

# Ensuring the effectiveness of the taxi order service by mathematical modeling of its work

A.N. Danilov<sup>1</sup>, N.A. Andriyanov<sup>2</sup>, P.T. Azanov<sup>3</sup>

<sup>1</sup>Taxi Ulyanovsk, pr-t Narimanova, 1, Ulyanovsk, Russia, 432071

<sup>2</sup>Ulyanovsk State Technical University, ul. Severniy Venets, 32, Ulyanovsk, Russia, 432027

<sup>3</sup>Tango Telecom, Krasnaya street, 122, Izhevsk, Republic Udmurtia, Russia, 426057

**Abstract.** We can't imagine modern science without mathematical modeling. At the same time, ideas related to mathematical modeling find themselves in various fields. One of the interesting applications of mathematical modeling of random processes is the work of the taxi order service. The article describes the description of the operation of the taxi order service using autoregressive models with multiple roots of characteristic equations of different multiplicity. Also attempts were made to approximate and predict the time series of various important parameters of the taxi order service. Algorithms for identifying model parameters are proposed.

**Keywords:** modeling of requests in taxi service, time series analysis, taxi ordering service, mathematical modeling.

## 1. Introduction

Modern information technologies have radically changed the market of taxi services. It will never be the same again. But even such as today, the situation can not remain. The spread of smartphones running iOS and Android has already led to the spread of applications related to the taxi order service. The taxi market is so competitive that the emergence of a monopoly on it is an impossible fact.

At the same time, it is possible to achieve lower prices and win customers, at the expense of almost 100% automation of service execution, perfect software, outsourcing by outsiders, etc. It should be noted that the automation requires a lot of tasks. For example, it is necessary to track in real time the tracking of drivers, their distribution by area. Timely analysis of this information allows you to quickly vary prices.

An attempt was made to approximate the data on the order distributions and predict future values. Analysis of the results of forecasting has shown that models based on random processes allow more efficiently to describe data of this kind.

The article discusses the possibility of using the mathematical modeling apparatus in analyzing information about the work of the taxi order service, shows how and what can be predicted, what opportunities the application of modeling and forecasting provides for influence on prices.

## 2. Modern taxi order services in Russia

The tasks of analyzing the efficiency of the taxi order services should be decided by comparing a set of criteria and indicators: from subjective customer feedback to accurate values, for example, the average waiting time in a conditional "rush hour." To do this, you need to perform an analysis of the known taxi order services and their capabilities.

### 2.1. Yandex.Taxi

Yandex.Taxi is online taxi order service implemented as mobile application or the website taxi.yandex.ru. It is the largest aggregator of taxi services in Russia in terms of the number of orders. The service sends the order to the driver who can arrive fastest taking into account the user's location and traffic situation.

As of July 2017, the service operates in 126 major cities in Russia, Belarus, Moldova, Ukraine, Armenia, Georgia and Kazakhstan. In total, more than 200,000 drivers are connected to Yandex.Taxi [1].

Available rates vary from city to city. The full lineup includes Economy, Comfort, Comfort +, Business and Minivan. In the comments to the order, you can specify several options: yellow registration numbers that allow taxi to travel along the lanes for public transport, a non-smoking driver, air conditioning, animal transportation, a receipt for payment and child seats (one or two) of three types: for children weighing from 9 to 18, from 15 to 25 and from 22 to 36 kg.

An analysis of the reasons that allowed the Yandex.Taxi service to reach high rates for such a short period of time shows that the efficiency of the service is largely related to the work of the IT department, including the decisive task of machine learning for optimal delivery of machines [2], and also with the accepted price policy and the work of marketers.

In addition, as a promising task related to image processing, it is worth mentioning the task of determining driver fatigue by the frequency of their blinking solved by Yandex.Taxi [3]. The Yandex.Taxi service is developing a new function that will monitor the eyes of drivers in real time and determine the degree of their fatigue in the frequency of blinking. Indeed, the frequency of blinking is a fairly accurate way of determining the driver's condition. Yandex.Taxi notes: "On average, a person blinks once every four seconds, if a person is tired or if he is drunk, the blinking frequency becomes significantly higher." [3].

