Control of a queuing system with hidden Markov state

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Abstract. A single-server finite-buffer queuing system is considered on a fixed time interval. The server accepts a non-stationary Poisson stream of incoming packets for further transmission through a communication channel governed by a hidden Markov chain. Round-trip times for sent packets are described by the Markov counting process which is observed directly. The service rate is proportional to the transmission rate with a channel-dependent factor. The transmission rate is to be optimized within the class of feedback control policies given two performance characteristics: the average number of lost packets and the mean level of energy consumption. The approach proposed for control optimization is based on the optimal filter equations, the complete-information control algorithm, and Monte Carlo simulation.

Keywords: queueing system, hidden Markov model, optimal control.

1. Introduction

For many of data transmission networks there is a problem of incomplete information about the current state of communication links, queuing systems, servers' load, etc. Basically, the actual information available for network monitors consists of round-trip times and loss messages in the packet traffic. Despite the fact that these data cannot fully characterize the network state, one can elaborate efficient methods for congestion control in noisy communication lines and partially observed queuing systems [1–3]. The problem of incomplete information is especially important for the communication networks with onboard units acting autonomously in the presence of randomly changing environment [4–6]. Such an equipment aims at achieving several conflicting objectives: minimum losses in incoming useful traffic; decrease of delays in data processing; maximum battery life. These applications motivate further development of optimal control methods for info-telecommunication systems given noisy observations.

In this paper we consider a single-server finite-buffer queuing system on a fixed time interval. Incoming packets enter the system in a nonstationary Poisson stream for further transmission through a loss-free communication channel. The channel state affects only delays in transmission of packets: the worse the state, the more the time spent for sending a packet. Losses of incoming packets occur only if the queue's buffer is full. Similarly to the Gilbert model [7], the communication channel state is of few variants such as "good", "normal", and "bad". But the current value of the state is not known so its dynamics is modelled by a hidden continuous-time Markov chain. Its transition rates are supposed to be time-dependent in order to take into account changing environment conditions caused by motion of the mobile transmitter according to the known itinerary. Indirect information about the channel state is available from the stream of round-trip times (RTTs) for sent packets and is described by the Markov counting process.

Both the service rate in the queuing system and the intensity of the observation stream are proportional to the controlled transmission rate with channel-dependent factors.

Our goal is to propose an approach for constructing feedback control policies and analyzing their performance for the queuing system with hidden Markov state. The synthesis of the transmission rate control is based on the optimal filter equations for the unobserved channel state and the aggregated optimization of two performance indices: the average number of lost packets and the mean level of energy consumption. The performance analysis of the constructed policies is performed using the techniques of Monte Carlo simulation.

2. Model formulation and problem statement

Consider two continuous-time Markov chains X_t and Y_t defined on a fixed time interval $[0,T]$. The controlled state of the queuing system is described by X_t while the hidden state of the communication channel is governed by Y_t. Let N be the largest number of packets that can be placed in the queue and K be the number of aggregated states for the channel. Following the martingale approach in description of stochastic control systems [8], we assume that the state spaces for X_t and Y_t are comprised of unit column-vectors:

$$\mathbb{S}^X = \{e_0, e_1, \ldots, e_N\} \subset \mathbb{R}^{N+1} \quad \text{and} \quad \mathbb{S}^Y = \{f_1, \ldots, f_K\} \subset \mathbb{R}^K. \tag{1}$$

Since X_t is a birth-death process, its model is completely specified by two transition rates:

- the arrival rate $a_{i,i+1} = \alpha(t)$ for transition $e_i \to e_{i+1}$, $i = 0, 1, \ldots, N-1$,
- the service rate $a_{i,i-1} = m\langle d, y\rangle$[1] for transition $e_i \to e_{i-1}$, $i = 1, \ldots, N$, where $m = \mu(t)$ is the controlled transmission rate and $\langle d, y\rangle$ is a factor dependent of the channel state $y = Y_t$.

The components of $d \in \mathbb{R}^K$ are known positive quantities, which define the following empirical rule: the better the state of the communication channel, the less the average time spent for sending a packet. So we suppose the states $f_1, \ldots, f_K$ are arranged in accordance with $d_1 > \ldots > d_K$.

