Conditions of implementing dynamical regimes with strange chaotic attractors in attitude dynamics of multi-rotor spacecraft

A.V. Doroshin[1]

[1]Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086

Abstract. The conditions of implementing chaotic regimes in the attitude dynamics of multi-rotor spacecraft are found. These chaotic regimes correspond to strange chaotic attractors in the phase space of spacecraft dynamics described by the components of the angulr velocity of the main body of spacecraft. At the fulfilment of these conditions and at the creation of corresponded external and internal torques, the system realizes the dynamical regime along the phase trajectory of the strange chaotic attractor. Such regimes can be applied to intentional chaotization of the angular motion of the spacecraft to solve the tasks of the attitude reorientation.

1. Introduction

As it was previously shown [1-5], the chaotic regimes can be presented in the dynamics of the angular motion of the spacecraft, and, moreover, such chaotic regimes can be intentionally initiated by the creation of external and internal torques having simple regular forms [3-5]. In cases of the chaotic regimes initiation, the complex effects of the dynamical deterministic chaos arise, including the complex aperiodic chaotic oscillations of all dynamical parameters.

To expand the results of the previous research [3, 4] in this work the search of possibilities of creating initial conditions to generate strange chaotic attractors in the dynamics of multi-rotor spacecraft is fulfilled. This search includes the initial conditions for the angular velocities of the main spacecraft body and all its internal rotors, the concrete structure of the internal and external torques, and also the conditions to escape from the chaotic regime after achievements of goals of the regimes, e.g. after chaotic reorientation [7].

On the base of the strange chaotic attractors of Wang-Sun [5] and Chen-Lee [6], the synthesis of the initial conditions for all dynamical parameters is realized. Also the scheme of implementing preliminary procedures of the internal rotors spin-up by the way of using the internal torques with their predefined modules and durations. After these preliminary procedures, the external (jet-engines) and internal (rotors electro-motors) torques are actuated to proceed to chaotic regime along the strange chaotic attractor in the phase space of the components of the angular velocities of the main spacecraft body (x, y, z). In the following figure the example of such attractor is presented – this attractor corresponds to Wang-Sun type, which has the new coefficients of the dynamical system. The corresponding modeling is also fulfilled in the work to investigate the main aspects of the chaotic regimes.

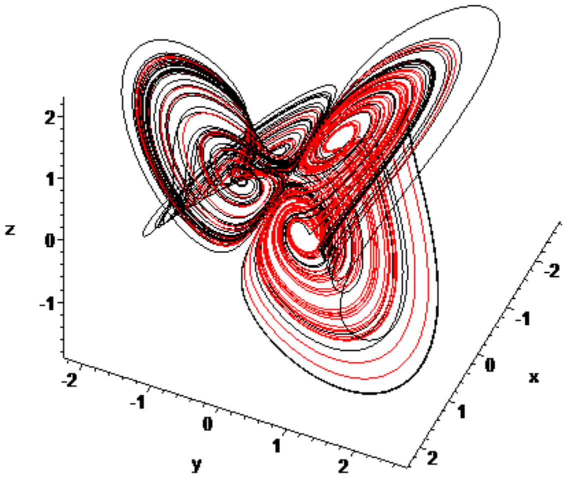

Figure 1. The new strange chaotic attractor of the Wang-Sun type (red) and the classical Wang-Sun attractor (black) in the phase space of the attitude dynamics of multi-rotor spacecraft.

2. Mathematical model

As it was described in [3, 4], the multi-rotor or, that is the same, the multi-spin spacecraft (MSSC) has the mechanical structure depicted at the fig.2 and the equations of the attitude motion:

$$\begin{cases} \hat{A}\dot{p}+\dot{D}_{12}+\left(\hat{C}-\hat{B}\right)qr+\left[qD_{56}-rD_{34}\right]=M_x^e; \\ \hat{B}\dot{q}+\dot{D}_{34}+\left(\hat{A}-\hat{C}\right)rp+\left[rD_{12}-pD_{56}\right]=M_y^e; \\ \hat{C}\dot{r}+\dot{D}_{56}+\left(\hat{B}-\hat{A}\right)pq+\left[pD_{34}-qD_{12}\right]=M_z^e; \end{cases} \tag{1}$$

$$\begin{cases} \dot{D}_{12}=M_{12}^i+M_{12}^e; \\ \dot{D}_{34}=M_{34}^i+M_{34}^e; \\ \dot{D}_{56}=M_{56}^i+M_{56}^e, \end{cases} \tag{2}$$