The solution to this problem is possible, because now all taxi drivers possess smartphones and all smartphones have cameras. Moreover, for a long time there are algorithms that very accurately allow you to notice the eyes, find them in the picture.

However, despite the foregoing, according to the financial statements, the Yandex.Taxi service shows a loss, which is also due to fairly low prices, which often the drivers do not want to work. Adjusted loss for 2016 amounted is 2.1 billion rubles, for the first quarter of 2017 it is 1.25 billion rubles.

### 2.2. Uber

Uber Technologies Inc. is American international company from San Francisco, who created the same mobile application for searching, calling and paying taxi or private drivers [4]. As of August 2017, service through the application is available in more than six hundred cities worldwide [5]. With the help of the Uber application, the customer reserves the car with the driver and tracks its movement to the specified point [6], payment is made using the bank card data or in cash. In most cases, drivers use their own cars, as well as car taxis or partners. In most countries 80% of the payment goes to the driver, 20% of payment is transferred to Uber.

Also worth noting the process of merging Uber and Yandex.Taxi. On July 13, 2017, Yandex and Uber signed an agreement on combining business and services on online taxi order in Russia, Azerbaijan, Armenia, Belarus, Georgia and Kazakhstan. In addition, Yandex will invest \$ 100 million in the new structure, and Uber will invest \$ 225 million. After this, the new company will be valued at

3.7 billion dollars. It will belong to Yandex at 59.3 percent, Uber will have 36.6 percent, and 4.1 percent to employees of the new company.

### 2.3. *Gettaxi.ru*

This service is also available for any device running on Android.

Gettaxi.ru's search of addresses can be dragged on for a few minutes, which is inconvenient for those who are in a hurry. This is the moment of not the most effective work of the service.

In the rest the work of the service is excellent. The design is beautiful, the filling is thought through to the smallest detail. Data about the car is very detailed. The number of the taxi, the stamp, the color of the car, the photograph and the full name of the driver are determined.

The delivery of the machine is operational, it takes 6-9 minutes on the territory of Moscow and St. Petersburg, you can also track its movement on an electronic map. The peculiarity of the service is the calculation of the time for a return trip. That is, you can calculate the total time on the road.

Service and payment in this case is a separate conversation. In addition, that every machine «Gettaxi.ru» is a good not Russian car, which differs in an ideal condition, free Wi-Fi in the salon and sockets for charging phones are available for all passengers!

All drivers of this service have excellent knowledge of the geography of their work, many of them do not even use the navigator to track the route. In "Gettaxi.ru" a mandatory dresscoat for drivers is a suit and a tie.

### 2.4. *TaxiOnline.ru*

The TaxiOnline.ru service was created by Tango Telecom LLC, a member of the 2T holding company. The idea of creating a service was born in early 2010, and practical implementation began in 2011 as part of a project to organize and bring to market a single taxi order service in various cities of Russia. Since 2008, one of the activities of the holding company is the production of modern software for taxi services, as well as the direct organization of such services in the regions of the Russian Federation. Currently, service drivers carry customers in 20 cities, performing more than 4,000 orders every day. The need to create a service similar to the TaxiOnline.ru was a logical continuation of the concept of the level of service provided to clients, and, undoubtedly, the requirement of time. The huge accumulated experience and wide geography of activity allow TaxiOnline.ru's specialists to create a truly convenient product.

TaxiOnline.ru is an easy way to go by taxi, ordering a car using the Internet. And you can make an order both from a computer and from a simple mobile phone connected to the Internet. Those who use taxi services often can download the mobile application named TaxiOnline and literally carry a taxi in your pocket, ordering the car at the touch of a button of your mobile phone.

