Correction of the interpolation effect in modeling the process of estimating image spatial deformations

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***Abstract*—The paper proposed a technique for removing the influence of image interpolation artifact in the estimation algorithm of image spatial deformations. The technique is considered on examples of several similarity measures using bilinear and bicubic interpolation.**

Keywords—simulation, interpolation, target function, similarity measures, spatial deformations.

1. Introduction

When developing algorithms to estimate the parameters of image spatial deformations [1], the study of the accuracy of developed algorithms and their performance is carried out, as a rule, on simulated images. Similar algorithms are used in determining the dynamic deformation of a scene from an image sequence [2], which is usually reduced to a sequential inter-frame estimation of deformations. In the study, one image $\mathbf{Z}=[z_{i,j}]$ (reference) can also be real, and the second $\widetilde{\mathbf{Z}}=[z_{\tilde{i},\tilde{j}}]$ (deformed) is formed from it according to the given parameters of deformations using some kind of interpolation, where $z_{i,j}$ and $z_{\tilde{i},\tilde{j}}$ pixels of reference and deformed images, and (i,j), $(\tilde{i},\tilde{j})$ coordinates of the regular grid on which they are given, in a single coordinate system. Then these given parameters are estimated under different conditions, in particular, the influence of noise. However, when forming a deformed image, interpolation makes changes in the correlation properties of the interpolated image, which affects the adequacy and correctness of the study.

2. Problem Statement

Let us assume that the origin of the coordinate coincides with the $(0,0)$. Then the coordinates $(\tilde{i},\tilde{j})$ of the nodes of the deformed image reference grid are determined by the deformation model adopted. In particular, when using the similarity model [3], which includes the parameters of shift $\bar{h}=(h_x,h_y)^T$, rotation angle φ and scale factor κ:

$$\tilde{i}=i\cos\varphi+j\sin\varphi+h_x,\ \tilde{j}=i\sin\varphi+j\cos\varphi+h_y\ . \quad (1)$$

The development of an algorithm for estimating the deformation parameters implies specifying some target function (TF) that characterizes the quality of estimation. Different similarity measures of images can be used as TF [4]. In this work, three most common similarity measures were used: mean square of inter-frame difference (MSID), inter-frame correlation coefficient (ICC), and mutual information (MI) (in the given examples Shannon's MI was used [5,6]).

The pixel intensity $z_{\tilde{i},\tilde{j}}$ of the interpolated image can be predicted, as already noted, through some interpolation:

$$\tilde{z}_{\tilde{i},\tilde{j}}=\sum_{n=a}^{c}\sum_{m=b}^{c}a_{nm}z_{nm}\ , \qquad (2)$$

where $a_{n,m}$ are coefficients determined by the type of interpolation [7]; (n,m) are coordinates of reference image pixels in the local coordinate system, the center of which (point $(n=0,m=0)$) corresponds to integer values of coordinates $(\tilde{i},\tilde{j})$. The fractional (subpixel) component $(\Delta x,\Delta y)$ of these coordinates determines the $a_{n,m}$ coefficients. For $b=0,c=1$, we obtain bilinear interpolation, for $b=-1,c=2$, we obtain bicubic one. Thus, in bilinear interpolation: $a_{0,0}=(1-\Delta x)(1-\Delta y)$, $a_{0,1}=(1-\Delta x)\Delta y$, $a_{1,0}=\Delta x(1-\Delta y)$, $a_{1,1}=\Delta x\Delta y$.

Fig. 1. Satellite image of the Ulyanovsk state technical university

Due to the correlation of the reference image pixels, the inter-pixel correlation coefficient of the deformed image does not change at the points corresponding to the reference image nodes and increases at the points between the nodes. In this case, the maximum increase is achieved by a half shift of the counts grid along both coordinates. This leads to a distortion of the TF shape as a function of the deformation parameters. An example of such distortions when simulating a parallel shift of the image Fig. 1 along one of the coordinate axes is shown in Fig. 2. Fig. 2a corresponds to the use of MSID, Fig. 2b – ICC, Fig. 2c – MI. The red curve corresponds to the undistorted TF, the blue curve to the TF under bilinear interpolation, and the green curve to the bicubic interpolation. We see that interpolation affects different TFs variously. The largest distortions are noticeable at MSID, the smallest – at MI. This is explained by the fact that different TFs reflect different characteristics of images. Namely, MSID shows brightness characteristics, ICC shows covariance one, MI shows entropic one. Note also that different deformation parameters also have various effects on TF. For example, at the rotation angle, the distortions of TF depend on the location of the rotation center. The distortions will be minimal when the center of rotation coincides with the center of the image.

