma^{(mk)}}\zeta_{i-f_2^{(m)}}\xi_{i-f_3^{(k)}}\cdot\Delta^{\gamma^{(m'k')}}\zeta_{i-f_2^{(m')}}\xi_{i-f_3^{(k')}}\right) = \sigma_\zeta^2\sigma_\xi^2 \lim_{N\to\infty}\frac{1}{N}\left(\sum_{j=0}^{N-1}\begin{pmatrix}\gamma^{(\mathrm{mk})}\\ j-f_2^{(k)}\end{pmatrix}\begin{pmatrix}\gamma^{(\mathrm{m'k'})}\\ j-f_2^{(\mathrm{k'})}\end{pmatrix}\frac{N-j}{N}\right)$$

With the known structure of the model, which means that orders r, r_1, r_2, $r_3^{(k)}$, $f^{(m)}$, $f_1^{(m)}, f_2^{(m)}, f_3^{(k)}$, $\alpha^{(m)}, \beta^{(m)}$, $\gamma^{(mk)}$ are determined, the following criterion can be applied to get strongly consistent estimates of parameters:

$$\min_{a,b,c}\sum_{i=1}^{N}\frac{\left(a^T\varphi_y^{(i)} - b^T\varphi_w^{(i)} - c^T\varphi_{wy}^{(i)}\right)^2}{\sigma_\varsigma^2 + \sigma_\xi^2 b^T H_\alpha b + \sigma_\zeta^2 a^T H_\beta a + c^T H_\gamma c}. \tag{2}$$

Theorem. Suppose that a random process $\{y_i, i = \ldots -1,0,1\ldots\}$ *is described by equation (1) with zero initial conditions and the assumptions 1-5 are met. Then estimate* $\hat{\theta}(N)$, *determined by expression (2) with probability 1 when* $N\to\infty$, *exists and is a unique and strongly consistent estimate, i.e.*

$$\hat{a}\xrightarrow[N\to\infty]{}a_0,\ \text{a.s.}$$

$$\hat{b}\xrightarrow[N\to\infty]{}b_0,\ \text{a.s.}$$

$$\hat{c}\xrightarrow[N\to\infty]{}c_0.\ \text{a.s.}$$

4. Conclusions

The algorithm (2) compared to Least Square (LS). The dynamic system is described by the equations

$$z_i = 0.5\Delta^{0.9} z_{i-1} - 0.2\Delta^{1.7} z_{i-1} + \Delta^{0.7} x_i - 0.2\Delta^{1.4} x_i + 0.3\Delta^{0.2} x_i z_{i-1}. \quad y_i = z_i + \xi_i,\ w_i = x_i + \zeta_i, \tag{3}$$

The coefficients of the bilinear system were given by

$$\theta_0 = \begin{pmatrix} 0.7 & -0.4 & 0.3 & 0.7 & 0.2 & 0.2 \end{pmatrix}^T.$$

The noise-free inputs x_i is modelled as

$$x_i = -0.8 \cdot x_{i-1} - 0.6 \cdot x_{i-2} + \zeta_i + 1.7 \cdot \zeta_{i-1} + 0.5 \cdot \zeta_{i-2},$$

where ζ_i - is are zero mean, white signal.

The estimates of the parameter vector are given in Table 1, as well as the normalized root mean square error, defined as

$$\delta\theta = \sqrt{\left\|\hat{\theta} - \theta_0\right\|^2 \Big/ \left\|\theta_0\right\|^2} \cdot 100\%.$$

The results are based on 50 independent Monte-Carlo simulations. The number of data points N in each simulation was 2000. For each method we have given the sample mean and sample standard deviation denoted by SD.

Table 1. Monte-Carlo simulation results.

Noise-signal ratio		Mean square error ± SD	
σ_ξ / σ_z	σ_ζ / σ_z	$\delta\theta$,%	$\delta\theta_{LS}$,%
0.25	0.25	2.94±2.27	11.57±2.37
0.50	0.50	8.49±5.14	25.81±2.20
0.75	0.75	16.58±10.85	38.31±2.58

5. Conclusions

An approach for the identification of dynamic bilinear discrete-time errors-in-variables system models has been developed. A Monte-Carlo simulation study compares two realizations of the proposed approach with least square technique. The results obtained demonstrate the relatively high accuracy and the robustness against noise of the algorithms proposed.

