Hyperfine structure of P-states of light muonic atoms

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Abstract. Precise calculation of hyperfine structure of P -states in muonic ions of lithium, beryllium and boron in quantum electrodynamics is performed. We calculate different corrections of orders α^5 and α^6 due to the vacuum polarization, nuclear structure and recoil in first and second orders of perturbation theory. The tensor method of projection operators for the calculation of the hyperfine structure of P-states with definite quantum numbers of total atomic momentum F and total muon momentum j in muonic ions is used. The obtained values of hyperfine splittings can be used for a comparison with future experimental data.

1. Introduction

In our previous works [1, 2], we calculated the Lamb shift and hyperfine structure of S-states in muonic ions of lithium, beryllium, and boron. The purpose of this paper is to investigate the HFS of the P-states in these ions, that is, in the precise calculation of various corrections and obtaining reliable estimates for the HFS intervals, which could be used for comparison with experimental data. The initial parameters that determine the values of the corrections in the HFS of muonic ions, are the masses of the nuclei, their spins, magnetic moments and charge radii.

2. General formalism

Our approach to the calculation of hyperfine splittings is based on quasipotential method in quantum electrodynamics in which the two-particle bound state is described by the Schrödinger equation [3]. In this work we use the approach to the calculation of hyperfine structure of muonic ions based on tensor representation of P-wave projection operators describing bound states. First we show on an example of calculating the leading order contributions how a tensor formalism helps investigate the hyperfine structure of the spectrum. It is useful to work in momentum representation where we can write the wave function of muonic ions 2P-state in the tensor form:

$$\psi_{2P}(\mathbf{p}) = (\varepsilon \cdot n_p)\, R_{21}(p), \tag{1}$$

where ε_δ is the polarization vector of orbital motion, $n_p = (0, \mathbf{p}/p)$, $R_{21}(p)$ is the radial wave function in momentum space. Then the energy shifts are presented in integral form:

$$\Delta E^{hfs} = \int (\varepsilon^* \cdot n_q)\, R_{21}(q) \frac{d\mathbf{q}}{(2\pi)^{3/2}} \int (\varepsilon \cdot n_p)\, R_{21}(p) \frac{d\mathbf{p}}{(2\pi)^{3/2}} \Delta V^{hfs}(\mathbf{p}, \mathbf{q}). \tag{2}$$

In the leading order the hyperfine potential ΔV^{hfs} is constructed by means of one-photon interaction amplitude $T_{1\gamma}$.

Writing the amplitude $T_{1\gamma}$ we refer to it a part of the bound state wave function related to orbital motion:

$$T_{1\gamma}(\mathbf{p},\mathbf{q}) = 4\pi Z\alpha\,(\varepsilon^*\cdot n_q)\left[\bar{u}(q_1)\left(\frac{p_{1,\mu}+q_{1,\mu}}{2m_1}+(1+a_\mu)\sigma_{\mu\epsilon}\frac{k_\epsilon}{2m_1}\right)u(p_1)\right](\varepsilon\cdot n_p)\,D_{\mu\nu}(k)\times \quad (3)$$

$$\varepsilon^*_{d,\rho}(q_2)\Big\{g_{\rho\sigma}\frac{(p_2+q_2)_\nu}{2m_2}F_1(k^2)-\frac{(p_2+q_2)_\nu}{2m_2}\frac{k_\rho k_\sigma}{2m_2^2}F_2(k^2)+(g_{\rho\lambda}g_{\sigma\mu}-g_{\rho\mu}g_{\sigma\lambda})\frac{k_\lambda}{2m_2}F_3(k^2)\Big\}\varepsilon_{d,\sigma}(p_2),$$

