Classification of multidimensional element types in automatic regulation systems

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Abstract. This study examines the possibility of applying the A.A. Krasovsky method of complex coordinates and complex transfer functions for multidimensional element types modeling. As it is demonstrated, the method can be taken as the basis for multidimensional regulation system design along with the extended version of classification suggested by A.A. Krasovsky.

1. Introduction

Currently, developers of modern multidimensional multi-connected technical regulation systems have to solve the problem of physically real connections present between the elements of developed systems even when designing mathematical models. These connections for some systems significantly affect their dynamics. Opto-electronic angle and range tracking system of moving objects is an example of above-mentioned complex system [1].

The most common methods for the mathematical description of multidimensional systems are matrix methods [2, 3]. Matrix methods are convenient for representation of high degree complexity in multi-connected technical regulation systems and perfectly suited for numerical solution by computer.

However, with these methods there is a problem to obtain qualitative estimation of matrix equations solution if applied to the system as a whole. Moreover, the greater the dimension of the system, the greater the number of connections between channels in the system, the more difficult it is to study such systems.

When conducting research on two-dimensional systems, Academician A.A. Krasovsky first suggested the method of complex coordinates and complex transfer functions [4]. In these works, he was also the first one to suggest a classification of two-dimensional systems with cross-connections. The method of A.A. Krasovsky was successfully applied and further developed in [1, 6].

The purpose of this study is to explore the possibility of reducing the complexity of modeling and research of multi-connected four-dimensional automatic regulation systems by applying the A. A. Krasovsky method of complex transfer functions.

2. Method description

In [7–9] the authors presented an obtainment of complex-valued and quaternion transfer functions for two-dimensional and four-dimensional systems respectively.

Consider the scheme given in [6] of generalized matrix-complex-valued unit of the control system from the A.A. Krasovsky classification point of view (Figure 1).

Here $y_1(t)$, $y_2(t)$ - are input signals; $x_1(t)$, $x_2(t)$ - are output signals; $W_1(p)$, $W_2(p)$ – are transfer functions.

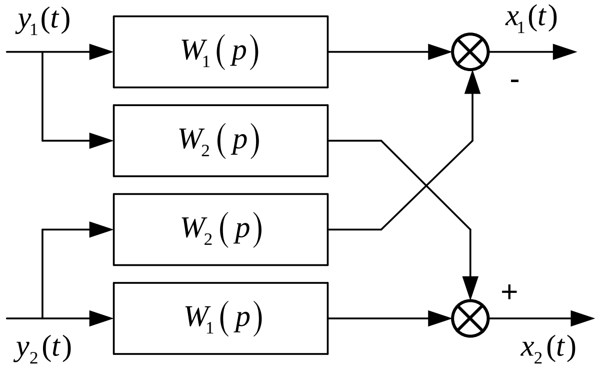

Figure 1. Generalized marix-complex-valued element.

In terms of the A.A. Krasovsky classification it is a two-channel element of control system with identical channels and antisymmetric cross- connections.

The matrix equation of the dynamics for this element will take form as shown below:

$$X(t)=W(p)Y(t) \tag{1}$$

Here $X(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ - is a column-vector of output signal; $Y(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$ - is a column-vector of input signal; $W(p)=\begin{bmatrix} W_1(p) & W_2(p) \\ -W_2(p) & W_1(p) \end{bmatrix}$ - is a transfer function matrix; $W_1(p)$, $W_2(p)$ - are transfer functions of the 1-st and the 2-nd channels respectfully.

The equation of given element in complex coordinates can be written:

$$\overline{x}(t) = \overline{W}(p)\overline{y}(t) = \big(x_1(t) + ix_2(t)\big) = (w_1(p) + iw_2(p)(y_1(t) + iy_2(t)) \tag{2}$$

Here $\overline{x}(t)$, $\overline{y}(t)$, $\overline{W}(p)$ – are output, input signals functions and transfer function in complex coordinates (this means that the name “complex-valued” can be applied in this case). After calculations (2) can be represented as:

$$x_1(t) + ix_2(t) = \big(w_1(p)y_1(t) - w_2(p)y_2(t)\big) + i(w_1(p)y_2(t) + w_2(p)y_1(t))$$

or coordinate form:

$$\begin{aligned} x_1(t) &= w_1(p)y_1(t) - w_2(p)y_2(t), \\ x_2(t) &= w_1(p)y_2(t) + w_2(p)y_1(t) \end{aligned} \tag{3}$$

It is easy to see that the coordinate forms of the matrix equation (1) and the equations in complex variables (2) are identical and make it possible to build an element based on them as in Figure 1.

