Direct and inverse problems to study the process of ion solutions filtering in porous medium

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Abstract. Solution of actual problem associated with technological process of filtering and dehydrating liquid solutions from fine-dispersed particles is discussed in the paper. Technological process is carried out in the dehydration and purification of chemical solutions, drinking water, pharmaceuticals, liquid fuels, products for public use, etc. A mathematical model has been developed to study the process, to determine the basic parameters of the object and the operating modes of the filtering aggregates to make management decisions; this model may take into account different operating modes of the filtering aggregate and physical-chemical properties of solutions. In the paper it is noted that when investigating the filtering process, it is rather difficult to determine the main parameters of the object under consideration and their ranges of changes to control the operating objects. Collecting data takes a lot of time; to conduct a series of experiments in laboratory conditions takes a lot of labor power and time; it is difficult to find the relationship between the parameters of the filter and technological process based on a limited experimental sampling. Urgent problems of determining the basic parameters and their ranges of changes leading to a decrease in the loss of valuable raw materials, an increase in filters performance, an improvement in the quality of the resulting product, etc. are solved in the paper. Based on the analysis of the conducted numerical experiments, conclusions are drawn that serve as the basis for making appropriate management decisions.

1. Introduction

The processes of separation of individual components or phases of complex mixtures are an integral part of numerous industrial fields. The process of separation and filtering of multi-component fluids and ionized solutions is an important stage, which plays an important role in manufacturing of food, oil and fat, pharmaceutical, ore-mining and processing, engineering, agro-industrial products.

Providing the population with drinking water, purification of water used in various production infrastructures and containing harmful impurities (especially heavy metals), refining and processing of oil, mineral processing, production of rare elements (gold, tungsten, uranium, silver, platinum and others) and many other aspects can serve as an examples of the use of separation technology and filtering. These processes differ from each other not only by the composition of separated phases or components but also by the methods and technological equipment used to achieve the goal. The desired result may be achieved using a single stage or by a multi-step process (for example, separation of natural gas, purification of drinking water).

Depending on physical and mechanical properties of the components, as well as on the volume of the processed raw material and liquid solutions, various methods and devices have been developed for filtering and separating individual components of the mixtures.

The improvement of existing and the development of new efficient methods and devices for the separation and filtering of complex mixtures, based on the results of a full-scale experiment, require substantial investments, and still are not always feasible. In this regard, it is advisable to refer to the methods of mathematical modeling and the capabilities of modern information technologies. This requires a thorough analysis of technological processes and mathematical models corresponding to them. The development of reliable mathematical model and the use of effective numerical methods for solving practical problems often allow not only to analyze the technological process, but also to successfully manage it; and moreover, to offer a new, effective way of organizing the technological process and determine its optimal operating parameters.

As follows from laboratory experiments and analyses of technological processes of filtering mixtures (TPFM) the initial stage of the process is accompanied by a certain period of time during which the formation of a sediment layer occur above the surface of filtering channels of the unit. During this period of TPHM solid particles contained in the solution inevitably skip. After the formation of stable filtering layers over the channels, a direct filtration period begins with the formation of a pure filtrate at the outlet of the filtering unit. The purity of the output solution during this period will depend firstly, on the stability of the filtering arches in relation to the pressure effect of suspension being filtered; secondly, on the stability of the filtering arches in relation to the possible vibrations of the filtering partition.

All filtering stages depend on the pore size in the filtering partition: first, the time of formation of the arches, and, consequently, the value of the initial skip of solid particles of the suspension; secondly, the strength of the formed arches; and, finally, the quality of the subsequent steady-state filtering process. At the stage of steady-state filtration process, the quality of the filtrate (its purity) depends on the porosity of the layer formed from the retained particles. Particles that are not retained by a sediment layer, managed to penetrate through the filtering arches have the ability to adhere to the inner surfaces of the filtering channels due to adhesion forces and reduce the flow cross section of the filtering partition.

