How to read the trendless sequences: the "universal" set of quantitative parameters

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Abstract. We want to demonstrate a set of "universal" parameters that help to read quantitatively any trendless sequence (TLS). This set will be very useful in order to select the "pattern" noise from the tested one and thereby to solve the problem of calibration of random fluctuations and express some qualitative inputs in terms of these "universal" parameters. This set of quantitative parameters allows to compare the TLS(s) of different nature (acoustic, mechanical, electrochemical, vibrational and etc.) with each other. Using the algorithm, we analysed the acoustic noise recorded from the frictionless bearings (FB) in a normal state and noise from the FBs with artificially created defects. The proposed algorithm allows detecting the desired defect that initially had a qualitative description only. We do suppose that the proposed "universal" scheme free from uncontrollable errors can find a wide application in solution of many practical problems.

1. Introduction

The problem of extraction of information from trendless fluctuations (always accompanied with the registered responses from open systems) became very important from the middle of the last century. Before, these fluctuations were not considered, or generally were served as a "marker" of poor-quality measurements. As an example of this approach, one can serve as a noise reduction system, given in the most known book [1]. Nowadays, the improvement of measurement and processing systems, as well as the work of many researchers [2-7], made possible to look at this problem from a different angle: "Noise is a source of information." However, the analysis of the modern methods [7-16] shows that now a "universal" approach for analysis of the trendless fluctuations is absent. In many cases, the authors use the "old" methods (Fourier-transform, Wavelet-decompositions and other methods) or introduce additional processing algorithms that are based presumably on conventional methods. These methods work for solution of specific tasks and, from our point of view, they are not universal. For example, the Fourier method carries in itself unjustified supposition about "a priori" known periodicity of a random signal [7,17-19]. In addition, the F-transform creates some set of the calculated frequencies that do not belong to the system considered. Wavelet method does not contain the general criterion for selection of an optimal set of wavelets that is optimal to consideration of the chosen TLS [7, 19-21], but contains uncontrollable errors especially related to application of the specific types of wavelets to the chosen random sequence.

Many methods contain the unjustified suppositions and uncontrollable errors [2, 4], however, the analysis of fluctuations as a small part of an accompanying signal requires an accurate and specific approach, at least in its preliminary analysis. We are not able to give all literature related to this subject (many papers are listed in book [2] and review [3] because each researcher dealing with specific noise tries to develop its own method. This preliminary analysis put forward a problem that can be formulated as follows: *is it possible to suggest some "universal" set of quantitative parameters that will be useful for treatment of large massive of data?* These proposed parameters should have: (a) clear interpretation, (b) free from the treatment errors and (c) can be applicable for treatment of different data, containing large number of data points. In addition, these parameters should be based on some simple principle.

In this paper, taking into account the previous attempts we want to define some simple set of quantitative parameters that can be applicable to consideration of a wide set of trendless sequences. We avoid deliberately the application of some fitting functions that are turned to be helpful for description of the SRAs [22, 23], however, the fitting error should be under a researcher control. All fitting functions with their parameters can be considered as the quantitative parameters of the second order. The basic idea, that allows introducing this universal set is based on the consideration of trendless fluctuations as a specific "struggle" between positive and negative tendencies (amplitudes). This principle/idea helps to add some new parameters to the conventional parameters as the mean value and the standard deviation. Usually a researcher-practitioner solves the following general problem: If some input predominant parameter is changed monotonically then is it possible "to notice" and express quantitatively this monotone change from the registered output fluctuations? The proposed algorithm (confirmed on the mimic and real data) allows to find the positive answer.

2. Description the proposed algorithm

As it had been mentioned in section 1, the idea was based on reasonable supposition that a "noise" is not a "disturbing factor"; it is used as a source of additional information. It was formulated thanks to papers [2, 4, 6], where scientists tried to extract information from random fluctuations. However, from the results obtained by them, it is impossible to select a universal idea for working with "noises". Also, we want also to find a positive answer on the following question:

Is it possible to find a set of simple "universal" parameters that can characterize the behavior of any trendless sequence irrespective to the main characteristic as the probability distribution function, which in the most cases is not known?

We must also bear in mind that the number of these parameters should be minimal and they should be rather "universal". These parameters should not contain uncontrollable treatment errors and accurately reflect a nature of the considered fluctuations. Before, we should give some definitions for better understanding the algorithm proposed below.