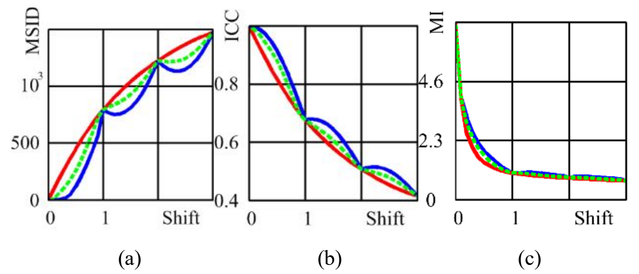

Fig. 2. TF when shifting the interpolated image

3. Correction of Interpolation Effect

To compensate for the interpolation effect, we used the addition of independent Gaussian noise to the interpolated image pixels, the intensity of which is a function of the deformation parameters. This functional dependence can be found from the following assumptions. Denote the variance of the reference image by σ_z^2. The pixel brightness at the node of the interpolated image is determined by the chosen type of interpolation, for example, (2), or other. To determine the variance of interpolated pixels it is necessary to use the same dependencies but take into account the correlation of the summands. For this purpose, the correlation function $R(l)$ of the reference image is used, where l is the distance between the corresponding pixels of the reference image. The value $\sigma^2_{\theta_{ij}}$ of the noise variance to be added to the pixels of the interpolated image can be found from the condition that the variances of all pixels $\{z_{\tilde{i},\tilde{j}}\}$ of the interpolated and reference images are equal.

$$\sigma_\theta^2(\tilde{i},\tilde{j}) = \sigma_{\tilde{z}}^2(\tilde{i},\tilde{j}) - \sigma_z^2 . \quad (3)$$

Let us give an example for a simple shift deformation h_x of the image along the basic axis. Then the intensity at node $(\tilde{i},\tilde{j})$ of the image under bilinear interpolation is determined by the expression:

$$z_{\tilde{i},\tilde{j}} = z_{i,j}(1-\Delta h_x) + z_{i,j}\Delta h_x , \quad (4)$$

and at the bicubic:

$$\begin{aligned} z_{\tilde{i},\tilde{j}} &= (2-\Delta h_x)(1+\Delta h_x)(z_{i,j}(1-\Delta h_x) + z_{i+1,j}\Delta h_x)/2 - \\ &-(\Delta h_x(1-\Delta h_x))(z_{i-1,j}(2-\Delta h_x) + z_{i-2,j}(1+\Delta h_x))/6 \end{aligned} , \quad (5)$$

Accordingly, for the variance $\sigma_\theta^2(\tilde{i},\tilde{j})$ with bilinear interpolation we obtain

$$\sigma_\theta^2(\tilde{i},\tilde{j}) = \sigma_{\tilde{z}}^2(\tilde{i},\tilde{j}) - \sigma_z^2 = -2(\Delta h_x^2 - \Delta h_x)(1-\rho(1))\sigma_z^2 , \quad (6)$$

In particular, for the image of Fig. 1 the interpixel correlation coefficient is $R(l) = 0.678$, and the variance is 1212.83. Substituting these values into (6), we obtain the value for finding the variance of added Gaussian noise. The plots of the target functions similar to Fig. 2, obtained with correction, are shown in Fig. 3. The results show that the interpolation effect is corrected and practically unnoticeable.

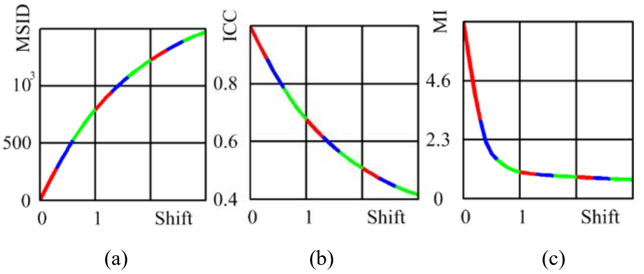

Fig. 3. TF after correction of the interpolation effect

4. Conclusion

The proposed method to correct the interpolation effect on TF quality for the problem of estimating the geometric deformation parameters of images has shown high efficiency. The method was considered by using bilinear and bicubic interpolation, for three similarity measures of images: MSID, ICC, and Shannon MI. However, it is not limited to these measures and is also applicable to others. Also, the method implies the possibility of using any interpolations: spline, using Lagrange polynomials, Newton, power functions and others.

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