This process progresses over time if a porous partition is used. Based on the above, it follows that when filtering a suspension, many parameters of different weights contribute to technological process. Deviation of these parameters from the normal ones leads to qualitative and quantitative changes in considered technological process as a whole. Therefore, the definition of the main parameters and their ranges of change is one of the main issues in the theory of research and control of technological processes.

It should be noted that the study of the process of filtering the suspension and the definition of the main parameters of the process under consideration and their ranges of changes to control them at operating facilities is rather difficult: first, data collection on the process takes a lot of time; secondly, it is necessary to conduct a series of experiments in the laboratory that take a lot of labor power and time; thirdly, it is difficult to find the relationship between the parameters of the filter and the technological process for a limited experimental sampling.

Considering the above, we can say that with the implementation of TPFM the study addresses an urgent tasks: the choice of technologies and devices and their modes of operation, the definition of the main parameters and their ranges of changes leading to a reduction in the loss of valuable raw material, an increase in filter performance, an improvement in the quality of the resulting product. etc.

Therefore, to achieve the best possible result in cleaning the final product from unwanted impurities, TPFM should be organized as a technological cycle with optimal parameters, including the characteristics of separators and filters and their operating modes.

M. Chraibi et al. [1] have considered a mathematical model of the process of multicomponent medium filtering. In the proposed model, the authors have replaced the relative phase permeability of the gas phase with a new expression that takes into account the effect of viscosity, density, and the capillary effect of the mixture.

In [2] the authors have proposed a mathematical model for the process of development of oil and gas fields, which takes into account the probability distribution of the parameters of the process under study.

In [3] the study is focused on the derivation of a numerical model of gas filtration in porous inhomogeneous media based on the finite element method and using fractional time derivatives. The authors consider the Caputo and Riemann-Liouville fractional derivatives. A numerical analysis using experimental input data is carried out.

The authors in [4] have proposed a numerical method for solving the inverse problem of determining well flow rates using specified bottom hole pressures for a multidimensional model of flow of a weakly compressible fluid in elastically deformable porous medium. The approximation in spatial variables was performed by the authors using the finite element method, which allowed the use of unstructured grids with inspissation (condensation) in the vicinity of well location. Time discretization is constructed using the implicit difference approximation. The authors present the results of numerical solutions for two- and three-dimensional statements of this problem.

In [5], an equation is proposed for describing the process of separating industrial suspensions at the initial period of time to study and establish experimental regularities of the processes of separation of suspensions and dehydration of resulting sediment, as well as identifying the main factors affecting the performance and efficiency of industrial filters and filter units. The empirical dependence of the Kozeni-Karman coefficient on the average particle diameter of solid phase of the divided concentrates and the empirical equation for the kinetics of dehydration of sediments of the divided suspensions were obtained, as well as the dependence for determining the air flow rate on the vacuum filters during the process of dehydration of sediments on the filter partitions taking into account the properties of the processed suspension and its solid phase.

In [6] the theoretical foundations of methods for calculating hydro-mechanical processes in filtering and centrifuging suspensions with Newtonian and non-Newtonian rheology for a wide class of separation equipment have been developed. To achieve this goal, the problem associated with the calculation of the flow of heterogeneous media with a solid phase is solved.

Mathematical models have been developed and numerical calculations have been made of the process of filtering heterogeneous media with (and without) forming sediment at its thin-layer flow over permeable surfaces of arbitrary shape, taking into account the initial section, and of the process of filtering the suspension on rotating permeable surfaces of arbitrary shape.

The main laws governing the motion of dispersed inclusions are investigated in the flow of a heterogeneous medium with phase separation under the action of mass forces, as well as taking into account the nonlinearity of its rheological state. In a multi-criteria statement the task of optimizing the operation of a drum vacuum filter with a descending working belt is solved.

In [7] a mathematical description of the filtering processes of suspensions formed during the machining of gallium arsenide plates through flat metal porous partitions is given. Formulas for mathematical calculation of suspension filtration through porous metal partitions are given. To develop the methods for calculating manufacturing plants, an analysis of the dependencies given in literature and obtained as a result of mathematical processing of experimental data is carried out.