Under the TLS ($Dy_j = y_j - \langle y \rangle$) we understand a set of fluctuations that oscillates relatively the horizontal OX axis. If the initial sequence has a trend then one can find the smoothed trend with the help of the POLS [24] and after its subtraction one can obtain the desired TLS again.

A single fluctuation is defined as a random deviation of an amplitude relative OX axis and, therefore, can be positive or negative. From this point of view, the given TLS can be considered as a sum of amplitudes having two opposite tendencies in positive and in negative directions, respectively. Therefore, one can put forward a simple principle reflecting a "specific struggle" between positive and negative amplitudes and based on this idea one can define the following parameter:

$$p_1 = \mathrm{Rg}(Dy) = \max(Dy) - \min(Dy). \quad (1)$$

The value defines the range of the given sequence (Dy: Dy_j, j=1, 2, …, N). Parameter p_1 is always *positive* and corresponds to the maximal intensity of the given TLS.

$$p_2 = \mathrm{Rg}(|Dy|) = \max(Dy_+) - |\min(Dy_-)|. \quad (2)$$

Parameter p_2 defines the relative contribution of amplitudes that are located in the opposite sides of the TLS. If $\mathrm{Rg}(|Dy|) \approx 0$, it corresponds to an "ideal" balance between positive and negative amplitudes. In the opposite cases, when $\mathrm{Rg}(|Dy|) > 0$ (<0) we observe a specific "spike" of positive (negative) amplitudes in the given TLS relatively each other.

$$p_3 = DN = N(x_+) - N(x_-). \quad (3)$$

Parameter p_3 determines the number of amplitudes located in the opposite sides of the trendless sequence. If DN > (<) 0 then the number of positive amplitudes exceeds the number of negative amplitudes and (vice versa).

$$p_4 = Nv = \frac{2\cdot N}{N(roots)}, 2 < Nv < \frac{N}{3}. \quad (4)$$

The value *N*(*roots*) determines the number of points (roots) that can cross the OX axis. This important parameter p_4 determines the number of data points corresponding to one oscillation. If *Nv* close to 2 then the oscillations of the given sequence have clearly expressed the HF character, while *Nv* becomes close to *N*/3 we receive finally a single LF oscillation. If *Nv* >*N*/3 then the desired oscillations are *absent*. One (two) crossing points are not sufficient for creation of a single oscillation. The parameter *N*(*roots*) allows to evaluate approximately the period $T \approx 2length(x)/N(roots)$ and frequency $\omega=2\pi/T$ of the mean oscillation.

$$p_5 = DS = S(y_+) + S(y_-), S(y_{+,-}) = \sum_{j=1}^{N_{+,-}} y_{(+,-)j}. \quad (5)$$

This parameter determines the total/cumulative contribution of all amplitudes corresponding to positive/negative amplitudes, respectively.

For better understanding of the meaning of these parameters that follows from the “struggle” principle, we listed the combination of their signs in Table 1.

Table 1. The combination of the parameters signs explaining a specific “struggle” between positive/negative tendencies.

p_1 = Rg(*Dy*) and p_2= Rg(│*Dy*│)	p_3 = *DN*	p_4 = *Nv*	p_5 = *DS*	Comments
Rg(│*Dy*│) ≈ 0 “ideal” sequence	*DN* > 0	2 < *Nv* < 20 HF fluctuations	*DS* > 0	Excess of positive amplitudes is predominant
Rg(*Dy*) < Dy_c Dy_c critical range.	*DN* < 0	20 < *Nv* < 50 Tendency to HF fluctuations	*DS* > 0	Excess of positive amplitudes is remained predominant
Rg(*Dy*) > Dy_c Critical behavior	*DN* < 0	*Nv* < *N*/10 Tendency to LF fluctuations	*DS* < 0	Excess of negative amplitudes is predominant
Rg(│*Dy*│) < (>) 0 Spike of negative (positive) amplitudes	*DN* > 0	*N*/10 < *Nv* < *N*/3 LF fluctuations	*DS* < 0	Excess of negative amplitudes is remained predominant

Besides these parameters, one can add some parameters that follow from the fitting of the bell-like curve (BLC) by beta-distribution function [23]:

$$Y \cong Bd(x; A, B, \alpha, \beta) = A(x - x_0)^{\alpha}(x_N - x)^{\beta} + B. \quad (6)$$