Thus, the generator of the non-homogeneous Markov process X_t is defined by the matrix $A(t, y, m) = \{a_{i,j}(t, y, m)\}_{i,j=0,1,\ldots,N}$ parameterized by channel state y and transmission rate m. Let $B(t) = \{b_{k,l}(t)\}_{k,l=1,\ldots,K}$ be the generator of the Markov process Y_t. Its transition rates are also assumed to be time-dependent, but the reason of this assumption is to take into account mobility of the transmitter which leads to time-varying behavior of the communication channel.

Indirect information about the hidden state Y_t is contained in the observation counting process R_t, where R_t is equal to the number of packets whose successful delivery is confirmed up to instant t. The intensity of R_t depends on the channel state and reflects the following natural assumption: the better the state of communication, the higher the reliability of transmission. So if $\langle c, Y_t\rangle$ stands for the current intensity of the observation process, then the components of $c \in \mathbb{R}^K$ satisfy inequalities $c_1 > \ldots > c_K > 0$.

The dependence between all the three processes has a statistical sense only: any changes in the one process never lead to an immediate response in the other. So jumps ΔX_t,[2] ΔY_t, or ΔR_t cannot occur at the same time.

Thus, the stochastic control system can be represented in the form[3]

$$\begin{cases} dX_t = A^*(t, Y_t, \mu(t)) X_t\, dt + dM_t^X, \\ dY_t = B^*(t) Y_t\, dt + dM_t^Y, \\ dR_t = \langle c, Y_t\rangle\, dt + dM_t^R, \end{cases} \tag{2}$$

[1] $\langle \cdot, \cdot \rangle$ is the inner product.
[2] $\Delta X_t = X_t - X_{t-}$.
[3] A^* means transpose of A.

where M_t^X, M_t^Y, M_t^R are pairwise orthogonal martingales [8].

A random process $\mu(t)$ is called a *control with complete information* if it:

(i) takes values from a prespecified segment $[\underline{m}, \overline{m}]$, where $\underline{m} > 0$;

(ii) has piecewise continuous paths;

(iii) is predictable with respect to filtration generated by X_t and Y_t.

Analogously, $\mu(t)$ is called a *control with incomplete information* if conditions (i) and (ii) are fulfilled but assumption (iii) is replaced with the following:

(iv) $\mu(t)$ is predictable with respect to filtration generated by X_t and R_t.

The classes of controls with complete and incomplete information are denoted by $\mathcal{C}$ and $\mathcal{I}$, respectively. The assumption $\mu \in \mathcal{C}$ means that the current value of $\mu(t)$ is fully determined by the prehistory $\{X_s, Y_s \colon s \in (0,t)\}$ of the queue system and the communication channel. But for the case $\mu \in \mathcal{I}$ we do not have direct information about the channel state, so the control $\mu(t)$ depends functionally on the known dynamics of the queue $\{X_s, s \in (0,t)\}$ and the observations of the counting process $\{R_s, s \in (0,t)\}$.

Consider two performance indices:

$$J_0[\mu] = \int_0^T \mathsf{P}\{X_t = N\}\alpha(t)\,dt \quad \text{and} \quad J_1[\mu] = \int_0^T \mathsf{E}\{\mu(t)\}\,dt, \tag{3}$$

where J_0 is equal to the average number of losses while J_1 stands for the mean level of power consumption.

Now we can formulate the optimal control problem:

$$J_0[\mu] \to \min_{\mu\in\mathcal{C}} \quad \text{subject to} \quad J_1[\mu] \le \overline{J}_1, \tag{4}$$

where $\overline{J}_1$ is a given bound such that $\underline{m} < \overline{J}_1/T < \overline{m}$. The goal of (4) is to find a control $\hat{\mu}$ that gives the minimum level of lost packets among all controls with complete information and limited energy consumption over the time horizon T. Analogously, the optimal control with incomplete information is defined as a solution to (4), where $\mathcal{C}$ is replaced with the class $\mathcal{I}$.