In dissertation by Dyachenko E. N. [8] the problems associated with the nonstationary process of filtering a suspension through porous media are solved. Static and dynamic models of porous media formation have been developed. Scientific results are obtained on the behavior of particles and structures at the micro level; previously they were unavailable due to insufficient computational power of computers and using the classical approaches to mechanics of continua. Based on the method of discrete elements, a numerical model of suspension filtration through filled-in filters has been developed. Results of filtering suspension simulation have been obtained; they show the ways to increase filters performance and optimize their operation.

In [9] a mathematical model of the filtration process in zinc production has been developed, based on the equations of material balance for the flow rates of process media and the concentrations of individual components. The model allows one to determine the costs and quantities of the resulting products of the process, to assess its current state and predict future states.
In [10], the dead mode of suspension filtering in the channel is considered. A numerical model of the process of non-stationary filtration of suspensions has been developed, which makes it possible to optimize the operating modes of filtration devices and reduce the costs of research and experimental design works on creation of industrial installations. A model of extrusion was accepted for the flow of suspension in the channel. On the inner surface of the channel there is a filtering partition through which the filtrate passes due to trans-membrane pressure. The filtrate flow depends on the hydraulic resistance of the filtering partition and the sediment layer on its surface.
P.J.Monteiro, Ch.H.Rycroft, and G.I.Barenblatt [11] have developed a mathematical model of fluid filtration in nano-porous rocks. In deriving the process model, it is assumed that filtration layer consists of two components: a fractured porous medium and specific organic inclusions consisting of kerogen; the model is based on the hypothesis that the permeability of inclusions substantially depends on the pressure gradient.
F.Boyer et al. [12] describe some aspects of modeling a diffuse flow of incompressible media consisting of three immiscible components, without phase transformations; the process of simulation of three-phase flows is supplemented by incorporating the Cahn-Hillard and Navier-Stokes equations. In this case, the surface tension is taken into account through capillary volume forces. Discretization of equations is performed by the time and space variables. The attention of the authors is drawn to the fact that most of the basic properties of original model, such as volume conservation and energy estimation, should be maintained at a discrete level. To obtain a solution in case of very thin moving inner layers, an adaptive refinement method is used with the limitation of total number of nodes in a grid of discrete model.
A.Raeini, M.Blunt and B.Bijeljic [13] have presented a stable numerical scheme to model multiphase flow in porous media, at characteristic size of the flow region from micron to millimeters. The numerical method has been developed for the effective modeling of multiphase flows in porous media with a complex nature of phase boundary motion and irregular solid boundaries. The Navier-Stokes equations are discretized using a finite volume approach, while the fluid volume method is used to find the interface locations. Capillary forces are calculated using the model of semi-permeable surface force, when the transition region for capillary pressure is effectively limited to a single mesh point.
M.A.Trapeznikova, N.G.Churbanova and A.A. Lyupa [14] have developed a mathematical model of the flow of weakly compressible fluid in a porous medium constructed by analogy with a quasi-gas-dynamic system of equations. The model is generalized for the case of three-phase fluid and supplemented by the energy conservation equation, which allows its use in modeling advanced thermal methods of oil production.
Based on above-stated, we can say that for a comprehensive study of the object, for determination of basic parameters and operating modes of filter units, it is necessary to create an easily implementable tool. From this point of view, the most appropriate method for solving the problem posed is mathematical modeling (MM) and computational experiment (VE) which may lead to appropriate solution.

2. Statement of the problem

Mathematical model of the filtering process in a dimensionless form has the form [15–17]:

$$\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial x} = -Eu\frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial^2 W}{\partial x^2} + \frac{W}{1-\theta}\frac{\partial \theta}{\partial t} - \frac{1}{Re_1}\frac{W}{(1-\theta)(1-\delta)^2}, \tag{1}$$

$$\frac{\partial\theta}{\partial t}+\frac{1}{m}\frac{\partial W\theta}{\partial x}+\frac{\partial\alpha}{\partial t}+(1-m_0)\frac{\partial\delta}{\partial t}=\frac{\mu_0\alpha_\tau}{H_0^2}\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial x}\right)^2, \tag{2}$$

