FEM analysis of the biaxial cyclic loading of the elastoplastic plate with concentrators: asymptotic states

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Abstract. Elements of structures which work in real conditions quite often are affected by variable temperatures and loadings. Nowadays due to the growth of interest to the knowledge of asymptotic behavior of the inelastic structure subjected to cyclic loading, direct and incremental methods of stabilized state determining begin to develop. If loadings vary and the body deforms elastically, then its durability is defined by fatigue characteristics of material, failure comes after a large number of cycles. If the body experiences elasto-plastic deformation, at loadings below limit, achievement of a dangerous state at rather small number of cycles is possible. In the present study results of finite-element (FEM) calculations of the asymptotical behavior of an elastoplastic plate with the central circular and elliptic holes under the biaxial cyclic loading for three different materials are presented. Incremental cyclic loading of the sample with stress concentrator (the central hole) is performed in the multifunctional finite-element package SIMULIA Abaqus. The ranges of loads found for shakedown, cyclic plasticity, ratcheting are presented. The results obtained are generalized and analyzed. Convenient normalization is suggested. The chosen normalization allows us to present all computed results, corresponding to separate materials, within one common curve with minimum scattering of the points. Convenience of the generalized diagram consists in a possibility to find an asymptotical behavior of an inelastic structure for materials for which computer calculations were not made should be included.

Keywords: shakedown, cyclic plasticity, ratchetting, finite element method, incremental analysis, cyclic loading, asymptotic behavior, inelastic structure.

1. Introduction. Cyclic Loading of Inelastic Structures: Incremental Loading, Asymptotic States and Direct Methods

Asymptotic states of inelastic structures subjected to cyclic loading cause the particular interest in Solid Mechanics. Response of the inelastic structure under periodic mechanical (or termomechanical) loading is complicated and may contain inelastic (plastic, viscoplastic) strains. The reason of these difficulties in structure behavior description is that we need calculations, including the whole history of loading.

Elements of structures under real serviceable conditions are often subjected to variable temperatures and loadings. If the body is elastic, then the durability is defined by the fatigue material data and fracture comes after large number of cycles. If the body is elastoplastic, the dangerous state can be reached after rather small number of cycles, even when the loading is less than limit value. It is necessary to distinguish two cases. The first one is when fracture comes owing to alternation of the plastic strains sign (e.g. after plastic stretching comes plastic compression etc.). It is alternate plasticity (plastic or low-cyclic fatigue). The second one is when plastic strains increase with every cycle, which leads to its inadmissible accumulation (progressing plastic deformation - ratcheting) [1].

In structural design shakedown is considered to be a safe regime, ratcheting should be avoided. In this regard, requirement in knowledge of the asymptotic behavior of the inelastic structure after a large number of loading cycles on the early steps of configuration design becomes actual [2].

Two classes of methods are developed in order to determine the asymptotic state: direct methods [2]–[3] and incremental methods [4].

In recent work incremental step-by-step biaxial loading of the elasto-plastic plate with the central hole is carried out. The applied load P_1 is cyclic. Three types of asymptotic behavior of the plate are revealed during the analysis of FEM calculations. Domains of loadings for three asymptotic regimes of an inelastic structure are found. Calculations are carried out for three different materials. The obtained ranges of shakedown, alternate plasticity and ratcheting were analyzed and general patterns of different regimes were revealed.

2. Computing experiments

Incremental methods are time-consuming and demand considerable number of numerical experiments [1-4]. The aim of direct numerical methods is to overcome shortcomings of incremental analysis, however the developed theory is far from completion and universal practical use. Owing to the specified reasons, in the recent work incremental loading of elasto-plastic plate with central hole was carried out in order to identify different asymptotic behavior of inelastic structure and determine loads, which lead to shakedown, cyclic plasticity and ratcheting. Implementation and research of the three asymptotic regimes were held out on the example of a simple structure – the square plate with central hole. The geometry of the plate is shown in figure1.