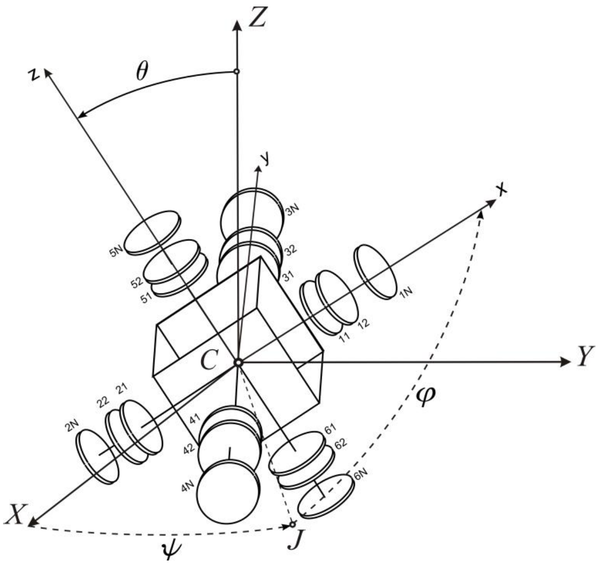

Figure 2. The mechanical structure of the MSSC.

where $\boldsymbol{\omega}=[p, q, r]^{\mathrm{T}}$ – is the vector of the absolute angular velocity of the main body in the connected frame $Cxyz$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ – are the general inertia moments of the spacecraft main body, M_x^e, M_y^e M_z^e – are the external torques applied to the spacecraft main body, σ_{kl} is the relative angular velocities of rotors;

I_l is the longitudinal and J_l is the equatorial inertia moments of the l-layer-rotor relatively the point O, N–is the quantity of layers of rotors, $M^e_{jlx}, M^e_{jly}, M^e_{jlz}$ are external torques acting only on the jl-th rotor, and M^i_{jl} is the torque from internal forces acting between the main body and the jl-th rotor, created by the internal electro-motors. Also the following expressions for the inertia parameters are actual:

$$\hat{A} = A - 2\sum_{j=1}^{N} I_j; \quad \hat{B} = B - 2\sum_{j=1}^{N} I_j; \quad \hat{C} = C - 2\sum_{j=1}^{N} I_j;$$

$$A = \tilde{A} + 4\bar{J} + 2\bar{I}, \ B = \tilde{B} + 4\bar{J} + 2\bar{I}; \ C = \tilde{C} + 4\bar{J} + 2\bar{I}; \ \bar{J} = \sum_{l=1}^{N} J_l; \ \bar{I} = \sum_{l=1}^{N} I_l$$

The rotors' summarized angular momentums are the following:

$$\begin{cases} D_{12} = \sum_{j=1}^{N}\left[\Delta_{1j} + \Delta_{2j}\right], \ D_{34} = \sum_{j=1}^{N}\left[\Delta_{3j} + \Delta_{4j}\right], \ D_{56} = \sum_{j=1}^{N}\left[\Delta_{5j} + \Delta_{6j}\right]; \\ \Delta_{1j} = I_j\left(p + \sigma_{1j}\right); \ \Delta_{2j} = I_j\left(p + \sigma_{2j}\right); \ \Delta_{3j} = I_j\left(q + \sigma_{3j}\right); \\ \Delta_{4j} = I_j\left(q + \sigma_{4j}\right); \ \Delta_{5j} = I_j\left(r + \sigma_{5j}\right); \ \Delta_{6j} = I_j\left(r + \sigma_{6j}\right). \end{cases} \tag{3}$$

The summarized internal (i) and external (e) torques applied to rotors are:

$$\begin{cases} M^i_{12} = \sum_{l=1}^{N}\left(M^i_{1l} + M^i_{2l}\right); \ M^i_{34} = \sum_{l=1}^{N}\left(M^i_{3l} + M^i_{4l}\right); \ M^i_{56} = \sum_{l=1}^{N}\left(M^i_{5l} + M^i_{6l}\right); \\ M^e_{12} = \sum_{l=1}^{N}\left(M^e_{1lx} + M^e_{2lx}\right); \ M^e_{34} = \sum_{l=1}^{N}\left(M^e_{3ly} + M^e_{4ly}\right); \ M^e_{56} = \sum_{l=1}^{N}\left(M^e_{5lz} + M^e_{6lz}\right). \end{cases} \tag{4}$$

To initiate the above noted strange chaotic attractors inside the dynamics of MSSC described by the equations (1) and (2) we will create controlling internal and external torques in the following forms:

$$M^i_{12} = \alpha_p \dot{p}; \qquad M^i_{34} = \beta_q \dot{q}; \qquad M^i_{56} = \gamma_r \dot{r}; \tag{5}$$

$$M^e_x = m_x + \alpha_1 p; \ M^e_y = m_y + \beta_1 q; \ M^e_z = m_z + \gamma_1 r, \tag{6}$$

with the following set of the controlling constant $\left\{m_x, \ m_y, \ m_z, \ \alpha_1, \ \beta_1, \ \gamma_1, \ \alpha_p, \ \beta_q, \ \gamma_r\right\}$.