This function fits BLC *Y*(*x*) that, in turn, is obtained after the integration of the SRA (the sequence of the ranged amplitudes) with preliminary elimination of its mean value. We want to stress here that the BLC (> 0) describes a *maximal* fluctuation located in the given interval $[x_0, x_N]$. The meaning of these fitting parameters (*A*, *B*, α, β) and their calculations are explained in papers [22,23]. In addition to these four parameters, one can add the maximal value of the BLC: $p_6 = Y_{mx}$, $p_7 = x_{mx}$ (this characteristic point separates the positive set of amplitudes from the negative one), then the similar measures of asymmetry:

$$p_8 = Dx = \frac{1}{2}(x_0 + x_N) - x_{mx}, p_9 = \frac{\alpha}{\beta} \equiv r = \frac{x_{mx} - x_0}{x_N - x_{mx}} \quad (7)$$

in vertical direction and a small parameter *B* that indicates a possible asymmetry in the horizontal direction. Therefore, summarizing the parameters determined above one can propose at least 10 quantitative parameters (including also the mean value of *y*, p_0=mean(*y*)) pretending on the “universal” description of the given TLS. We want to stress here that parameter p_0 determines the specific “equilibrium” line separating the set of positive amplitudes from the negative ones. Parameters *A*, $p_6 = Y_{mx}$ is intensity of fluctuations. *Ymx* of the BLC separates the positive set of amplitudes from the negative ones. *B* is asymmetry in horizontal direction with respect to OX axis. $p_7 = x_{mx}$ and $p_8 = Dx$, where $Dx = \frac{x_0+x_N}{2} - x_{mx}$ have two cases. If Dx close to zero – BLC is symmetrical. If $Dx > 0$ (< 0) excess of negative amplitudes, (excess of positive amplitudes). $p_9=r$, where $\frac{\alpha}{\beta} \equiv r = \frac{x_{mx}-x_0}{x_N-x_{mx}}$ also have two cases. If $r \approx 1$ – BLC is symmetrical. If $r < 1$ (> 1) excess of negative amplitudes (excess of positive amplitudes). $S(\alpha,\beta)=\alpha+\beta$, $0< S(\alpha,\beta) <1$ is the most of amplitudes is located near the mean value, where $1< S(\alpha,\beta) <2$ fractal property of the TLS. In this research, we will use only the parameters p_0-p_9. As it follows from examples given below these parameters are sufficient for the solution of the problems formulated in the text.

As it has been mentioned in Introduction, we deliberately avoid the application of the FFT (the fast Fourier transform). This transformation is based on *a priori* (and, in the most cases, invalid) supposition that the given TLS is pure *periodical* and, in addition, it contains the excess of frequencies that do *not* exist in the given TLS reflecting the output of the studied system. That is why in the most cases the FT of the given TLS is not used as a *fitting* function [17,18] and is considered as an "independent" source of information. The same situation is related to application of the wavelets [19-21, 25, 26]. At present time, we do not have the justified criterion for selection of an *optimal* wavelet that fully corresponds to the analysis of the given TLS. Besides, the selected wavelet family contains some uncontrollable errors and application of two and more types of wavelets taken from other families can lead to different/contradictory results.

Finishing this section, we should show a possible way of application of these 10 parameters in analysis of a large amount of data. As it was mentioned above, the most part of researches solve the following problem: there is one predominant and external factor F (for example, pressure P, temperature T, humidity H, intensity of radiation I, substance concentration C and etc.) that is changed monotonically in the certain range $F_k= a\cdot k + b$ (k=0,1,…, K). For each fixed value of F_k we have some rectangle matrix of measurements $N\times M$ ($N \geq M$), where (j=1,2,…N) determines the number of data points and M (m=1,2,…,M) determines the total number of repeatable (or statistically similar) measurements. We imply that this initial matrix contains only fluctuations (serving as an additional source of information) expressed in the form of the TLS(s). Having these huge massive of data is it possible to reduce them to the minimal number of quantitative parameters reflecting some common features of the phenomenon studied? The solution can be divided on some steps:

S1. We transform each TLS to the reduced sequence containing only n (in our case n=10) quantitative parameters. Therefore, we obtain the reduced matrix $n\times M$. We transpose this matrix ($n < M$) and obtain the matrix $M\times n= (n\times M)^T$, where number of measurements (rows) exceeds the number of the reduced parameters n (columns). Each column ($p_{m,l}$: m=1,2,…,M; l=1,2,…,n) for the fixed l has a different statistical meaning and in some cases it has a sense to make them statistically similar to each other with the help of transformation:

$$Nrm_l = -1 < \frac{p_l - mean(p_l)}{max(p_l) - mean(p_l)} < 1, l = 1,2,\ldots,n. \quad (8)$$

Here p_l (l=1,2,…,n) determines the vector of one of the reduced parameters belonging to the reduced set n.

S2. Then one can apply the reduction procedure to the Nrm_l vertical vector having m=1,2,…,M data "points". After that, we obtain the square matrix $n\times n$ containing only reduced parameters taken over all measurements.

S3. Finally, for the remaining reduced matrix $Pr_{n,n}$ one can apply the singular valued decomposition (SVD) operation [27] (widely used in the PCA [28]) and find the eigenvalues of this matrix. These eigenvalues (associated with the principal components) located in the descending order ($Ev_1 > Ev_2 > \ldots > Ev_n$) can characterize the initial rectangle matrix $N\times M$ forming n-possible functions $Ev_l(k)$ depending on the external factor F_k. From these functions one can select the most sensitive and monotone function that reflects the monotone behavior of the predominant factor F_k. We should stress that this approach is rather "universal" and can be applied to a wide set of TLS(s).

3. Quantitative "reading" of real sequences

One of us (RRN) received an admission to real data. These real data were related to detections of defects in the frictionless bearings (FB(s)) from acoustic recordings that were performed in the same experimental conditions (the fixed recording distance from the source of a noise and registered microphone, temperature, humidity, pressure were conserved during the whole experiment) for a normal FB and the chosen bearing with defects. The types of defects are explained and listed in Table 2.

Table 2. Artificially created defects in the frictionless bearings (FB).

Number of defect	Description of the defect	Realization technique
1	Separator defect	Separator was unclamped. One rivet was removed.
2	Separator defect	Separator was unclamped Two rivets were removed in random order.
3	Defect of the bearing ball	The defect was created by the electron discharge method. The hole diameter on the defect of the FB ball was varied in the interval 1.0 up to 1,5 mm. The measured depth of the hole was 0.2 mm.
4	Defect created on the external side of a bearing race	The defect was created by the electron discharge method. The hole diameter on the external FB race was varied in the interval 1.0 up to 1.5 mm. The measured depth of the hole was 0.2 mm.
5	Defect created on the internal side of a bearing race	The defect was created by the electron discharge method on the internal surface. The hole diameter on the internal FB race was varied in the same interval 1.0 up to 1.5 mm. The measured depth of the hole was 0.2 mm.
6	Bad lubrication	In the normal oiling metal chips in proportion (30 % chips) were added.

Comments to Table 2. Each acoustic recording at the *same* experimental conditions for normal and defect FBs was repeated 15 times. The total number of data points for each recording includes 44100 data points.

Each experiment for the given FB was repeated 15 times, the total number of the data points in each recording was N = 44100. To work with the data the algorithm was offered, which includes in itself the following steps:

S1. For each defect we have the rectangle matrix of the size $N\times M$ (N=44100, M=15). With the help of the proposed algorithm we reduce the initial matrix to the reduced matrix $n\times 3$ (where n=10 and it includes the set of parameters (p_0-p_9) defined in section 2). From each measurement we took only three basic parameters (s=1 (mean(m)), s=2 (stdev(m)) and s=3 (range(m)=max(m)-min(m)), where m=1,2,…M (M=15). Therefore, after the first step we receive seven matrices of the size $n\times s$ (including the normal FB and associated with 6 defects listed in the Tables 3-5.

Table 3. The total set of matrices obtained for all types of defects (defects 1-2).