The mission of TaxiOnline.ru is the maximum consolidation of resources to provide the most convenient taxi service.

In its work, the TaxiOnline.ru service is guided by the principles of professionalism, responsibility and efficiency. Professionalism means that the service staff thoroughly know their business and have a successful experience of doing it in practice. Following the second principle, TaxiOnline.ru is responsible for the quality and effectiveness of its work, the fulfillment of its obligations and the wisdom of the decisions taken. Achieving the task at optimal costs is an integral part of the service and indicates its effectiveness.

The purpose of TaxiOnline.ru service is to provide any interested person with the opportunity to use taxi services as quickly and comfortably as possible. This means that you can call a taxi in the most convenient way and get the service as soon as possible. The matter is that the order through TaxiOnline.ru will be processed first of all in comparison with telephone applications. And the search for the car will be performed not by one, but by all taxi services of any city registered in the service, in which the TaxiOnline.ru service works.

### 2.5. Summary

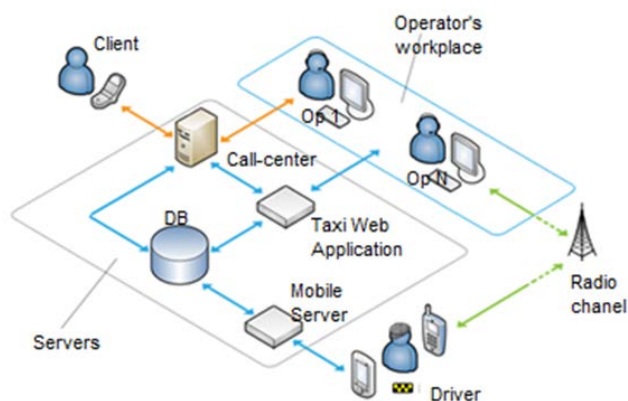
Thus, the taxi order service currently combines the efforts of programmers, marketers, web designers, taxi service managers, economists, accountants, seo-specialists, lawyers and simply managers who perform not the most creative, but from this no less important work. However, the development of such areas as Big Data, requires the selection of new specialists to achieve maximum efficiency. Among such specialists, one can already single out programmers-analysts, or engineers of Data-Mining. A complete and adequate analysis of the data is much more understandable if it is possible to select a mathematical description for these data.

### 3. Taxi service statistics collection

However, first, let us consider the task of obtaining data in the taxi order service. To store information about calls, a database server is used, for example, MySQL [7]. However, in our article we will not consider the data of telephony due to the fact that this method of ordering the taxi is gradually losing its popularity. Tariffs are set up using a separate module called Tarifficator, which is programmed to use it on the web.

Thus, it is advisable to use virtualization methods to separate different servers, including a telephony server, a telephony database server and a web server. In addition, an application server is needed, through which information is transferred from the contact center to the drivers. This is provided by a special program for taxis. And here it is recommended to use one more database server to store order information.

Fig. 1 shows the complete architecture of the considered service.



**Figure 1.** Detailed diagram of the taxi order service.

The application for the Taxi-program can have versions running on smartphones with Android and iOS.

The order database is updated, when a particular driver take the order. For example, information about driver's car, time of order picking, etc. is registered. It can be used to inform the client about the assigned car.