3. Method and results

Following [9], the solution $\hat{\mu}$ to the optimal control problem (4) can be constructed in the form $\hat{\mu} = \mu^o(\hat{\lambda})$, where $\mu^o(\lambda)$ is the optimal control with respect to the augmented functional:

$$L[\mu, \lambda] = J_0[\mu] + \lambda J_1[\mu] \to \min_{\mu\in\mathcal{C}}, \tag{5}$$

and $\hat{\lambda}$ is the Lagrange multiplier found from the dual optimization problem

$$L[\mu^o(\lambda), \lambda] \to \max_{\lambda\ge 0}. \tag{6}$$

The unconstrained control problem (5) can be solved using the dynamic programming approach. The solution to the dynamic programming equation consists of the cost functions

$$\varphi_{i,k}(t,\lambda), \quad i = 0, 1, \ldots, N, \quad k = 1, \ldots, K,$$

whose initial values $\varphi_{i,k}(0,\lambda)$ coincide with the optimum $L[\mu^o(\lambda),\lambda]$ calculated under the initial conditions $X_0 = e_i$, $Y_0 = f_k$. The optimal control is defined by the rule

$$\mu^o(t,\lambda) = m^o_{i,k}(t,\lambda) \quad \text{if} \quad X_{t-} = e_i, \quad Y_{t-} = f_k,$$

where the set of policies

$$m^o_{i,k}(t,\lambda), \quad i = 0,1,\dots,N, \quad k = 1,\dots,K,$$

are found from minimization of the right-hand side of the dynamic programming equation.

In order to construct a control with incomplete information, we have to solve the optimal filtering equation [2, 5] for the hidden state Y_t given the observation process R_t. It can be done on-line together with applying the control policies. Let $\pi_t = \mathsf{E}\{Y_t \mid \mathcal{R}_t\}$ be the conditional expectation of the hidden state with respect to sigma-algebra $\mathcal{R}_t$ generated by the counting observations $\{R_s,\ s \in [0,t]\}$. Since $Y_t \in \mathbb{S}^Y$, the vector π_t consists of conditional probabilities

$$\langle \pi_t, f_k\rangle = \mathsf{P}\{Y_t = f_k \mid \mathcal{R}_t\}, \quad k = 1,\dots,K.$$

We propose to define a control with incomplete information in the following form:

$$\widetilde{\mu}(t) = \sum_{i=0}^{N}\sum_{k=1}^{K} m^o_{i,k}(t,\hat{\lambda})\mathrm{I}\{X_{t-} = e_i\}\langle \pi_t, f_k\rangle,$$

where $\mathrm{I}\{\dots\}$ is the indicator of random event $\{\dots\}$. To explain the structure of this control it suffices to note that the policy

$$\widetilde{\mu}_i(t) = \sum_{k=1}^{K} m^o_{i,k}(t,\hat{\lambda})\langle \pi_t, f_k\rangle$$

coincides with the conditional expectation of the optimal control with complete information $\hat{\mu}(t)$ given the known state $X_{t-} = e_i$ and available observations $\{R_s,\ s < t\}$.

The numerical experiment was preformed with the following parameters:

$$N = 8, \quad K = 2, \quad T = 100, \quad \overline{J}_1 = 143, \quad \underline{m} = 0.5, \quad \overline{m} = 8, \quad d = \mathrm{col}[1, 0.5], \quad c = \mathrm{col}[1, 0.1].$$

The arrival rate is shown in Fig. 1. Two channel states f_1 and f_2 will be referred to as "good" and "bad", respectively. The rate of transition "good"→"bad" and the rate of inverse transition are shown in Fig. 2.

The optimal control policies are shown in Fig. 3, 4. If the channel is in state "bad", the optimal way for control of the queuing system is to decrease the transmission rate to its lower bound $\underline{m}$ ($\hat{\mu}(t) = \overline{m}$ only if the queue is empty). For the good state, the optimal transmission rate should be used at the highest level $\overline{m}$ on the most part of the time segment and, only in the end, it should be switched to the lowest level $\underline{m}$.