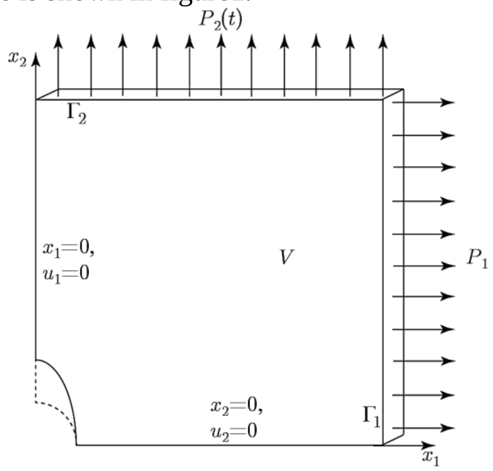

Figure 1. Geometry of the plate with the central hole.

The length of a side of the plate L is $16dm$. The plate's side length to its thickness ratio is 0.02. The semi major axis of the elliptical hole is $a = 3dm$, and the semi minor axis is $b = 2dm$.

Let's consider cases when the copper plate has different mechanical properties:

a) density is $\rho = 5400 kg / m^3$, Young's modulus is $E = 4.3\times10^8 kg / m^2$, Poisson's ratio is $\nu = 0.28$ [5];
b) density is $\rho = 8920 kg / m^3$, Young's modulus is $E = 13\times10^9 kg / m^2$, Poisson's ratio is $\nu = 0.28$ [6];
c) density is $\rho = 8920 kg / m^3$, Young's modulus is $E = 1\times10^{10} kg / m^2$, Poisson's ratio is $\nu = 0.3$ [6];

The plate is subjected to the biaxial loading P_1 and P_2 (figure 1). The applied load $P_2(t)$ is cyclic. The applied loading is schematically shown in figure 1.

3. Elasto-plastic Analysis

In this section the finite-element method solution of the cyclic loaded elasto-plastic plate with central hole problem is given. The purpose of the present finite element analysis is to define load amplitudes (boundaries of shakedown, cyclic plasticity and ratcheting domains) when these regimes are realized. Let's consider a body, occupying volume V (the plate with central hole, the quarter of which is given in figure 1). The boundary conditions are: when $\vec{x} \in \Gamma_2$, $P_2(t) = k_2 \sigma_T \sin(2\pi t / T)$, see figure 2. When $\vec{x} \in \Gamma_1$, $P_1 = k_1 \sigma_T$, where σ_T is material yielding strength; k_1, k_2 are constants of proportionalities, which are used to vary the amplitude of the applied load.

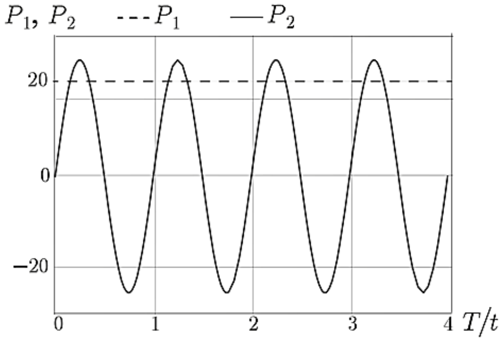

Figure 2. Periodic law for the load.

The boundary conditions are: periodic load $\sigma_{22} = P_2(t)$ on $\vec{x} \in \Gamma_2$ and static load $\sigma_{11} = P_1$ on $\vec{x} \in \Gamma_1$. Boundary conditions on the symmetry planes of the quarter of a plate are shown in the figure 1.

4. Finite Element Analysis

The series of calculations were carried out in multifunctional finite-element package SIMULIA Abaqus. Plastic material properties of the model are set by the table 1. The first value in the first column is the material yielding strength, and the last one in the first column is the ultimate strength of material [5]–[6].

Table 1. Stress – plastic strain dependence for copper.