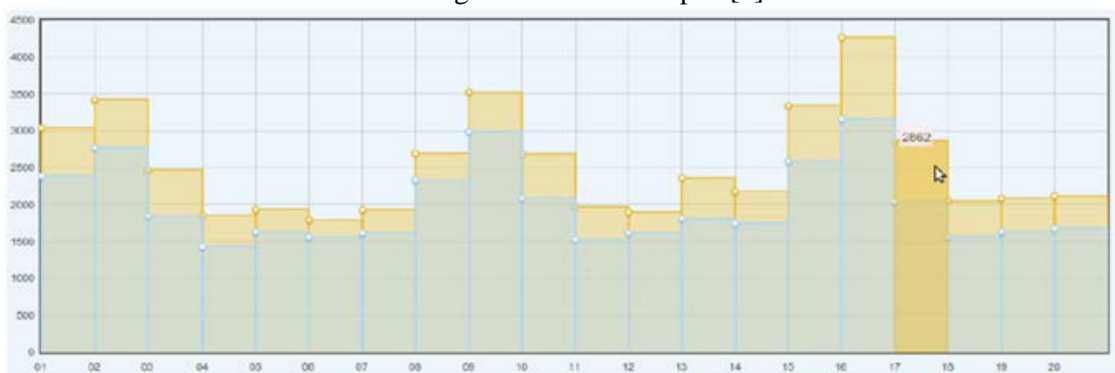
Using the programming languages PHP and JavaScript, a web-based interface for analyzing order data was developed [8]. This interface can be conditionally called Tarifficator and allows you to obtain various statistical characteristics, as well as implement database modifications that are aimed at changing prices. In addition, here you can view statistics on orders in real time.

Another revision, written in PHP and JavaScript, is the calculator of complex routes. This development allows you to calculate the cost of an order in the case when the driver passes several points in sequence, for example, goes first from point A to point B, and then from point B to point C.

Statistics are collected using database servers, but the presentation of information in a convenient form is obtained using the Tarifficator, which allows you to display statistics either in a text document or in an excel format document.

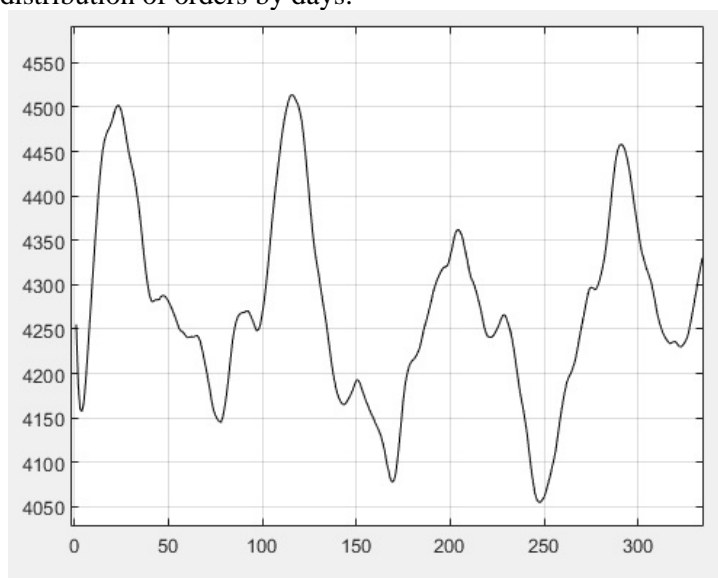
A module for data analysis has been improved for the convenience of operating with different statistics in PHP and JavaScript languages. This module, in addition to constructing various statistical graphs using the flot.js library. Fig. 2 shows the statistics of orders by hours in the day. It also allows you to make changes in the database related to setting prices. In addition, using the Tarifficator you can view statistics on orders in real time.

To work with statistics, a module also provides features that allows real data to be adjusted using statistical random sequence models. For example, the identification of parameters for the distribution of orders by days is realized, a common autoregressive model is calculated. We also made a forecast for the following days. A doubly stochastic autoregressive models allow us describe the nonstationarity in the distribution of data such as surges on weekends and downturns on Monday. For the latter algorithms for parameter identification based on a combination of pseudo-gradient search algorithms and nonlinear recurrent filtering have been developed [9].



**Figure 2.** Example of presenting statistics in the Tarifficator program.

Fig. 3 shows the distribution of orders by days.



**Figure 3.** Distribution of orders by day (smoothed graph)

Analysis of the curve in Fig. 3 shows that the distribution of orders has a quasi-periodic nature. There are several peaks that characterize orders improvement for holidays.

