VIII Международная конференция и молодёжная школа «Информационные технологии и нанотехнологии» (ИТНТ-2022)
Том 1. Компьютерная оптика и нанофотоника

Dynamics of two-photon Tavis-Cummings model with Kerr media

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***Abstract*—We investigated the entanglement dynamics between two qubits not-resonantly interacting with a one-mode field of lossless cavity with Kerr media through degenerate two-photon transitions. We obtained the exact solution for time-dependent density matrix and calculated on its basis the qubit-qubit entanglement parameter – concurrence. The results show that Kerr media and detuning greatly affected the entanglement behavior. More interestingly, that for initial entangled qubits states Kerr media and detuning avoids the entanglement sudden death effect.**

Keywords— qubits, cavity, two-photon transitions, detuning, Kerr media, entanglement, concurrence.

1. INTRODUCTION

Studies of qubit-qubit entanglement in many-particle systems nowadays are still of vital importance because non-local correaltions or entanglement is at the core of quantum technologies, quatum computing and quantum communications. The entanglement can be observed in various physical systems such as neutral atoms, superconducting circuits, spin systems, trapped ions, quantum dots, Bose–Einstein condensates and so on [1]. Superconducting (SC) circuits are eventuality good prospects to execute quantum qubits, and provide a convenient controlling on qubits in diverse domains of quantum calculations and information communication [1]. The superconducting qubit interacted with a quantized fields of coplanar 1D resonators is considered as one of the excellent examples which can be theoretically characterized through the Jaynes Cummings model [2]. It is well known that JCM is the simplest physical model that describes the interaction of a natural or artificial two-level atom (qubit) with a single-mode cavity field, and was used to understand a wide variety of phenomena in quantum optics and informatics. During last decades the numerous generalizations of the JCM have been investigated. A large number of authors (see Refs in [3]) studied the dynamics of two-level atoms interacting with field of one-mode cavity with a Kerr-like medium. In practice, Kerr nonlinearities in atomic systems χ are, however, often small in comparison to photon loss rate κ, making the observation of these non-classical states of light difficult. As an alternative approach, the strong photon-photon interaction in superconducting quantum circuits chains in a cavity with $\chi / \kappa :$ 30 can readily be realized experimentally [4].

As well known, the interactions between qubits degrade the quality of the entanglement. But it can also induce entanglement. In particular, Kim et al. [5] investigated the atom-atom entanglement in the system of two identical two-level atoms with one--photon transitions induced by a single-mode thermal field. They have shown that a chaotic field with minimal information can entangle atoms prepared initially in a separable state. The entanglement between two identical two-level atoms through a nonlinear two-photon interaction with one-mode thermal field has been studied by Zhou et al. [6]. They have shown that atom-atom entanglement induced by a nonlinear interaction is larger than that induced by a linear interaction. Resently, we considered the dynamics of two superconducting qubits interacting with one-mode electromagnetic field of an lossless cavity with a Kerr medium via one-photon transition and showed that the Kerr nonlinearity can enhance the maximum degree of entanglement induced by the Fock and thermal cavity field [3].

It is of interest to investigate the entanglement between two identical qubits interacting with a Fock, coherent or thermal cavity field through two-photon interaction. In this paper, we study the influence of a detuning and Kerr nonlinearity on atomic entanglement for system consisting of two identical qubits nont-resonantly interacting with one-mode cavity field by means of two-photon transitions.

2. MODEL

We consider two identical qubits not-resonantly interacting with one-mode cavity field. We suppose that qubit--field coupling constants are equal. The suppose also that there is an additional Kerr medium in the cavity. The interaction Hamiltonian for considered system under rotating wave approximation can be written as

$$H = \sum_{i=1} \hbar \Delta \sigma_i^z + \sum_{i=1}^{2} \hbar g (\sigma_i^+ a^2 + \sigma_i^- a^{\dagger 2}) + \hbar X \left(a^\dagger \right)^2 a^2 , \qquad (1)$$

where σ_i^z is the quasi-spin inversion operator, $\sigma_i^+ = |+\rangle_{ii}\langle -|$ and $\sigma_i^- = |-\rangle_{ii}\langle +|$ are the transition operators between the excited $|+\rangle_i$ and the ground $|-\rangle_i$ states in the i -th qubit ($i = 1,2$), $a^\dagger$ and a are the creation and the annihilation operators of photons of the cavity mode, g is the coupling constant between qubits and the cavity mode, X is the Kerr nonlinearity and Δ is the detuning between the qubit transition frequency ω_0 and twice cavity mode frequency ω : $\Delta = \omega_0 - 2\omega$.

The initial qubits state is assumed to be separable

$$|\Psi(0)\rangle_Q = |+,+\rangle, \ |\Psi(0)\rangle_Q = |+,-\rangle, \ |\Psi(0)\rangle_Q = |-,-\rangle, \qquad (2)$$

or entangled

$$|\Psi(0)\rangle_Q = \cos\theta\,|+,-\rangle + \sin\theta\,|-,+\rangle. \qquad (3)$$

Here the parameter $0 \le \theta \le \pi / 2$ defines the amount of initial atomic entanglement. Value $\theta = \pi / 4$ corresponds to maximum degree of atomic entanglement.

