Estimation of signal-to-clutter-plus-noise ratio in presence of clutter clipping

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Abstract. This paper presents the mathematical model of a clutter clipping in radars with quasicontinuous mode of transmission and reception of signals with pseudorandom amplitudeand phase-shift keying. The estimation of signal-to-clutter-plus-noise ratio in presence of clutter clipping is derived using the distribution histogram of the received signal amplitudes. The modelling example of a pinpoint target locating is given. The mathematical model assumes the clipping of the time-distributed clutter.

Keywords: pseudorandom amplitude- and phase-shift keying signals, clutter clipping, signal-to-clutter-plus-noise ratio.

1. Introduction

Clipping is a very effective method of an impulse noise amplitude normalizing. It is usually used in a wideband signals processing [1]. An advanced radar mode (quasicontinuous mode), which uses probing signals with pseudorandom amplitude- and phase-shift keying (APSK) has the feature of return signals partial overlap in time [2]. In quasicontinuous mode time intervals with no high-power returns occur in additive mixture of return signals. Clipping of input signal normalizes the high-power signal level, that interfere with weak valid APSK signal. A part of valid signal pulses is also suppressed during clipping. However, valid signal pulses that do not clipped can be coherently processed.

With clipping, the degree of suppression of high-power return signals depends on the length of the target and varies according to average duty cycle and minimum pulse duration of probing APSK signal. An estimate of the weak signal detection reliability increase for the indicated dependencies is shown in [3]. However, the analysis of signal-to-clutter-plus-noise ratio (SCNR) was carried out assuming that the radar has the spaced antennas and the processed signal represents an additive mixture of a weak valid signal and equal power clutter with different delays. In a realistic interference situation, the distribution of interference intensity is complex. In addition, when operating with single antenna for transmission and reception, the estimate of the SCNR should take into account the energy loss due to receive path blanking.

The aim of the paper is to estimate the SCNR with clutter clipping in a radar with quasicontinuous mode of wideband signals transmission and reception. The mathematical modeling of additive mixture

of APSK signal, clutter and noise allows us to construct a distribution histogram of the amplitude values of the received signal before and after the clipping. The obtained probability of the received signal clipping makes it possible to form an estimate of the SCNR after correlation APSK signal processing with clutter clipping.

2. Mathematical model of clutter clipping

The complex envelope u(t) of the probing APSK signal of length $T=Nt_b$, consisting of N elementary pulses $u_0(t)$ of duration t_b , is determined by the discrete ternary sequence $w_n=x_nz_n$, $w_n \in \{0,\pm 1\}$, n=0,1..N-1, where z_n and x_n are binary sequences $(z_n \in \{\pm 1\}, x_n \in \{0,1\})$, which determine the pseudorandom law of phase-shift keying and the amplitude-shift keying of u(t) accordingly.

$$u(t) = \sum_{n=0}^{N-1} w_n \cdot u_0 (t - n \cdot t_b)$$
(1)

The APSK signal is described by the average duty cycle C = K/N, where $K = \sum_{n=0}^{N-1} |w_n|^2$.

Figure 1 shows an example of the keying sequence w_n of a length N=2048 and an average duty cycle C=20%.



Figure 1. Ternary keying sequence w_n of APSK signal.

When radar is operating with single antenna for transmission and reception, the receive path is blanked during the transmission of probing pulse by the signal

$$x_{\rm r}(t) = \sum_{n=0}^{N-1} (1 - x_n) \cdot u_0(t - n \cdot t_{\rm b}).$$
⁽²⁾

Let the received signal $s(t) = [s_c(t) + \xi(t) + \eta(t)] \cdot x_r(t)$ be the linear sum of three signals:

1) The valid signal $s_c(t)=A_cu(t-\tau_c)\exp(j2\pi F_ct+\varphi_c)$ with amplitude A_c , delay τ_c , Doppler frequency shift F_c and random initial phase φ_c ;

2) The clutter, that is formed by the targets in the radar's coverage area:

$$\xi(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(\tau, F) \sum_{n=0}^{N-1} w_n u_0 (t - \tau - nt_b) \cdot \exp(j2\pi Ft) d\tau dF , \qquad (3)$$

where the statistical averaging $|\rho(\tau,F)|^2$ determines the distribution density of the total interference power along delay τ and the Doppler frequency shift *F*;

3) The noise $\eta(t)$ with the normal distribution law and power P_0 in the signal band.

We assume that the clutter component $\xi(t)$ with a delay, equal to the valid signal delay τ_c , has low power and differs from the valid signal by the Doppler frequency shift.