$$\frac{\partial\delta}{\partial t}=\lambda_0\left(\theta-\gamma\delta\right), \quad \theta=\frac{\alpha}{1-\delta}, \tag{3}$$

$$\left.\begin{array}{l} W=1\theta=e^{\frac{-\lambda m H_0(1-m_0)x}{W_o}}=\varphi\left(x\right), \quad \delta=0 \ at \ t=0 \\ \dfrac{\partial W}{\partial x}=\dfrac{H_0^3}{Hk_0}\left[P_0-\dfrac{W}{(1-\delta)^2}\right], \quad \theta=1 \ at \ x=0 \\ \dfrac{\partial W}{\partial x}=0, \quad \dfrac{\partial\theta}{\partial x}=\dfrac{mH_0\lambda\left(1-m_0\right)}{W_0}\left(\gamma_0\delta-\theta\right), \ at \ x=1 \end{array}\right\} \tag{4}$$

where

$$\lambda_0=\lambda\alpha_\tau, \quad \gamma_0=\tfrac{\gamma\delta_0}{\theta_0}, \quad P_0=-\tfrac{k_0}{W_0H_0\mu}\tfrac{\partial P}{\partial x},$$

$$R_\ell=\tfrac{\rho H_0W_0}{\mu}, \quad R_{\ell_1}=\tfrac{\rho Hk_0W_0}{\mu H_0^2} \ -is\,the\,Reynolds\,number; \ P_0^*=\tfrac{\partial P}{\partial x}= \ const,$$

$Eu=\frac{P^*}{\rho W_0}--is\ the\ Euler\ number\ W$ is the filtration rate; θ is the volume concentration of suspended matter in flowing mixture; δ is the concentration of suspended sediment mass in filter pores; α is the concentration of suspended particles; ρ and μ are the density and viscosity of suspensions; P is the pressure in the column of the unit; $H,\ H_0$ are the height of the filter column of the unit and the filter thickness, respectively; k_0 is the permeability coefficient of the filter before operation; β is the effective constant of exchanging ions; χ is the longitudinal diffusion coefficient; λ is the kinetic coefficient; γ is the coefficient of dispersion; β is the effective rate constant of exchanging ions; m_0, m are the initial porosity and filter porosity.

Input parameters k_0, λ, γ are experimental parameters determined in laboratory observations. However, given the fact that mathematical model is simplified and includes a term derived from the processing of experimental data, the laboratory data of these parameters may not be suitable for solving the direct problem of this process. Therefore, it is advisable to determine the parameters of a mathematical model mathematically, i.e. from the solution of inverse problem.

Experimental parameter – the permeability coefficient is extremely important when calculating the laminar flow of a fluid through a porous medium. Therefore, the determination of the value of this parameter in laboratory conditions is given much attention to. Here, a particular attention is paid to its dependence on the size of soil particles (filter), on fluid viscosity and its temperature, etc. In [5] a technique for determining the permeability of a porous medium at gas and fluid filtering is given. It is found that the permeability determined by gas is significantly higher than the permeability of the same rock determined by fluid. In addition, it is found that the permeability of the medium depends on fluid properties. According to these authors, the difference in numerical values of the permeability of rock when filtering fluid and gas occurs as a result of physicochemical processes of fluid interaction with rock.

In laboratory measurements of the permeability coefficient, due to extreme complexity of the experiment, errors are often made (these can be the errors made by experimenter the errors of the instruments), which lead to an inaccurate value of the parameter. Consequently, the obtained numerical values of the direct problem with such an erroneous parameter can lead to a discrepancy of results of mathematical model with experimental data and production data. From this point of view, realization of additional calculations using other experimental data, i.e. solving the inverse problem is of particular interest.