The behavior of the control with incomplete information is much more complicated because it depends on fluctuations of conditional probabilities $\langle \pi_t, f_k\rangle$, $k = 1, 2$ (see Fig. 5). Nevertheless, the policies $\widetilde{\mu}_i(t)$ also coincide with each other on the most part of the time segment if the queuing system contains just one packet (if $i > 0$).

The values of performance indices J_0 and J_1 for the constructed controls are shown in Fig. 6. The optimal control with complete information $\hat{\mu}(t)$ provides the minimum average number of lost packets J_0 under the constraint on the mean level of energy consumption $J_1 \le \overline{J}_1$. At the same, the control with incomplete information $\widetilde{\mu}(t)$ makes possible to lose less packets than for $\hat{\mu}(t)$, however this result is obtained by means of a violation of the energy constraint: $J_1[\widetilde{\mu}] > \overline{J}_1$. In addition to the expected values $(J_0[\mu],\ J_1[\mu])$, we present the results of Monte Carlo simulation for sample versions $(J_0^{(s)}[\mu],\ J_1^{(s)}[\mu])$ of the both performance indices:

$$J_0^{(s)}[\mu] = \int_0^T \mathrm{I}\{X_t = N\}\alpha(t)\,dt, \quad J_1^{(s)}[\mu] = \int_0^T \mu(t)\,dt.$$

Thus, the scheme proposed for data transmission control in the absence of complete information needs an additional step of optimization to take into account different requirements posed on the desired control. The development of constrained optimization techniques for partially observed controlled queuing systems constitutes a direction of our further research.

4. Acknowledgment

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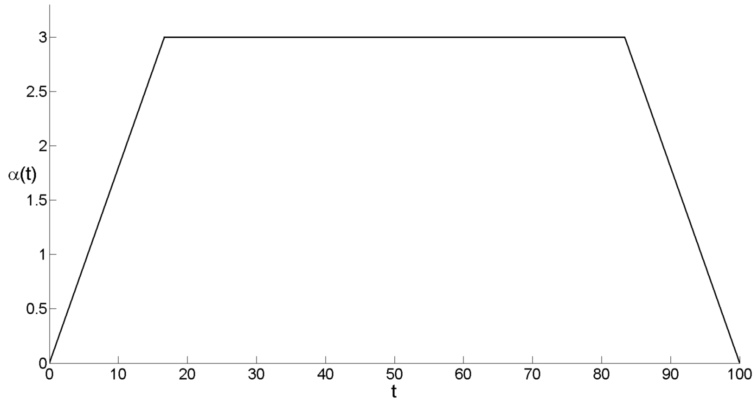

Figure 1. Arrival rate $\alpha(t)$.

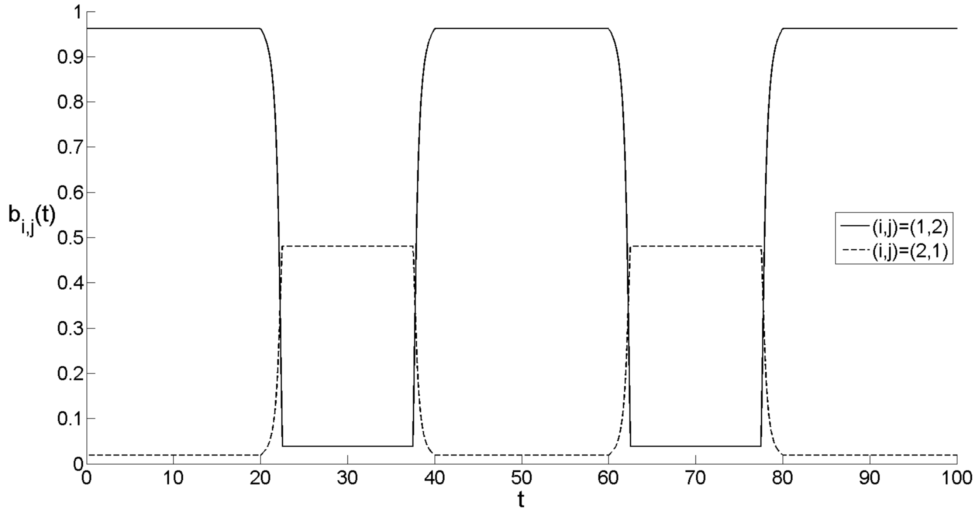

Figure 2. Transition rates $b_{1,2}(t)$ and $b_{2,1}(t)$.