**4. Autoregressions with multiple roots of characteristic equations**

Often the real signals that can appear at the output of various devices, for example, voltage sensors, have a smooth shape. The use of autoregressions to describe such signals requires that they be of

sufficiently high order, which in turn leads to an increase in computational costs. One way out of this situation is to use autoregressions with multiple roots of the characteristic equations [10-12]. The application of such models in conjunction with the proposed doubly stochastic sequences [13, 14] allows to adequately describe a sufficiently wide range of real signals.

Consider the following characteristic equation

$$z^m - \rho_1 z^{m-1} - \rho_2 z^{m-2} - \dots - \rho_m = 0 \tag{1}$$

On the basis of equation (1), an autoregressive model of the order  $m$  can be written as follows

$$x_i = \rho_1 x_{i-1} + \rho_2 x_{i-2} + \dots + \rho_m x_{i-m} + \xi_i, \tag{2}$$

where  $\xi_i$  is random additive with zero mathematical expectation and variance  $\sigma_\xi^2$ , which depends on the correlation coefficients  $\rho_1, \rho_2, \dots, \rho_m$ .

The characteristic equation (1) with the root  $z = \rho$  of  $m$ -multiplicity has the form  $(z - \rho)^m = 0$ , and the autoregressive equation (2) takes the following form

$$(1 - \rho z^{-1})^m x_i = \xi_i, \tag{3}$$

where  $z^{-k} x_i = x_{i-k}$ .

Using equation (3), we find in explicit form expressions for models of different multiplicity:

- $x_i = \rho x_{i-1} + \xi_i, m = 1;$
- $x_i = 2\rho x_{i-1} - \rho^2 x_{i-2} + \xi_i, m = 2;$
- $x_i = 3\rho x_{i-1} - 3\rho^2 x_{i-2} + \rho^3 x_{i-3} + \xi_i, m = 3;$
- $x_i = 4\rho x_{i-1} - 6\rho^2 x_{i-2} + 4\rho^3 x_{i-3} - \rho^4 x_{i-4} + \xi_i, m = 4.$

Consider the simulation of such random sequences for the different cases. Fig. 4 shows the first case when the model's correlation parameters provide the same correlation interval. Fig. 5 shows the second case when we use the same correlation coefficient in the modeling.