If there are experimental data $W(t_j)$ at $\Delta P = const$ or $\Delta P(t_j)$ at $W = const$, then solving the inverse problem, the numerical values of parameters k_0, λ, γ could be determined. It should be noted that to obtain these experimental data is easier than to determine the parameters. For example, in the filter column operating at $W = const$ a device is installed which records the changes in pressure on a special paper diagram (Fergana Artificial Chemical Fiber Plant). These diagrams can be used to solve the inverse problem.

$$\frac{1}{k_0(1-\delta)^2} = \frac{1}{k_0^{(s-1)}\left(1-\delta^{(s-1)}\right)^2} - \frac{k_0^s - k_0^{(s-1)}}{\left(k_0^{(s-1)^2}\right)^2\left(1-\delta^{(s-1)}\right)^2} - \frac{2}{k_0^{(s-1)}\left((1-\delta)^{(s-1)}\right)^3}\times$$
$$\times\frac{\partial\delta^{(s-1)}}{\partial\gamma}\left(\lambda^{(s)}-\lambda^{(s-1)}\right) - \frac{2}{k_0^{(s-1)}\left(\left(1-\delta^{(s-1)}\right)\right)^3}\frac{\partial\delta^{(s-1)}}{\partial\gamma}\left(\gamma^{(s)}-\gamma^{(s-1)}\right),$$

$$\frac{\partial\delta}{\partial\lambda} = \theta_0 e^{-\left(Bx+t\gamma^{(s-1)}\lambda^{(s-1)}\right)}\left[\left(\frac{Bx\lambda^{(s-1)}}{2\gamma^{(s-1)}} - \frac{1+t}{\gamma^{(s-1)}}\right)\right]\cdot\left(Bx+t\gamma^{(s-1)}\lambda^{(s-1)}\right) - \frac{Bx}{\lambda\gamma^{(s-1)}},$$

$$\frac{\partial\delta^{(s-1)}}{\partial\lambda} = \lambda^{(s-1)}e^{-Bx\lambda^{(s-1)}}\left(\frac{Bx}{2}-\frac{1}{\lambda^{(s-1)}}\right)\cdot\frac{1}{\gamma^{(s-1)}}\cdot\left[\frac{1}{\gamma^{(s-1)}} - e^{-(\lambda\gamma)^{(s-1)}}\left(\lambda^{(s-1)}t+\frac{1}{\gamma^{(s-1)}}\right)\right],$$

where

$$B = \tfrac{1-m_0}{W_0}.$$

Equations (2) – (3) can serve as a mathematical model for the inverse problem. Integrating this equation in the range from 0 to H_0, we get:

$$P(t_j) - P_0 = \frac{\mu H_0 W_0}{Hk_0}\int\limits_0^{H_0}\frac{dx}{(1-\delta)^2} - \rho H_0 W_0 \left.\frac{\partial\theta}{\partial x}\right|_{x=H_0}. \tag{5}$$

Then the parameters can be determined from the minimum of the functional

$$R = \int\limits_0^t \left|P(\tau, k_0, \lambda, \gamma) - \tilde{P}(\tau)\right|^2 d\tau = \min. \tag{6}$$

Functions δ in (5) can be calculated from the solution of the Yu.M. Shekhtman's equation of balance and kinetics. To do this, expand the function $I_0\left(2\sqrt{\lambda^2\gamma Bxt}\right)$ entering the series, restricted to two members and perform the integration operation

$$\delta(x,t) = \theta_o\lambda e^{-\lambda Bx}\left[e^{-\lambda Bt}\left(\frac{Bx}{\partial\gamma} - \frac{1+f}{\lambda\gamma}\right) + \frac{1}{\lambda\gamma} - \frac{Bx}{\lambda\gamma}\right], \tag{7}$$

Then the parameters may be determined from the system of equations

$$\left.\begin{aligned} &\frac{\partial R}{\partial k_0} = 0, \\ &\frac{\partial R}{\partial \lambda} = 0, \\ &\frac{\partial R}{\partial \gamma} = 0 \end{aligned}\right\}. \tag{8}$$

3. Methods of problem solution

The system of equation (8) is nonlinear with respect to unknowns k_0, λ, γ; its solution is associated with certain difficulties. However, applying the quasi-linearization method [**?**] to (5), (7) it can be reduced to a linear system. Assuming that all conditions of the theorem are differentiated under the integral sign and substituting these values in (5) we get:

$$P\left(t\right) = f_1 + f_2 k_0 + f_3\lambda + f_4\gamma,$$

where

$$f_i = f_i\left(t, k_0^{s-1}\right),\ \lambda^{(s-1)},\ \gamma^{(s-1)}, \delta^{(s-1)} = \delta\left(x, t, \lambda^{(s-1)}\right),\ \gamma^{(s-1)}.$$

Thus, the sought for parameters can be determined from system (8). The basic stability theorems for inverse problems were first formulated and proved by A.N. Tikhonov in [**?**].