Future work could encompass potential extensions to handle the case of the coloured output noise together with the recursive implementation of the algorithm.

6. Acknowledgments

The authors gratefully acknowledge the contributions of prof. O.A. Katsyuba in improving the article.

7. References

[1] Stiassnie, M. On the application of fractional calculus for the formulation of viscoelastic models / M. Stiassnie // Applied Mathematical Modelling. – 1979. – Vol. 3. – P. 300-302.

[2] Bagley, R.L Fractional calculus – a different approach to the analysis of viscoelastically damped structures / R.L. Bagley // AIAA J. – 1983. – Vol. 21. – P. 741-748.

[3] Reyes-Melo, M.E. Application of fractional calculus to modeling of relaxation phenomena of organic dielectric materials / M.E. Reyes-Melo, J.J. Martinez-Vega, C.A. Guerrero-Salazar, U. Ortiz-Mendez // Proc Int Conf Solid Dielectrics. – 2004. – Vol. 2. – P. 530-533.

[4] Vinagre, B.M. Modeling and control of dynamic system using fractional calculus: Application to electrochemical processes and flexible structures / B.M. Vinagre, V. Feliu // Proc 41-st IEEE Conf Decision Control, Las Vegas, NV. – 2002. – P. 214-239.

[5] Mathieu, B. Identification of non integer order systems in the time domain / B. Mathieu, L.L. Lay, A. Oustaloup // IEEE SMC/IMACS Symp Control, Optimization and Supervision. Lille, France. – 1996. – P. 952-956.

[6] Cois, O. Non-integer model from modal decomposition for time domain system identification / O. Cois, A. Oustaloup, E. Battaglia, J-L. Battaglia // Proc Symp System Identification, SYSID 2000, Santa Barbara, CA. – 2000. – Vol. 3. – P. 989-994.
[7] Cois, O. Fractional state variable filter for system identification by fractional model / O. Cois, A. Oustaloup, T. Poinot, J-L. Battaglia // Proc. European Contr. Conf., ECC2001, Porto, Portugal. – 2001. – P. 2481-2486.
[8] Malti, R. Tutorial on system identification using fractional differentiation models / R. Malti, M. Aoun, J. Sabatier, A. Oustaloup // 14th IFAC Symposium on System Identification (SYSID’2006), Newcastle, Australia. – 2006. – P. 606-611.
[9] Chetoui, M. EIV methods for system identification with fractional models / M. Chetoui, R. Malti, M. Thomassin, M. Aoun, S. Najar, A. Oustaloup, M.N. Abdelkrim // Proc. 16th IFAC Symposium on System Identification (SYSID). – Brussels. – 2012. – P. 1641-1646.
[10] Ivanov, D.V. Identification discrete fractional order linear dynamic systems with output-error / D.V. Ivanov // Proceedings International Siberian Conference on Control and Communications (SIBCON'2013). – Krasnoyarsk: Siberian Federal University, 2013. – CFP13794-CDR.
[11] Ivanov, D.V. Identification discrete fractional order linear dynamic systems with errors-in-variables / D.V. Ivanov // Proceedings of IEEE East-West Design & Test Symposium (EWDTS'2013). – Rostov-on-Don, 2013. – P. 374-377.
[12] Ivanov, D.V. Identification Discrete Fractional Order Hammerstein Systems / D.V. Ivanov // Proceedings International Siberian Conference on Control and Communications (SIBCON). – Omsk State Technical University, 2015. – CFP13794-CDR.
[13] Mohler, R.R. Nonlinear Systems: Applications to Bilinear Control. / R.R. Mohler // Prentice Hall, Englewood Cliffs, NJ. – 1991. – Vol. 2. − 192 p.
[14] Mohler, R.R. Bilinear control and application to flexible A.C. transmission systems / R.R. Mohler, A.Y. Khapalov // J. of Optimization Theory and Applications. − 2000. − Vol. 105(3). – P. 621-637.
[15] Mohler, R.R. An overview of bilinear system theory and applications / R.R. Mohler, W.J. Kolodziej // IEEE Trans. On Systems, Man, and Cybernetics. – 1980. − Vol. 10(10). − P. 683-688.