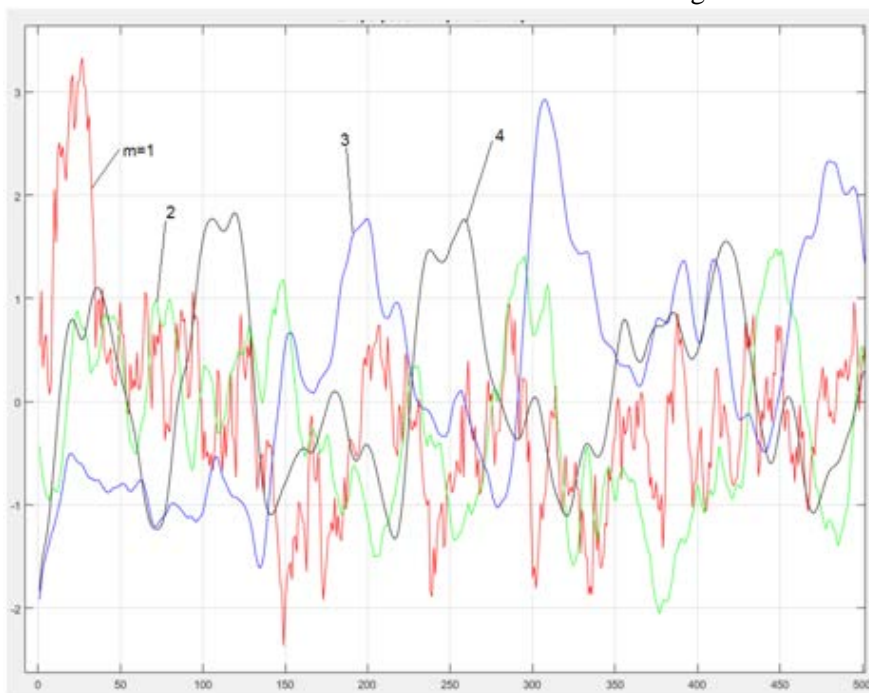
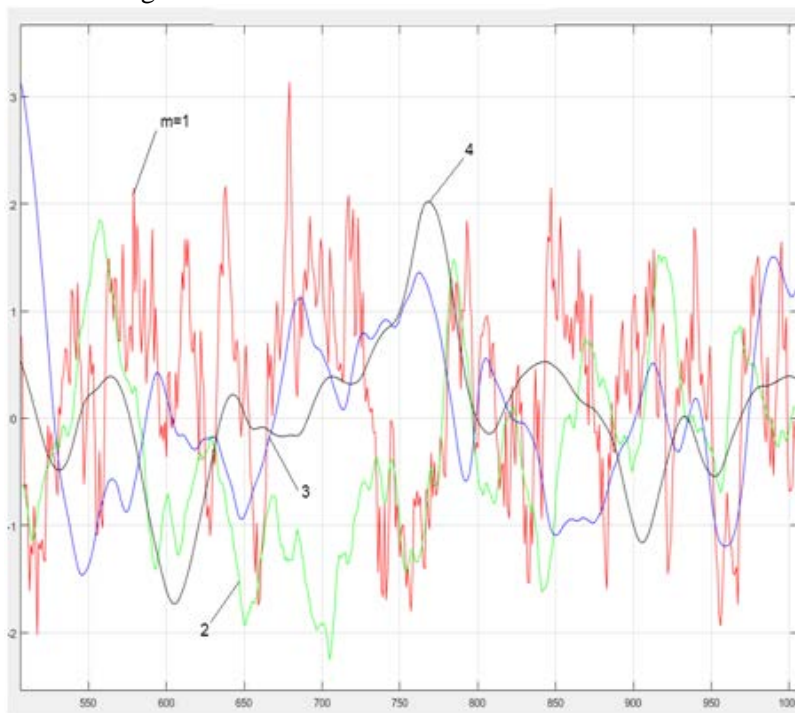


Figure 4. Autoregressions with the same correlation interval.

In Fig. 4 «m=1»-line indicates autoregression of the first order, «m=2»-line indicates autoregression of second order, «m=3»-line indicates autoregression of third order, and «m=4»-line indicates autoregression of fourth order. A similar correspondence of the «m=X»-line of the simulated sequences is preserved for Fig. 5.



**Figure 5.** Autoregression with the same coefficient  $\rho$ .

Analysis of the obtained random processes allows us to conclude that smooth characteristics can be obtained with the help of autoregressions of high orders. However, for a quasiperiodic order data sequence, a second-order model is intuitively better suited.

### 5. Approximation of taxi order service data and its forecasting

An important factor is the optimal forecasting of the taxi order service data. To approximate the data, we use a linear function, a polynomial function, and also an autoregressive model with multiple roots of the second order.

Fig. 6 shows the results of forecasting using various algorithms.

The analysis shows that the use of autoregressions with multiple roots ensures optimal forecast among the algorithms used. In this case, effective algorithms for detecting anomalies can be used for this model, which in the long term will allow finding various deviations in taxi order distributions. Such deviations may be a critically big number of rejected orders or a sharp decrease in the number of orders, which may indicate a malfunction in the software.

For the presented figure, it is obvious that approximation with prediction should be performed using quasi-periodic or generally random rather than polynomial functions. However polynoms can provide the good approximation it can't make a good forecast. And the linear approximation also can't provide a good forecast.

In this case, the reliability of approximation on the basis of a polynomial of the 6th degree is close to 1, while for linear approximation it tends to 0.