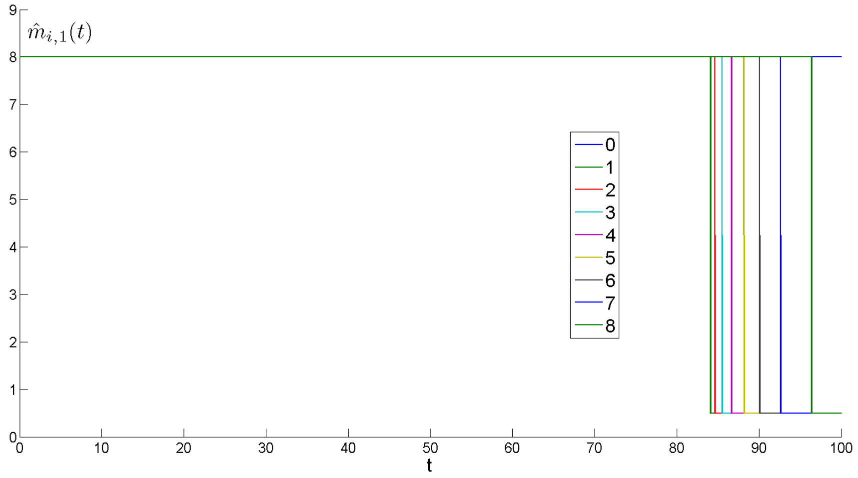

Figure 3. Optimal policies $\hat{m}_{i,1}(t)$ given channel state "good" and queue states $i = 0, 1, \ldots, 8$.

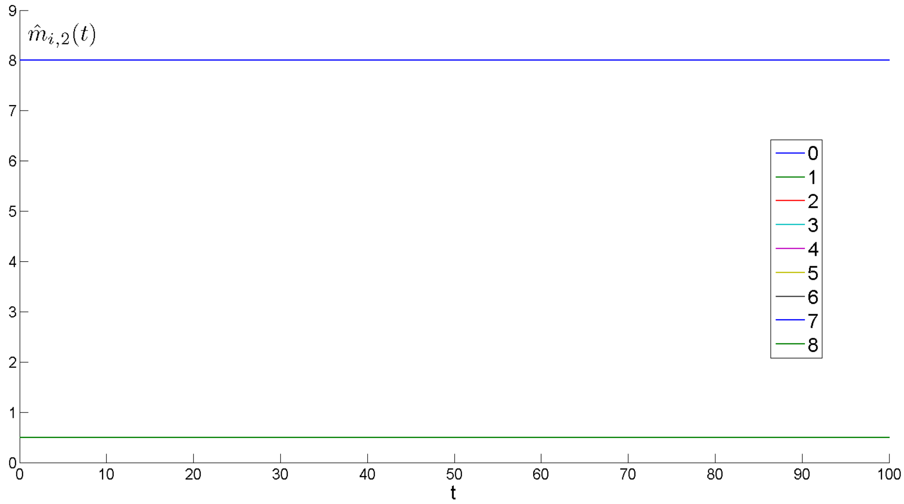

Figure 4. Optimal policies $\hat{m}_{i,2}(t)$ given channel state "bad" and queue states $i = 0, 1, \ldots, 8$.