Prms	Mean-nrm	Stdev-nrm	Range-nrm	Mean-def1	Stdev-def1	Rng def1	Mean-def2	Stdev-def2	Rng def2
p_0	-4.67025	0.47834	1.90576	-3.27902	1.58945	4.87139	-3.88459	0.66019	1.97109
p_1	1093.53	73.4428	301	2322.4	303.684	1174	11333.9	465.827	1408
p_2	70.0738	42.9153	176.835	130.558	124.894	413.191	1244.47	236.621	780.878
p_3	-145.467	147.4	602	39.2	97.2469	290	-448.2	116.953	400
p_4	11.6731	1.71566	5.00173	4.48757	0.1152	0.27368	4.54249	0.08892	0.28814
p_5	-70.0738	42.9153	176.835	-130.558	124.894	413.191	-1244.47	236.621	780.878
p_6	1.8098E6	139300	366160	5.37576E6	248281	584650	1.0829E7	436526	1.4148E6
p_7	21.9768	0.0737	0.301	22.0691	0.04862	0.145	21.8254	0.05848	0.2
p_8	0.07323	0.0737	0.30101	-0.0191	0.04862	0.145	0.22461	0.05848	0.2
p_9	0.9934	0.00662	0.02699	1.00174	0.00442	0.01319	0.97985	0.0052	0.01781

Table 4. The total set of matrices obtained for all types of defects (defects 3-5).

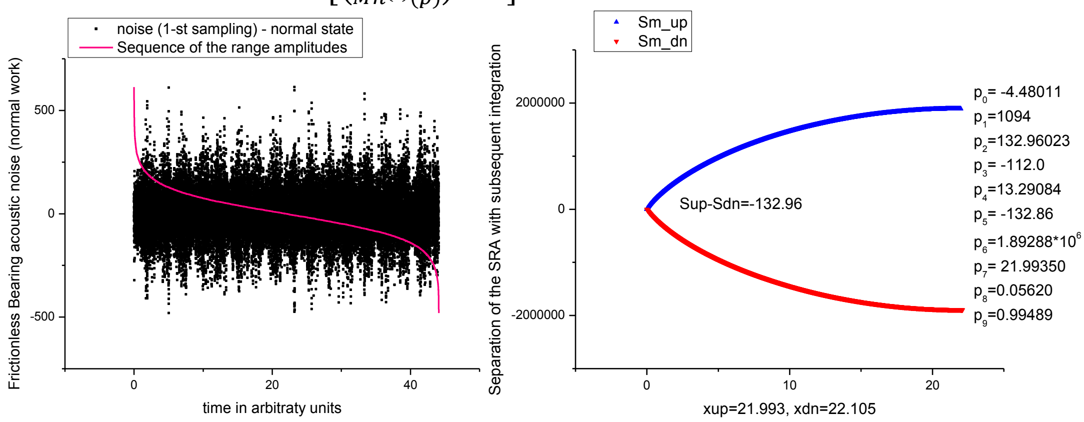
Prms	Mean-def3	Stdev-def3	Range-def3	Mean-def4	Stdev-def4	Rng def4	Mean-def5	Stdev-def5	Rng def5
p_0	-3.0545	0.7435	2.3494	-3.00654	0.5384	1.2912	-3.09632	0.2761	0.9883
p_1	2432.5	407.48	1328	2388	435.37	1415	1608.8	619.73	2086
p_2	210.809	241.52	803.45	267.013	282.60	976.40	-17.2073	134.98	510.94
p_3	-54.4	199.81	536	-244	155.22	526	-165.6	100.18	280
p_4	5.54224	0.0245	0.08	6.28622	0.2036	0.5228	6.40816	0.4357	1.12855
p_5	-210.80	241.52	803.45	-267.013	282.60	976.40	17.2073	134.98	510.94
p_6	3.870E6	270407	661720	3.49421E6	308128	728170	2.60445E6	480303	1.1019E6
p_7	22.0223	0.09991	0.268	21.9275	0.07761	0.263	21.9667	0.05009	0.14
p_8	0.0277	0.09991	0.26801	0.1225	0.07761	0.26301	0.0833	0.05009	0.14
p_9	0.99753	0.00905	0.02429	0.98898	0.007	0.02372	0.99248	0.00451	0.01261

Table 5. The total set of matrices obtained for all types of defects (defect 6).