The complex-valued equation (2) can be taken as the basis for the development of both a separate two-dimensional element of the projected system of any complexity, and the entire system as a whole.

It should be noted that a special case may occur when:

$$w_1(t) = w_2(t)$$

Then the system of coordinate equations (3) can be written in two forms:
the first one is:

$$\begin{aligned} x_1(t) &= w(p)y_1(t) - w(p)y_2(t), \\ x_2(t) &= w(p)y_1(t) + w(p)y_2(t) \end{aligned} \tag{4-a}$$

the second one is:

$$\begin{aligned} x_1(t) &= w(p)y_1(t) - w(p)y_2(t) = w(p)(y_1(t) - y_2(t)), \\ x_2(t) &= w(p)y_1(t) + w(p)y_2(t) = w(p)(y_1(t) + y_2(t)) \end{aligned} \tag{4-b}$$

These two different equation forms have two different structural schemes for the same control system element as in Figure2 and in Figure 3.

Given classification can be applied for the less known complex number system such as hyperbolic complex numbers [9].

For example, consider obtained in [8] generalized matrix-complex-valued control system for the hyperbolic numbers.

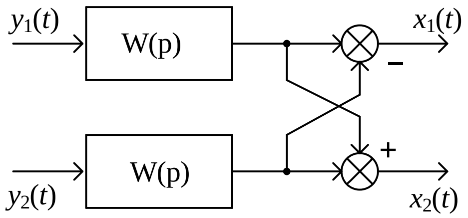

Figure 2. Two-dimensional element with output cross-connections.

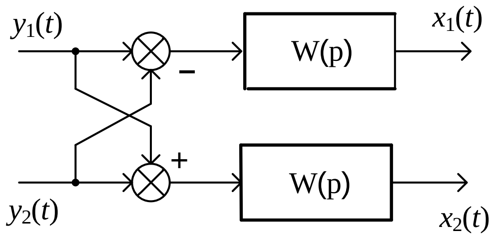

Figure 3. Two-dimensional element with input cross-connections.

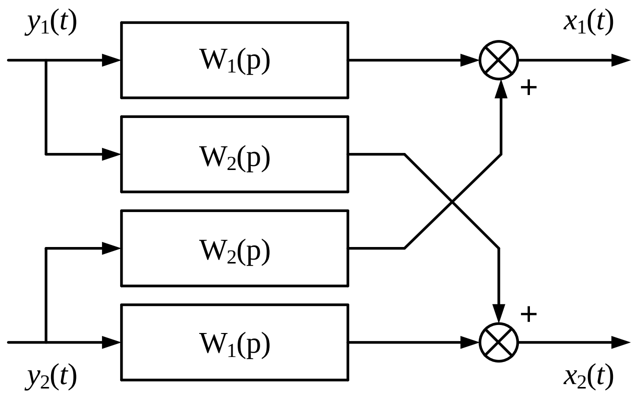

Figure 4. Generalized matrix-complex-valued control system for the hyperbolic numbers.

The result of complex coordinates method applied to given matrix-complex-valued control system is shown below:

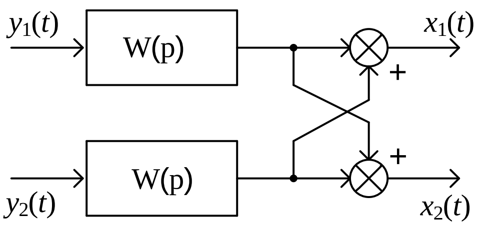

Figure 5. Two-dimensional element with output cross-connections.

Figure 5 demonstrates two-dimensional element with output cross-connections and equal in signs transfer functions. This is an expansion of the A.A.Krassovky classification, because A.A.Krassovky considered only antisymmetric cross-connections.

3. Conclusion

Complex coordinates method has been applied to the generalized matrix-complex-valued unit of the control system and the result of such approach is reducing complexity of multidimensional control systems modeling.

Given method allows using generalized structural schemes for the first stage of modeling, without examining the features of typical element. Nevertheless, if the developer chooses to apply, for example an oscillatory element as a typical element, then accordingly to the classification of A.A.

Krasovsky in Figure 2 will be a structural scheme of two-dimensional oscillatory element with cross-connections at the output, and in Figure 3 - a two-dimensional oscillatory element with cross-connections at the input and so on.

Same development approach can be applied for other element types such as integrating and differentiating. In addition, there are quaternion elements were presented in [7-9], but they require further study.

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