The solution of the inverse problem in this case is to determine such a numerical value (knowing the approximate values or not knowing them at all) that adequately corresponds to the mathematical models of production processes. Without indicated ?estimates? the sense of problem solution is lost.

Since $\tilde{P}_j = \tilde{P}\left(t_j\right)$ are obtained from experiment, they obviously have some errors (caused by vibrations of the apparatus, errors in reading diagram data, the human factor when entering the data into a computer, etc.).

If substitute experimental data P_j in (6) and replace the integral by the sum, then a method for the determination of k_0, λ *and* γ is obtained. But this method of determining the parameters, as indicated by many authors, will be unstable, as due to errors, a poorly conditioned linear system can be obtained. To solve the problem in such a statement, one can apply the A.N. Tikhonov regularization [**?**] and the problem can be solved to the end. However, for mass processing of experimental data, with sufficient accuracy, one can obtain a stable method for solving inverse problems using the following method.

Experimental data $\tilde{P}_j$ given in a table form are approximated by the Chebyshev polynomial. Estimates of dispersions of approximating polynomials of n and $n+1$ degrees are denoted by S_n and S_{n+1}, respectively. If $S_n > S_{n+1}$, then to make a final decision it is necessary to use the Fisher criterion, i.e., if

$$\tfrac{S_n}{S_{n+1}} > F_\nu\left(\overline{\psi}_1, \overline{\psi}_2\right)$$

(ν is the significance level; $\overline{\psi}_1$ *and* $\overline{\psi}_2$ are the degrees of freedom), then the degree of the approximating polynomial is assumed to be equal $n+1$, otherwise it is n. Reiterating this process for $n = 1, 2\ldots\ldots$ the degree of the approximating polynomial is found.

The approximating polynomial is represented as:

$$P\left(t\right) = \sum_{k=1}^{n} a_k\varphi_k\left(t\right),$$

where

$$a_k = \frac{\sum\limits_{i=1}^{m}\tilde{P}_i\varphi_k(t_i)}{\sum\limits_{i=1}^{m}\varphi_k^2(t_i)};$$

$\varphi_k\left(t\right)$ is the Chebyshev polynomial of k-th degree; a_k is the coefficient of approximate polynomial, determined by known recurrence relations.

The estimation of dispersion of approximating polynomial is done using the relationship

$$S = \frac{\sum\limits_{i=1}^{m}\left(\tilde{P}_i - P_i\right)}{m-(n-1)}.$$

After obtaining the approximating polynomial, the correlation ratio is calculated $\eta = \sqrt{1-\zeta_k}$, where

$$\zeta_k = \frac{r_n}{r_0};\ \ r_n = S_n\left(m-1\right);\ \ r_0 = \sum_{i=1}^{m} \tilde{P}_i^{\lambda} - \left(\sum_{i=1}^{m} \tilde{P}_i\right)^{\lambda}.$$

The correlation ratio η characterizes the closeness of the approximating polynomial to experimental data. If η approaches 1, then the closeness is good. In particular, if $\eta = 1$, then the approximating polynomial should have values at all points of the experimental data. Therefore, the following condition should always be held $0 < \eta < 1$.