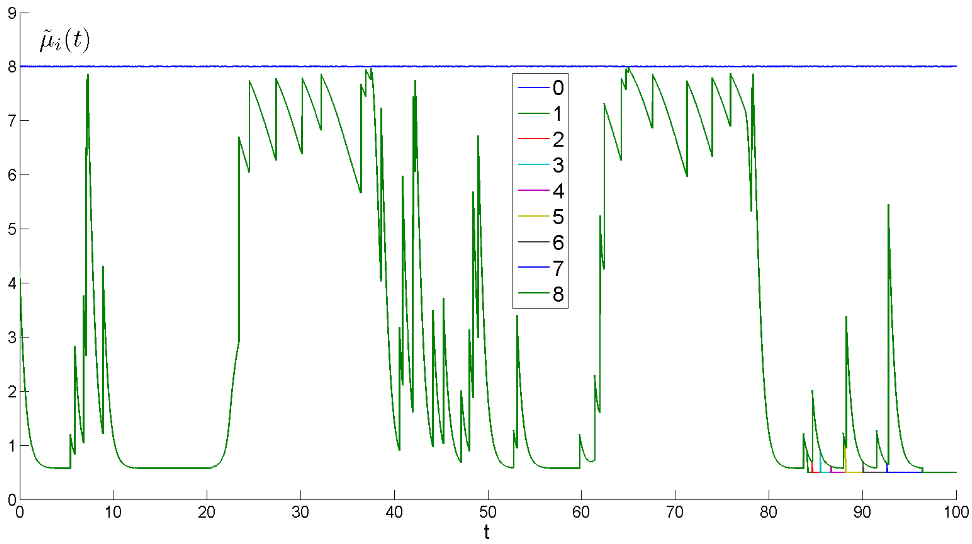

Figure 5. Policies $\widetilde{\mu}_i(t)$ with incomplete information given queue states $i = 0, 1, \ldots, 8$.

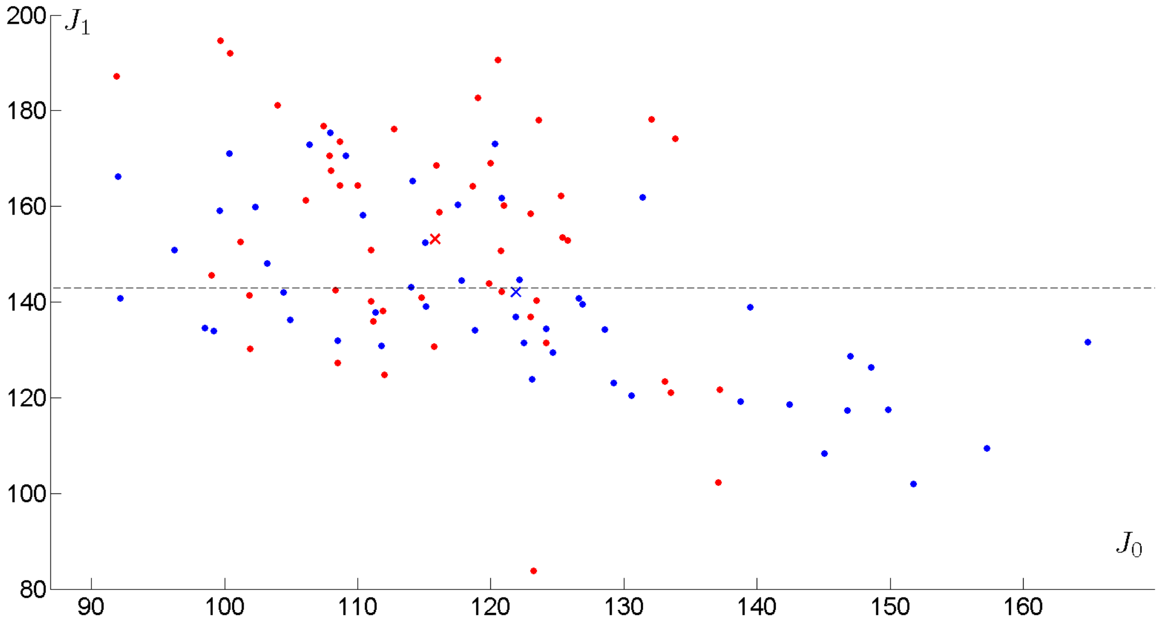

Figure 6. The expected values of losses J_0 and power consumption J_1 are shown as a blue cross for the optimal control with complete information $\hat{\mu}$ and as a red cross for the control with incomplete information $\widetilde{\mu}$. The sample values $(J_0^{(s)}[\mu], J_1^{(s)}[\mu])$ are shown as blue dots for $\mu = \hat{\mu}$ and as red dots for $\mu = \widetilde{\mu}$. The dash line corresponds to the upper bound for J_1.

5. References

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