Typically, the power of the valid signal is significantly lower than the total interference power, but, at the same time, it can exceed the noise power P_0 . In this case, the received signal envelope is defined

by the distribution of a clutter total power along the delay $p_{\xi}(\tau) = \int_{-\infty}^{\infty} |\rho(\tau, F)|^2 dF$.

In some cases, we have the high-level regions in $p_{\xi}(\tau)$, like it is depicted in figure 2. It leads to the instantaneous amplitude pulsed bursts of the received signal complex envelope |s(t)| as in figure 3a.

The impulse character of the complex envelope |s(t)| makes it possible to apply received signal clipping, providing a vast reduction of the clutter power at the input of correlation receiver.





Figure 3. The envelope of the additive mixture of the valid signal, clutter and noise: a) before clipping; b) after clipping.

After clipping of the received signal U_0 the complex envelope s(t) transforms to

$$s_{o}(t) = \begin{cases} U_{o} \cdot \exp[j \cdot \arg(s(t))], \ |s(t)| \ge U_{o} \\ s(t), \ |s(t)| < U_{o} \end{cases}$$
(4)

where $\arg(s(t))$ is the instantaneous phase of the received signal complex envelope.

The envelope $|s_o(t)|$ of the additive mixture of the valid signal, clutter and noise after blanking in the receive path and clipping is pictured in figure 3b. The same figure outlines the valid signal envelope $|s_c(t) \cdot x_r(t)|$, contained in the received signal s(t). Comparing the difference between $|s_o(t)| |u| / |s_c(t) \cdot x_r(t)|$, we can note, that the valid signal level is similar to the clutter pulses level after clipping, although the valid signal power before clipping was 44 dB lower than the total clutter power $(A)^{-2} \int_{0}^{\infty} u(t) dt > N$ and 14 dB higher that its sum price. It has possible to detect the solid

 $(A_c)^{-2} \int_{0}^{\infty} p_{\xi}(\tau) d\tau > N$, and 14 dB higher that its own noise. It became possible to detect the valid

signal due to a partial time overlap.

The signal is processed by a multichannel delay $\tau_m = m t_b$, m = 1,2,3,.. and the Doppler frequency shift $F_v = v/T$, $v=0,\pm 1,\pm 2,..$ correlation receiver. The output response at the (m,v) processing channel is described by

$$R_{m,\nu} = \left| \int_{0}^{T} s_{o}(t) u^{*}(t - m \cdot t_{b}) \exp(-j2\pi F_{\nu}t) dt \right|, \qquad (5)$$

where * indicates the complex conjugation operation. When the clipping is not applied we must take $s_0(t)=s(t)$ in equation (5).

The result of the response function simulation $R_{m,v}$ of the signal s(t) without clipping is shown in figure 4a and with clipping – in figure 4b. The envelope of the processed signal s(t) is given in figure 3a.

Note that the normalization of R_{max} corresponds to the maximum value of $R_{m,v}$ obtained by the correlation processing of s(t) without the clipping.

If the clutter is not clipped, then for a given APSK signal spectrum width its duration $T=Nt_b$ is insufficient to detect the valid signal against a noise background. The peak to average sidelobe ratio of the ambiguity function of the APSK signal of length N=2048 equals 10lg(N)=33.1 dB. This value is 11 dB less than the clutter-to-signal ratio at the input of the correlation receiver for the given example. The valid signal is not detected (figure 4a).

After the clipping the clutter power at the input of the correlation receiver decreases. The valid signal power loss is minor and the signal can be detected.



Figure 4. The correlation processing results: a) without clipping; b) with clipping.

The detection reliability of the valid signal with m time delay and v Doppler frequency shift after the clutter clipping is determined by SCNR at the (m, v) processing output.

3. SCNR estimate with applied clipping

In clutter clipping mode the estimate of SCNR averaged over all correlation processing channels can be obtained on the basis of the probability $D(/s(t)/\geq U_o)$ of complex envelope amplitude /s(t)/exceeding the clipping level U_o . In case of discrete observation with sampling interval $t_n=nt_b$, n=0..N-1, the value of D is estimated by the ratio $D=K_{s>Uo}/N$, where $K_{s>Uo}$ is the number of clipping threshold excess cases. The probability that the clipping threshold is not exceeded equals (1-D).

In quasicontinuous mode return signals are received during the transmission gaps, what leads to the loss of a part of the return signals, including those exceeding the clipping threshold. As a result of the receive path keying by the signal $x_r(t)$, the number of clipping threshold excess cases $K_{s>Uo}$ decreases.