Substituting in (6) the Chebyshev polynomial instead of experimental data $\tilde{P}_j$, we get

$$R = \int_{o}^{t} \left[P\left(\tau\right) - \varphi\left(\tau\right)\right]^{2} d\tau = \min \ \ or \ \ R = \sum_{i=1}^{m}\left[P_{n_i} + \sum_{i=1}^{m} \frac{\partial P_{ni}}{\partial \alpha_\ell} - \varphi_i\right]^2 = \min$$

here, given the continuous dependence of P on the parameters, the last expression is expanded into the Taylor series

$$P_{n+1} = P_n + \sum_{l=1}^{3} \frac{\partial P_n}{\partial \alpha_\ell} + 0\left(\alpha_\ell^{\lambda}\right);\ \ P_{n_i} = P_n\left(t_i\right) = P\left(t_i, k_0, \lambda, \gamma\right);$$

$$\frac{\partial P}{\partial k_0} = \frac{\partial P(t_i)}{\partial k_0};\ \frac{\partial P}{\partial \lambda} = \frac{\partial P(t_i)}{\partial \lambda};\ \frac{\partial P}{\partial \gamma} = \frac{\partial P(t_i)}{\partial \gamma};$$

$$\varphi_i = \varphi\left(t_i\right) = \sum_{l=1}^{m} \alpha_l \varphi_e\left(t_i\right).$$

Thus, the parameters k_0, λ *and* γ are found from the system of algebraic equations (8), the solution of which is not difficult to obtain. The iteration process continues until the following condition is fulfilled

$$\max_{\ell} \left|\alpha_\ell\right| < \varepsilon,\ \ \varepsilon > 0 \qquad (9)$$

(S is the number of iteration). The computational process is carried out according to the following scheme:

– The initial values of $k_o^{(0)},\ \lambda^{(0)},\ \gamma^{(0)}$ are set; the value of $\delta\left(t, k_0, \lambda, \gamma\right)$ is calculated;

– Then, from (2.32) the values of $k_o^{(1)},\ \lambda^{(1)},\ \gamma^{(1)}$ are found;

– Condition (9) is checked. If it is not met, already found values of $k_o^{(1)},\ \lambda^{(1)},\ \gamma^{(1)}$ are taken as initial values of the parameters $k_o^{(2)},\ \lambda^{(2)},\ \gamma^{(2)}$ obtained from (8), etc. If condition (9) is met at iteration, the values of $k_o^{(N)}$, $\lambda^{(N)}$, $\gamma^{(N)}$ are taken as sought for parameters.

According to the above scheme, calculations have been made on the following initial data: $W_0 = 0.0025 m/s$; $m_1 = 0.4$; $m_0 = 0.15$; $H_0 = 0.5$ m; $\rho = 25 g/cm$; $H = 1m$; $\mu = 980 cP$.

4. Results and discussion

Based on the above algorithm, numerical calculations have been carried out on a computer, the result are shown in Figures 1 – 4.

Figs. 1 – 4 show the curves constructed on these initial data and the corresponding Chebyshev approximating polynomials.

Analysis of obtained results of numerical calculations (Figs. 1 – 4) and their comparison with experimental data has shown that the calculation of true values of the process parameters plays a significant role in modeling the object of research and in determining their key indicators over time. As show numerical calculations carried out on a computer with an increase in calculation of true value of the process parameters, the degree of adequacy of developed MM of the object

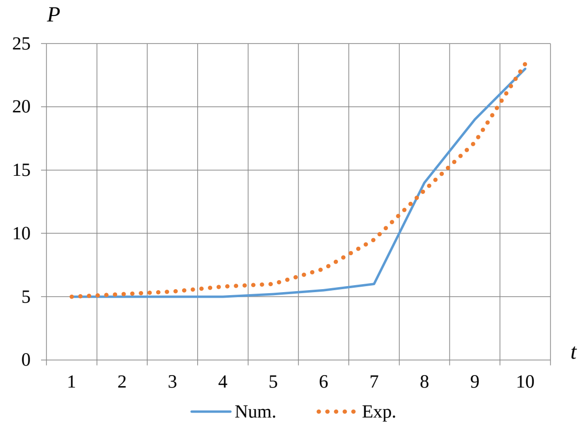

Figure 1. Increase in pressure change inside the filter column of the unit at $k_0 = 9,6\ Darcy$; $\lambda \cdot 10^{-3} = 0,24\ 1/sec$; $\gamma = 0,004$.