Prms	Mean-def6	Stdev-def6	Range-def6
p_0	-4.23894	1.07338	2.47347
p_1	1417.7	357.107	880
p_2	51.178	99.9625	378.901
p_3	-300	160.135	592
p_4	8.15484	2.25404	5.10398
p_5	-51.178	99.9625	378.901
p_6	2.427E6	554516	1.19996E6
p_7	21.8995	0.08007	0.296
p_8	0.1505	0.08007	0.29601
p_9	0.98647	0.00715	0.02644

S2. In order to see the possible differences between the parameters referred to normal FB and bearing with the given defect one can consider the ratio:

$$Rt_s(p) = \left[\left(\frac{Md^{(s)}(p)}{Mn^{(s)}(p)}\right) - 1\right], s = 1,2,3; p = 0,1,\ldots,9. \quad (8)$$



a) b)

Figure 1. (a). Typical noise record obtained for the FB in normal state. It corresponds to the first sampling. Solid red line shows the desired SRA that divides the positive set of amplitudes from the negative ones. If we divide this line relatively mean value and then integrate it, we obtain the curve showing the summarized contributions of these amplitudes. These curves are shown in the next figure. (b). These two curves determine the total contribution of the positive (blue line) and negative (red line) amplitudes. They are slightly differed from each other. This difference (-132.96) is shown inside in the figure. On the right-hand side we show the total set of the reduced parameters corresponding to the chosen sampling. The meaning of these parameters is given in the text and is explained in Tables 1 and 2.