(a)		(b)		(c)	
σ, kg/m^2	ε^{pl}	σ, kg/m^2	ε^{pl}	σ, kg/m^2	ε^{pl}
54×10^4	0	210×10^6	0	60×10^6	0
58×10^4	0.0006	240×10^6	0.0055	100×10^6	0.09
63×10^4	0.0080	280×10^6	0.015	140×10^6	0.15
69×10^4	0.0013	300×10^6	0.02	170×10^6	0.23
74×10^4	0.0018	320×10^6	0.025	200×10^6	0.36
78×10^4	0.0023	360×10^6	0.03	220×10^6	0.45

In the series of calculations constants of proportionality (k_1, k_2) varied from 0 to 1.3.

It is obvious that for different asymptotic regimes of the structure stresses always behave cyclically. This feature is the basis for the most part of direct methods which are used to determine the asymptotic state.

For the analyzed elasto-plastic plate all three regimes were obtained by changing the magnitude of the load and loadings that lead to the change of the regime were defined.

One can determine the asymptotic type of the structure after a number of loading cycles by the use of the plastic deformations character.

The obtained data were generalized and represented in the form of the diagram for both of the plates (figure 3) in which the domains of loadings corresponding to each type of an asymptotical behavior of a plate are defined. The type of structure behavior was defined for the most deformed finite element of the plate. The location and number of this element varied according to the exact loading case.

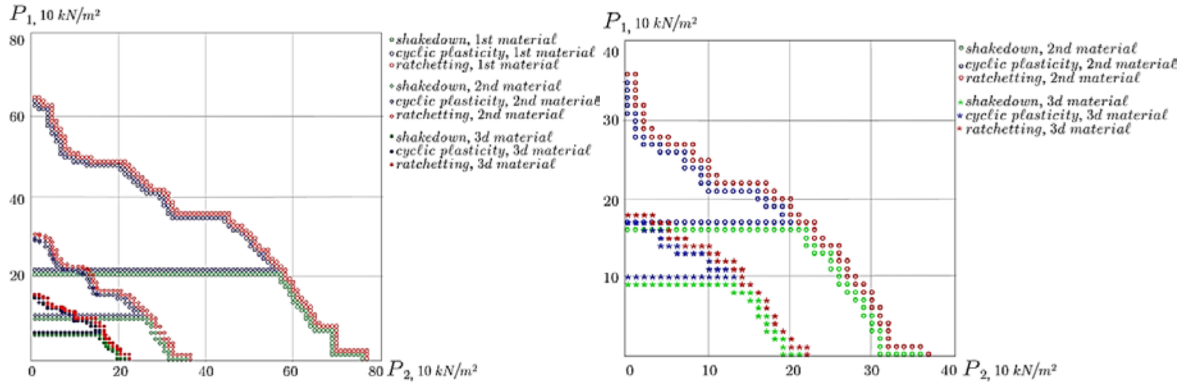

Figure 3. Results of numerical analysis: the diagram of loading domains for different asymptotic regimes of the plate: left – with elliptic hole, right – with circular hole.

Boundaries of the loading limits P_1 and $P_{2\max}$ for ratcheting are marked with red color, for cyclic plasticity – with blue, for shakedown – with green. From the diagram it is obvious that without cyclic loading ratcheting comes at the magnitude of the load which is equal to the material ultimate strength.

This diagram allows us to define safe magnitudes of loads for the structure and avoid dangerous (unsafe) regimes.

From the figure 3 it is seen that for different materials limits of shakedown, cyclic plasticity and progressing plastic flow differ, but qualitative distribution of domains remains similar for all the materials.

Using this hypothesis, based on the supervision, we can introduce dimensionless parameters: $\pi_1 = P_1 / \sigma_B$, $\pi_2 = P_2 / \sigma_B$ and construct diagram P_1 and $P_{2\max}$ on the plane π_1, π_2 for all the three materials. Results are shown in the figure 4.

From the figure 4 it is seen that all curves defining domain boundaries match and lay down on the uniform curve. On the plane π_1, π_2 all normalized calculated points from the numerical experiment lie on the uniform curve. The variables π_1, π_2 can be interpreted as similarity variables.

Thus, numerical analysis showed independence of characteristic domains from mechanical properties of materials. Therefore there is no need to calculate domains for every distinct material.