where $p_{1,2}=\frac{m_{1,2}}{(m_1+m_2)}P\pm p$ are four-momenta of initial muon and nucleus, $q_{1,2}=\frac{m_{1,2}}{(m_1+m_2)}Q\pm q$ are four-momenta of final muon and nucleus. They are expressed in terms of total two-particle momenta P,Q and relative momenta p,q. $D_{\mu\nu}(k)$ is the photon propagator which is taken to be in the Coulomb gauge. We consider (3) as a starting point for a composition of orbital $\mathbf{L}$ momentum, the nucleus spin $\mathbf{s}_2$ (note that the spin of the nucleus is usually denoted by $\mathbf{I}$) and muon spin $\mathbf{s}_1$. In the first scheme of momentum composition we add firstly momenta $\mathbf{L}$ and $\mathbf{s}_1$ obtaining two muon states with angular momenta $j=1/2$ and $j=3/2$. In the Rarita-Schwinger formalism the wave function of the state with half-integer spin 3/2 is described by

$$\psi_{\mu,\alpha}(\mathbf{p},\sigma)=\sum_{\lambda,\omega}<\frac{1}{2}\omega;1\lambda|\frac{3}{2}\sigma>\varepsilon_\mu(\mathbf{p},\lambda)u_\alpha(\mathbf{p},\omega), \quad (4)$$

where $<\frac{1}{2}\omega;1\lambda|\frac{3}{2}\sigma>$ are the Clebsch-Gordon coefficients.

Basic contribution to hyperfine structure is determined by hyperfine part of the Breit Hamiltonian:

$$\Delta V_B^{hfs}(r)=\frac{Z\alpha(1+\kappa_d)}{2m_1m_2r^3}\big[1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\big](\boldsymbol{L}\boldsymbol{s}_2)-\frac{Z\alpha(1+\kappa_d)(1+a_\mu)}{2m_1m_2r^3}\left[(\boldsymbol{s}_1\boldsymbol{s}_2)-3(\boldsymbol{s}_1\boldsymbol{n})(\boldsymbol{s}_2\boldsymbol{n})\right], \quad (5)$$

where m_1, m_2 are the muon and nucleus masses, κ_N, a_μ are the nucleus and muon anomalous magnetic moments, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are the spin operators of muon and nucleus, $\boldsymbol{n}=\boldsymbol{r}/r$. The operator (5) does not commute with the muon total angular momentum $\mathbf{J}=\mathbf{L}+\mathbf{s}_1$. As a result there is the mixing between energy levels $2P_{1/2}$ and $2P_{3/2}$.

The corrections to the vacuum polarization (VP) of order α^5 are important to obtain reliable results. In the formulated framework these effects can be easily studied. In the first order perturbation theory one-loop vacuum polarization contribution to HFS is determined by the amplitude in Fig. 1. For its calculation in momentum representation which we use, the following replacement in the photon propagator should be done:

$$\frac{1}{k^2}\to\frac{\alpha}{3\pi}\int_1^\infty\frac{\rho(\xi)d\xi}{k^2+4m_e^2\xi^2},\quad \rho(\xi)=\sqrt{\xi^2-1}(2\xi^2+1)/\xi^4. \quad (6)$$

For a completeness, we analyze vacuum polarization corrections of order α^5 in second order perturbation theory, which are determined by the reduced Coulomb Green's function (see the amplitude in Fig. 2(a)):

$$G_{2P}(\boldsymbol{r},\boldsymbol{r}')=-\frac{\mu^2(Z\alpha)}{36z^2z'^2}\left(\frac{3}{4\pi}\boldsymbol{n}\boldsymbol{n}'\right)e^{-(z+z')/2}g(z,z'), \quad (7)$$

$$g(z,z')=24z_<^3+36z_<^3z_>+36z_<^3z_>^2+24z_>^3+36z_<z_>^3+36z_<^2z_>^3+49z_<^3z_>^3-3z_<^4z_>^3-$$
$$-12e^{z_<}(2+z_<+z_<^2)z_>^3-3z_<^3z_>^4+12z_<^3z_>^3[-2C+Ei(z_<)-lnz_<-lnz_>],$$

where $C=0.5772...$ is the Euler constant, $z=Wr,\ z_<=min(z,z'),\ z_>=max(z,z')$.