The initial cavity mode state are assumed to be a Fock with initial wave-function

$$|\Psi(0)\rangle_F = |n\rangle \ (n = 0, 1, 2...) , \quad (4)$$

coherent with initial wave-function

$$|\Psi(0)\rangle_F = \sum_n C_n |n\rangle, \quad (5)$$

where $C_n = e^{-\bar{n}/2}\bar{n}^n / \sqrt{n!}$, $\bar{n}$ is the coherent cavity mean photon number or thermal one-mode state with density matrix

$$\rho_F(0) = \sum_n p_n |n\rangle\langle n|, \quad (6)$$

where weight coefficients are $p_n = \bar{n}^n / (1+\bar{n})^{n+1}$,

$\bar{n}$ is the thermal cavity mean photon number $\bar{n} = \left(\exp\left[\hbar\omega / k_B T\right] - 1\right)^{-1}$, k_B is the Boltzmann constant and T is the equilibrium cavity temperature.

3. Concurrence calculations

Using the dressed states of the model with Hamiltonian (1), we derived the exact solution of Liouville equation for whole statistical operator $\rho_{Q+F}(t)$. By averaging the density matrix over the radiation parameters, we get two-qubits statistical operator needed to calculate the qubit-qubit entanglement criterion: $\rho_Q(t) = Tr_F \rho_{Q+F}(t)$.

We used the Wootter's criterion or concurrence to investigate the qubit-qubit time-dependent behavior [7]. Wootters proposed the quantitative measure of entanglement of two qubits. It is based on the application of the so-called "spin-flip" transformation, or the "inverted spins" matrix, which is defined as follows $\rho_Q = (\sigma_y \otimes \sigma_y)\rho_Q^*(\sigma_y \otimes \sigma_y)$,

where ρ_Q^* is the complex conjugation $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is the y -component of Pauli matrix. The concurrence can be define in the form

$$\mathrm{C} = \max\left\{\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, \ \ 0\right\}, \quad (7)$$

where λ_m are the eigenvalues of the $\rho_Q\tilde{\rho}_Q$, in descending order. Using the exact formulae for qubit-qubit density matrix $\rho_Q(t)$, we calculated the time dependece of concurrence (7) for all considered initial states of qubits and cavity fields.

4. Results and Discussion

Numerical calculations of concurrence for separable initial qubits (2) and different initial states of cavity field (4)-(6) showed that detuning and Kerr nonlinearity can greatly enhance the maximum degree of qubit-qubit entanglement. It is also shown that for initial qubits state $|\Psi(0)\rangle_Q = |+,+\rangle$ the qubit-qubit entanglement induced by the cavity field arises only in the presence of detuning or Kerr nonlinearity. For entangled initial qubits state (3) and thermal cavity field states (6) the detuning and Kerr media lead to the disappearance of the entanglement sudden death effect. Fig. 1 shows the time dependence of concurrence for entangled initial qubits state (3) with $\theta = \pi/4$ and thermal field with $\bar{n} = 3$. The solid line correponds to the model without detuning and Kerr nonlinearity and dashed line corresponds to model with $\Delta = \mathrm{X} = 3g$.

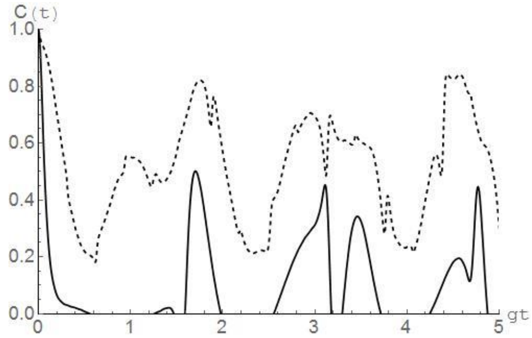

Fig. 1. Concurrence vs scaled time gt for entangled qubits state (3) with $\theta = \pi/4$ and thermal cavity field with $\bar{n} = 3$. Parameters $\Delta = \mathrm{X} = 0$ (solid) and $\Delta = \mathrm{X} = 3g$ (dashed)

5. Conclusion

We studied dynamics of two identical qubits not-resonantly interacting with one-mode cavity field through the degenerate two-photon transitions taking into account the kerr nonlinearity. The field is assumed to be in the Fock, coherent or thermal initial state and qubits are in different separable or Bell's type entangled initial state. We obtained the exact solution for the model under considerations. On its basis we calculated the qubit-qubit entanglement criteria – concurrence. The results showed that for separable initial qubits state the detuning and Kerr nonlinearity can greatly enhance the maximum degree of qubit-qubit entanglement. We also derived that for initially exited two qubits the qubit-qubit entanglement induced by the cavity field arises only in the presence of detuning or Kerr nonlinearity. For entangled initial qubits states and non-zero values of detuning and Kerr nonlinearity smoothes negativity amplitude fluctuations induced by the cavity field. In this case the effect of sudden death of entanglement dies away for thermal initial cavity state. These results may be useful for quantum information processing based on the entanglement.

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