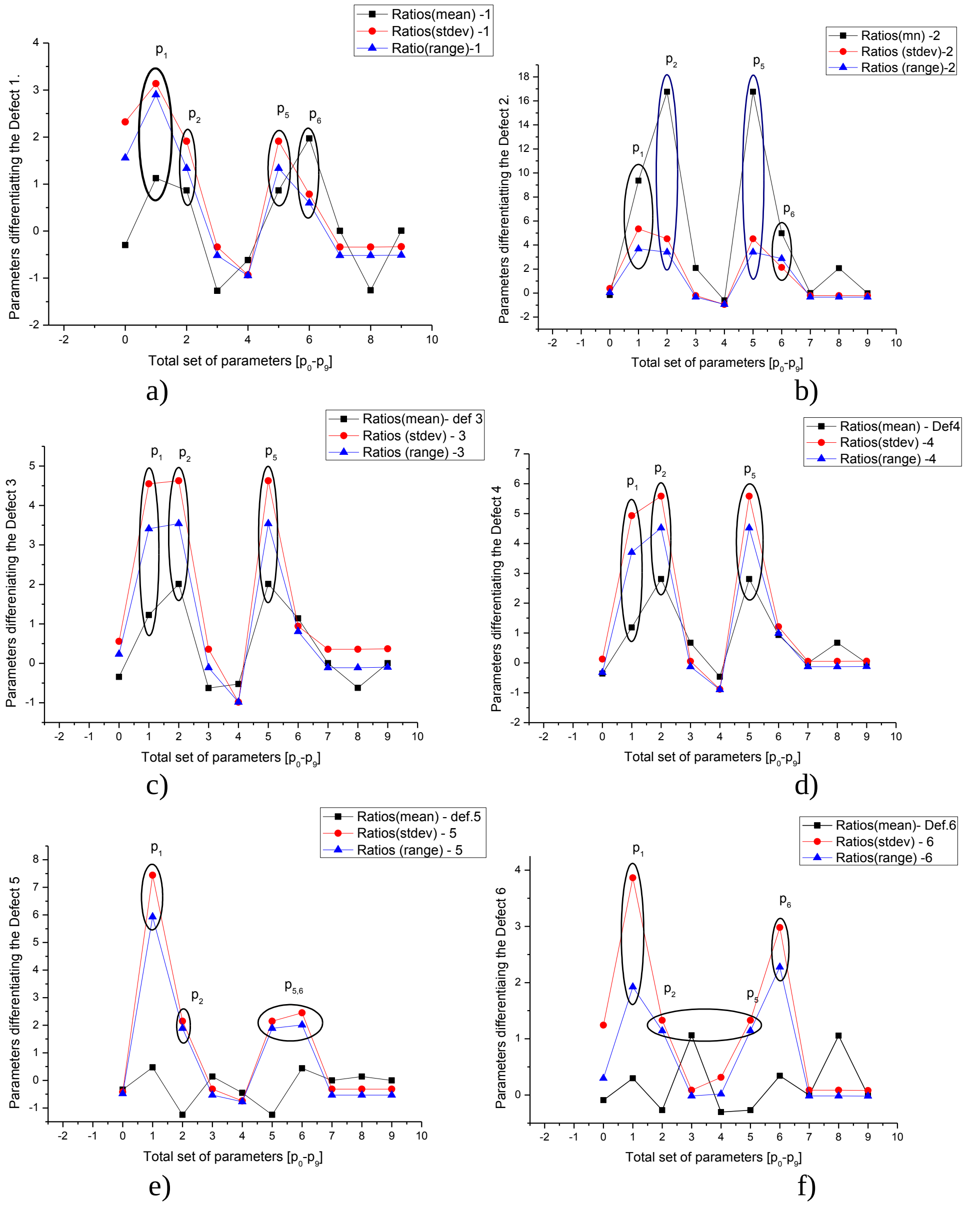


Figure 2. (a). The sensitivity of the function (5) to the presence of the defect 1 that corresponds to extraction of one rivet. As one see from this figure the parameters $p_{1,2,5,6}$ differentiate easily the presence of the first defect from the sound records corresponding to the normal work of the FB. (b). It is obvious that extraction of two rivets (defect number 2) should lead to more expressed distortions in comparison with the records corresponding the normal state. This figure demonstrate clearly this effect. The same parameters $p_{1,2,5,6}$ detect this defect, however the scale of deviations increases essentially. (c) Here we show the parameters $p_{1,2,5}$ that can differentiate the defect number 3. All three ratios enable to select this defect from the normal working state. (d) Comparing this figure with the previous figure corresponding to the defect 3 one can notice that the same parameters $p_{1,2,5}$ were involved in detection of the defect 4, however, these defects are different. (e) This defect can be differentiated only by two last ratios associated with standard deviations and ranges. The ratio associated with mean value becomes less informative and does not prove its efficiency. (f) This defect related to bad lubrication can be detected only with the help of two parameters p_1 and p_6. Other two parameters $p_{2,5}$ can be considered as the secondary indicators. This example shows that in practice in would be desirable to have a set of parameters having different sensitivity to the influence of the external factor. At least these 10 selected parameters are proved their efficiency in testing them on real data.

In the result of realization of these two steps, we obtain three functions depending on the 10 parameters (p_0-p_9) for each given defect. The subsequent analysis of these functions will be simple: if the function (8) exceeds the unit value then this ratio will have distinct feature that differentiates the defect from the normal working state. It is obvious that the set of parameters (p_0-p_9) will have different sensitivity to each defect and *a priori* to evaluate this sensitivity is impossible. Figure 5 shows the fixed sampling of the initial set of data associated with the recording of the FB in normal state. Other data associated with the chosen type of defect look similar and therefore are omitted.
Figures 2(a-f) corresponding to number of the listed defects present themselves the *key* figures that help to differentiate the sound files corresponding to the given defect from the normal sound file.
We constructed them in accordance with expression (8) in order to see the desired differences engendered by each defect.
As one can see from analysis of these six figures, the selected parameters (p_0-p_9) are sufficient to detect them all and they have different sensitivity to the presence of the given defect. Comparing all figures with each other, one can say that the defects one and two are differentiated more easily in comparison with last defect related with bad lubrication. In the last case only two parameters $p_{1,6}$ have the most distinctive deviations and two ratios only as the standard deviations and ranges enable to detect it.

4. Results and discussion

In this work, some universal “platform” for processing various types of data was demonstrated. In our opinion, it can attract the attention of many researchers working in various branches of engineering and the natural sciences. To summarize and focus on its distinctive features:
1. The proposed “platform” is based on a simple “struggle” principle between positive and negative amplitudes. It is free from the uncontrollable errors and does not use any unjustified supposition as the FFT or wavelet analysis;
2. The proposed platform reminds a “violin cleats” which allow you to quite flexibly customize our tool. If necessary, the sensitivity can be adjusted by the number of parameters. This makes it possible to increase the detection of the prevailing external factor;
3. The instructive example based on real data shows that the proposed platform enables to detect the defects that initially can be described only qualitatively.
Before, any researcher knew presumably only two basic “universal parameters” as the mean value (it can be interpreted as a specific equilibrium line separating the positive amplitudes from the negative ones) and the standard deviation (it evaluates approximately the deviations from this line). The proposed scheme can add another set of universal parameters based on specific tendencies of a “struggle” between positive and negative amplitudes.

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