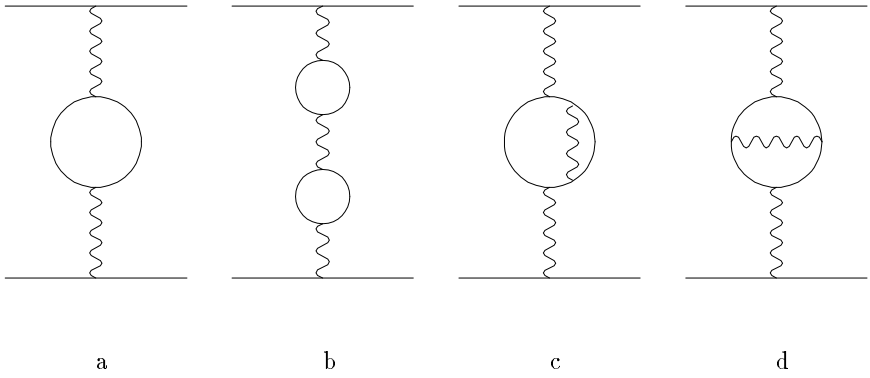

Figure 1. Vacuum polarization effects in one-photon interaction. The wavy line represents hyperfine part of the interaction.

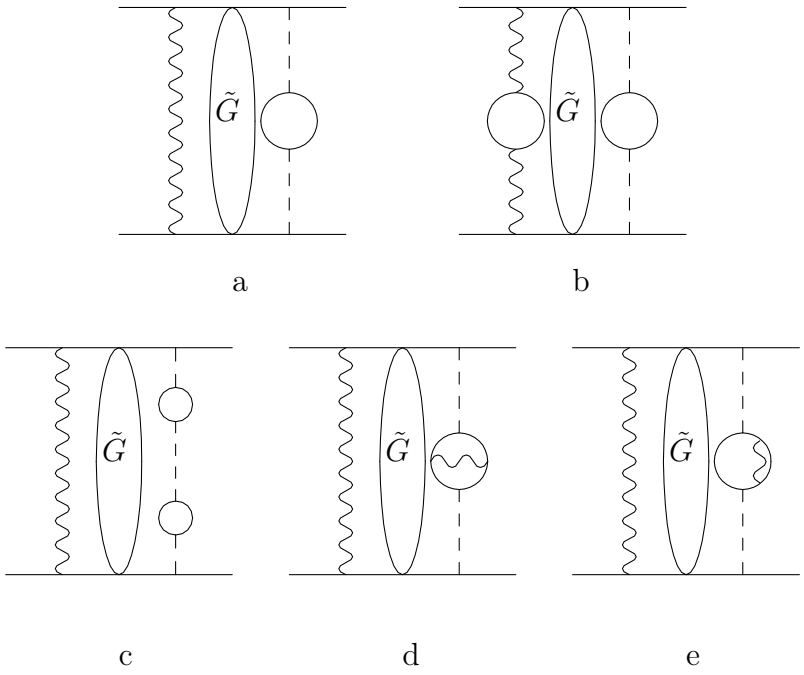

Figure 2. Vacuum polarization effects in the second order perturbation theory. Dashed and wavy lines represent correspondingly the Coulomb and hyperfine interactions.

3. Conclusion

In this work we investigate the hyperfine structure of energy levels related to the P-wave states of muonic ions of lithium, beryllium and boron on the basis of three dimensional quasipotential approach in quantum electrodynamics. To increase the accuracy of the calculation we take into account the leading order contribution and several basic corrections of order α^5 and α^6. These corrections are connected with the vacuum polarization effect, quadruple interaction, nuclear structure and relativistic effects. Some corrections are obtained in analytical form, but most part of contributions to the energy spectrum is presented first in integral form, and then calculated numerically. Obtained results improve the values of transition frequencies between energy levels 2P and 2S.

4. References

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Acknowledgments

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