If we use the pseudorandom keying sequence the probability of APSK signal pulse transmission is determined by duty cycle *C*, $x_n=1$. The probability that the receive path is opened in the *n* time delay equals $\Lambda=1-C$. The variable Λ for quasicontinuous mode is usually called the reception coefficient [2]. The (*C* Λ) product defines the probability of the arrival of the valid signal with *m* time delay in the *m* range processing channel. All range channels has the same statistical characteristics.

In the absence of clipping threshold excesses during *T*, every range channel will coherently accumulate (CNA) pulses in average. Because of the time overlap with the clutter pulses that exceed the clipping threshold, the number of coherently accumulated valid signal pulses decreases to the value [CNA(1-D)]. The power of the valid signal at the correlation receiver output matches the estimate on the basis of

$$P_{\rm c} = A_{\rm c}^2 \left[CN\Lambda \left(1 - D \right) \right]^2 \tag{6}$$

The power of signal s(t) pulses over the clipping threshold becomes U_o^2 . They interfere with the detection on the valid signal with power less than U_o^2 during the correlation processing. The valid signal pulses, which exceed the clipping threshold, are suppressed. The probability of reception of the

clipped clutter in the m range channel is determined as the product of the clipping threshold excess probability D by the probability of reference signal non-zero value.

The signal s(t) pulses that do not exceed the clipping threshold contain both valid signal pulses, noise and clutter and zero-amplitude pulses formed as a result of receive path keying. Let's now estimate their contribution to the response power at the range channel output, tuned to receive the valid signal.

Let us denote the power of clutter pulses, that not exceed the clipping threshold, by $P_{\xi < U_0}$. The reception probability of such clutter pulses at the range channel, tuned to the valid signal reception, is the same as for noise and is determined by the $[C\Lambda(1-D)]$.

As a result, the clutter and noise power after clipping and correlation processing is estimated by

$$P_{\xi\eta} = CNDU_o^2 + CN\Lambda(1-D)(P_{\xi
⁽⁷⁾$$

Taking into account equation (6) and equation (7) the SCNR after clutter clipping and correlation processing is defined by

$$q = \frac{P_{\rm c}}{P_{\xi\eta}} = \frac{A_{\rm c}^2 C N \Lambda^2 (1-D)^2}{U_{\rm o}^2 D + (P_{\xi < U_{\rm o}} + P_{\rm 0}) \Lambda (1-D)}$$
(8)

The clipping threshold in equation (8) should ensure the maximum reduction of the clutter power with the minimum power loss of the valid signal.

The lack of information about the valid signal power and its SNR allows us to propose the clipping threshold according to expression

$$U_{o} = \left[\int_{0}^{T} M(t) |s(t)|^{2} dt / \int_{0}^{T} M(t) dt\right]^{1/2},$$
(9)
where $M(t) = \begin{cases} 1, & |s(t)|^{2} < P_{s} \\ 0, & |s(t)|^{2} \ge P_{s} \end{cases}$ is a mask signal, $P_{s} = \frac{1}{T} \int_{0}^{T} |s(t)|^{2} dt.$

Expression (9) determines the clipping threshold by the RMS value of the received signal without the high-power interference overshoots.

The amplitude distribution histograms of the additive mixture of depicted in figure 3 realizations of s(t) and $s_o(t)$ before and after clipping are given in figure 5. The clipping threshold equals U_o calculated according equation (9). The peak value at zero point equals to 0.2 and reproduces the keying of the receive path. The clipping threshold excess probability is 0.19.



The mathematical simulation showed that after clutter clipping the RMS signal level at the multichannel correlation receiver output reduced by 19 dB (figure 4b). The RMS signal level is set by the detection threshold. As a result, the SCNR after clipping was 18 dB, which corresponds with the estimate from the equation (8).

Let's note that in shown example when we decrease the clipping threshold to the noise level, the valid signal is also affected by clipping. At the same time the clipping threshold excess probability increases to 0.58. The signal level at the multichannel correlation receiver output is further reduced by 16.3 dB and the SCNR is increased to 22 dB.

4. Conclusion

The results of clutter clipping simulation demonstrated the clipping efficiency for detecting weak valid signals against a background of high-power clutter. The expression for resulting SCNR at the correlation receiver output can be used for further studies of clipping efficiency.

5. Acknowledgments

The work was supported by Russian Ministry of Education, project №8.7367.2017/8.9.

6. References

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