The solid thick line in Fig. 6 shows the actual data, solid fine line shows approximation and prediction by a linear function, the dashed line shows approximation and prediction by a polynomial

of the 6th degree, and dotted line shows the prediction by using of second order autoregression with multiple roots of characteristic equations.



**Figure 6.** Forecasting taxi service data.

## 6. Conclusion

A brief overview of the taxi order services in Russia has been completed. The forecasting of the order distribution sequence is investigated. It is shown that the use of autoregressions with multiple roots for the specific statistics collected led to a gain in predicting future values in comparison with polynomial and linear prediction. In this case, the gain was about 100-200% compared with linear approximation and even more compared with polynomial approximation.

## 7. Acknowledgment

The study was supported by RFBR, project № 17-01-00179.

## 8. References

- [1] Taxi for two. Yandex and Uber combine services for online booking of travels / RBC, NEWSPAPER. – 2017. – Vol. 123(2620). (in Russian).
- [2] [Electronic resource]. – Access mode: <https://rb.ru/news/ya-tochki-posadki/> (10.11.2017). (in Russian).
- [3] [Electronic resource]. – Access mode: <https://hitech.vesti.ru/article/681864/> (10.11.2017). (in Russian).
- [4] Goode, L. Worth It? An App to Get a Cab / L. Goode // The Wall Street Journal, 17.06.2011.
- [5] How Uber helps cities become better. [Electronic resource]. – Access mode: [www.uber.com](http://www.uber.com). – at the bottom of the page. (10.11.2017).
- [6] Rao, L. Uber Brings Its Disruptive Car Service To Chicago. TechCrunch / L. Rao // AOL. – [Electronic resource]. – Access mode: <https://techcrunch.com/2011/09/22/uber-brings-its-disruptive-car-service-to-chicago/>. (10.11.2017).
- [7] Azanov, P.T. Development of software system for analysis and optimization of taxi services efficiency by statistical modeling methods / P.T. Azanov, A.N. Danilov, N.A. Andriyanov // CEUR Workshop Proceedings. – 2017. – Volume 1904. – P. 232-238. DOI: 10.18287/1613-0073-2017-1904-232-238
- [8] Vasiliev, K.K. Estimation of the parameters of doubly stochastic random fields / K.K. Vasiliev, V.E. Dementiev, N.A. Andriyanov // Radio Engineering. – 2014. – Vol. 7. – P. 103-106. (in Russian).



- [9] Vasiliev, K.K. Efficiency of filtration of an autoregressive model with multiple roots of characteristic equations / K.K. Vasiliev, Yu.N. Gavrilina, N.A. Andriyanov // Modern problems of design, production and operation of radio engineering systems. – 2016. – Vol. 1(10). – P. 130-133. (in Russian).
- [10] Vasiliev, K.K. Analysis of autoregressions with multiple roots of characteristic equations / K.K. Vasiliev, N.A. Andriyanov // Radio Engineering. – 2017. – Vol. 6. – P. 13-17. (in Russian).
- [11] Vasiliev, K.K. Mixed model of images based on models with multiple roots and Habibi / K.K. Vasiliev, N.A. Andriyanov, V.E. Dementiev // Radioelectronic engineering. – 2015. – Vol. 2(8). – P. 119-122. (in Russian).
- [12] Andriyanov, N.A. A Brief Review of the Methods of Processing Doubly Stochastic Models / N.A. Andriyanov // Science of the XXI Century: Discoveries, Innovations, Technologies. Collection of proceedings on the materials of the International Scientific and Practical Conference. – 2016. – Vol. 3. – P. 18-20. (in Russian).
- [13] Vasiliev, K.K. Wiener filter for doubly stochastic random processes / K.K. Vasiliev, V.E. Dementyev, N.A. Andriyanov // Modern problems of design, production and operation of radio engineering systems. – 2016. – Vol. 1(10). – P. 119-122. (in Russian).