Then the summarized angular momentums of rotors take the form:

$$D_{12} = \alpha_p p + \alpha_0; \ D_{34} = \beta_q q + \beta_0; \ D_{56} = \gamma_r r + \gamma_0, \tag{7}$$

where $\alpha_0, \beta_0, \gamma_0$ – are also the controlling constants, defined by the initial conditions.

So, if find the appropriate values of the controlling constants, it is possible to initiate the strange chaotic attractors. The corresponded method to find such values is described in the paper [3]. E.g., to initiate the strange chaotic attractor, presented at the fig.1, we found the following values of the controlling constants:

$$\left\{\begin{array}{l} m_x = 0.0079, \ m_y = 0.0182, \ m_z = 0; \\ \alpha_1 = 16.0277, \ \beta_1 = -32.3408, \ \gamma_1 = -0.0220; \\ \alpha_p = -3.6975, \ \beta_q = 16.3246, \ \gamma_r = -49.9780; \\ \alpha_0 = 0, \ \beta_0 = 0, \ \gamma_0 = -0.4247. \end{array}\right\} \tag{8}$$

The values like (8) can be created by only one layer of rotors, and everywhere below we will consider the single-layer (N=1) structure of the MSSC, that allows to neglect the second indexes in the designations of relative angular velocities of rotors: $\sigma_{kl} = \sigma_k$. Now the question only is how we can implement the found constants in the framework of the natural dynamics of the MSSC.

3. The implementation of the controlling constants

To supply the predefined values of the controlling constants (e.g. the found values (8)), it is possible to suggest the following steps of the algorithm of the initiation of the motion of MSSC.

Firstly, the main rigid body of the MSSC must be preliminary stopped in its rotation relative inertial space, that corresponds to nullification of the angular velocity (p=q=r=0). This stopping can be fulfilled with the help of the creation of torques $M_x^e, M_y^e\ M_z^e$, formed by the main jet-engines of MSSC. After stopping the main body, we can take the initial condition of motion of the main body of MSSC as:

$$p(0)=p_0=0;\quad q(0)=q_0=0;\quad r(0)=r_0=0. \tag{9}$$

Secondly, we must stabilize the rest of the main body (p=q=r=0) by the main jet-engines, and must spin the rotors by the internal electro-motors $\left(M_j^i,\ j=2,4,6\right)$ up to the following relative angular velocities, which we will further consider as initial:

$$\begin{cases}\sigma_1(0)=\sigma_{10}=0;\quad \sigma_3(0)=\sigma_{30}=0;\quad \sigma_5(0)=\sigma_{50}=0;\\ \sigma_2(0)=\sigma_{20}=\alpha_0;\quad \sigma_4(0)=\sigma_{40}=\beta_0;\quad \sigma_6(0)=\sigma_{60}=\gamma_0.\end{cases} \tag{10}$$

Thirdly, the following laws of the relative rotation of the rotors are realized with the help of internal electro-motors and sensors of the angular velocity of the main body:

$$\begin{cases}\sigma_1(t)=k_1 p(t);\quad \sigma_3(t)=k_3 q(t);\quad \sigma_5(t)=k_5 r(t);\\ \sigma_2(t)\equiv\alpha_0;\quad \sigma_4(t)\equiv\beta_0;\quad \sigma_6(t)\equiv\gamma_0.\end{cases} \tag{11}$$

where

$$k_1=\frac{1}{I}\left(\alpha_p-2I\right);\quad k_3=\frac{1}{I}\left(\beta_q-2I\right);\quad k_5=\frac{1}{I}\left(\gamma_r-2I\right) \tag{12}$$

The checking the suggested algorithm can be realized by the direct substitution of the expressions (9)-(12) into formulae (5)-(7). So, starting with the initial values (10) and (9), the laws of controlling the rotors relative velocities (11) allow to implement the time-dependencies of summarized angular momentums (7), and to create the external torque (6), formed by the main jet-engines of the MSSC. Then at the preliminary found controlling constants (e.g. (8)) the MSSC will fulfill such angular motion, that has in the 3D-phase space {p=x, q=y, r=z} the trajectory, which corresponds to strange chaotic attractor, like at the fig.1.

4. Conclusion

As it is presented above, the possibility of the initiation of the strange chaotic attractors is possible in the phase space of the attitude dynamics of the multi-spin spacecraft. The fact of the existing of strange attractors in the motion of spacecraft is known [1-6], but the suggested in this paper algorithm of the strange chaotic attractors' initiation is new, including the synthesis of initial conditions and controlling laws. The spacecraft in this case implements the intentionally initiated chaotic dynamics, which can be used in the framework of the design of new non-traditional motion control schemes [e.g. 7].

5. Acknowledgments

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6. References

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