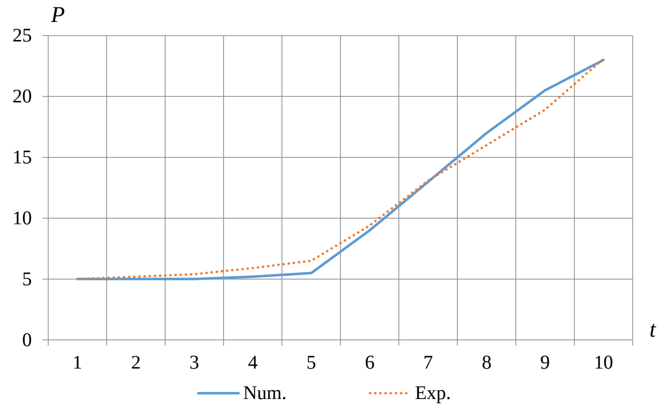

Figure 2. Increase in pressure change inside the filter column of the unit at $k_0 = 9,82\ Darcy$; $\lambda \cdot 10^{-3} = 0,24\ 1/sec$; $\gamma = 0,004$.

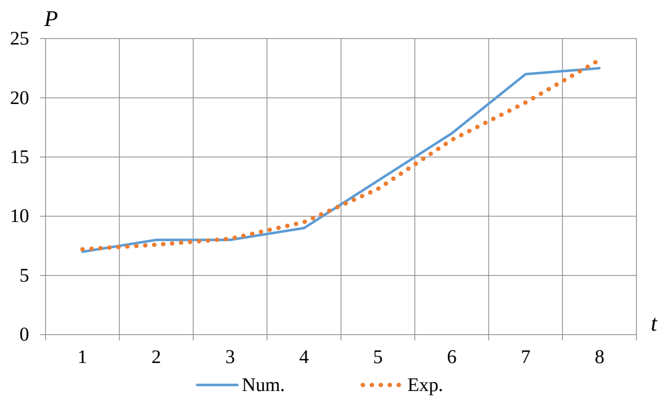

Figure 3. Increase in pressure change inside the filter column of the unit at $k_0 = 8,7\ Darcy$; $\lambda \cdot 10^{-3} = 0,21\ 1/sec$; $\gamma = 0,47$.

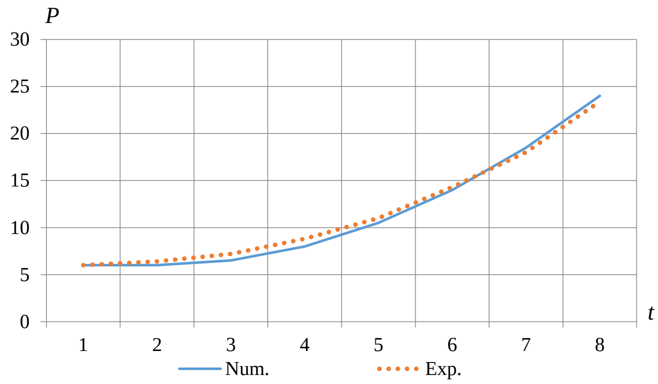

Figure 4. Increase in pressure change inside the filter column of the unit at $k_0 = 9,0\ Darcy$; $\lambda \cdot 10^{-3} = 0,21\ 1/sec$; $\gamma = 0,0043$.

increases (Figs. 1 – 4). This can be noted when comparing results obtained with model data and experiments (Fig. 4).

So, it can be stated that by solving inverse problems and substituting the found values of parameters into initial system of equations, the sought for functions W, δ, θ are calculated.

5. Conclusion

Firstly, in the course of the study, it is stated that the input parameters k_0, λ, γ obtained in experimental data processing, that is, laboratory data, are not applicable for solving the direct problem of the process. Therefore, it is more advisable to mathematically determine the parameters of mathematical model, i.e. from the solution of the inverse problem than by laboratory tests (Fig. 4). Secondly, the adequacy of the developed MM of the object can be achieved by calculating the exact values of the parameters of the TPFM under consideration.

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