The chosen normalization allows us to present all computed charts for certain materials within one curve with the minimum point dispersion. The convenience of the generalized diagram is in opportunity to know the asymptotic behavior of an inelastic structure for different materials without computing.

5. Conclusions

By means of varying k_1 and k_2 we can study the whole range of static and cyclic loads, applied to the structure. In the present paper large series of numerical computer modeling experiments for plate with central hole under cyclic loading (in the whole range values of coefficients k_1 and k_2) were carried out and interpreted. On the basis of the computer experiment ranges of loads for shakedown, cyclic plasticity and ratcheting were determined.

Types of asymptotic behaviors of non-elastic structures are discovered. The typical diagrams shown in figure 4 are constructed; their main and intrinsic features are defined. Borders of asymptotic behavior types of a certain structure subjected to identical structure loading, for different materials match and lay down on one curve.

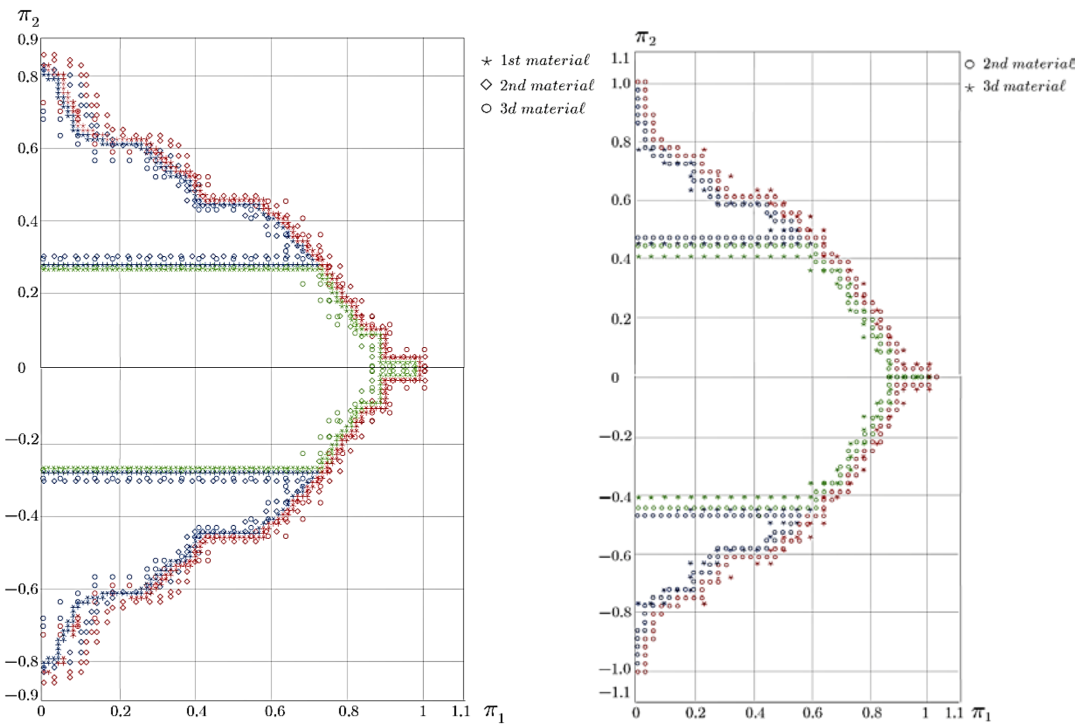

Figure 4. Results of numerical analysis: the generalized diagram of loading domains for different asymptotic regimes of the plate: left – with elliptic hole, right – with circular hole.

For three materials with different mechanical features domains are similar. Thus a convenient normalization of computing results is offered. Experimental data are presented in a generalized diagram of structure's asymptotic behavior.

The obtained incremental analysis results may be useful for checking the data received with the direct methods. Direct methods give us an opportunity to decrease computing expenses while determining ranges of safe working regimes of test samples. This will allow organizing design and repair of structures in a new way.

6. References

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