Cellular automata-based models of group motion and interaction of agents with memory

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Abstract

The paper is devoted to the construction of the motion and interaction model for agents with memory. Agents move on the landscape consisting of squares with different passability. We briefly described the cellular automata-based model with one common to all agents layer corresponding to the landscape and many agent-specific layers corresponding to an agent's memory. Methods for the random landscape generation and for the simulation of a communication system are developed. Also, we studied a connection between the discrete and the continuous formulation of the agent motion model and found several dependencies between parameters of the model.

Keywords: cellular automaton; motion model; conflict model; agent system; random landscape generation; landscape metrics

1. Introduction and definitions

Previously the author studied the cellular automaton-based models of motion [1] and communication [2]. In this paper I continue the previous work, propose cellular automaton that takes into account the history of the movement of agents and obtain few quantitative characteristics of the model. Give definitions according to the work [3].

Definition 1. *Let us call landscape $\mathcal{L}_l(n \times m)$ rectangle from $n \times m = N$ cells ω_{ij}, $(i, j) \in I \subset \mathbb{Z}^2$ with equal size belonging to l different classes and that to i-th class it belongs N_i cells, i.e. $\sum_{i=1}^{l} N_i = N$.*

Note that for landscapes generated for testing of path-finding algorithms, landscape cells will be divided into classes according to the maximum possible cell-crossing speed.

Definition 2. *Configuration entropy of the landscape $\mathcal{L}_l(n \times m)$ is defined as*

$$S(\mathcal{L}_l(n \times m)) = -\sum_{i=1}^{l} \frac{N_i}{N} \ln \frac{N_i}{N}$$

and characterizes landscape heterogeneity in whole.

Definition 3. *Total Edge is defined as the total number of the abutting edges of cells in $\mathcal{L}$, which are belonged to different classes. We will further denote the Total Edge of the landscape $\mathcal{L}$ as $TE(\mathcal{L})$.*

Definition 4. *Total Edge Density (TED) of the landscape $\mathcal{L}$ is defined as the ratio $TE(\mathcal{L})$ to the total cell quantity N in the $\mathcal{L}$*

$$TED(\mathcal{L}) = TE(\mathcal{L})/N.$$

Definition 5. *Denote Euclidean distance between $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$, $x, y \in \mathbb{R}^n$ as*

$$\|x - y\| = \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{1/2}.$$

2. Description of the automaton

Let the $Ag = \{ag_1, \ldots, ag_k\}$ is the system of agents, which move across the landscape $\mathcal{L}_l(n \times m)$, and initial and final cells of the landscape are specified for each agent. The idea of the article is that in the model in addition to the total for all agents "layer" corresponding to objective reality, each agent would have been "layer" corresponding to the information about the reality, which is known to this agent.

The behavior of the agents of the system is modeled by a cellular automaton in which the set of cells is $World = \{(i, j, id) | i, j \in \mathbb{Z}, id = \overline{0, k}\} \subset \mathbb{Z}^3$. In the set $World$ we will allocate $k + 1$ cell planes: a layer of the objective reality $OWorld = \{(i, j, 0) | i, j \in \mathbb{Z}\}$, and the layers of subjective reality of agent with identifier $ag = \overline{1, k}$ $SWorld_{ag} = \{(i, j, ag) | i, j \in \mathbb{Z}\}$. In this way,

$$World = OWorld \cup \left(\bigcup_{ag=1}^{k} SWorld_{ag}\right).$$

We assume that the rectangle $K \subset World$, $K = \{(i, j, id) | i = \overline{0, L_K}, j = \overline{0, L_K}, id = \overline{0, k}\}$ is selected and all cells are in the resting state outside of it.

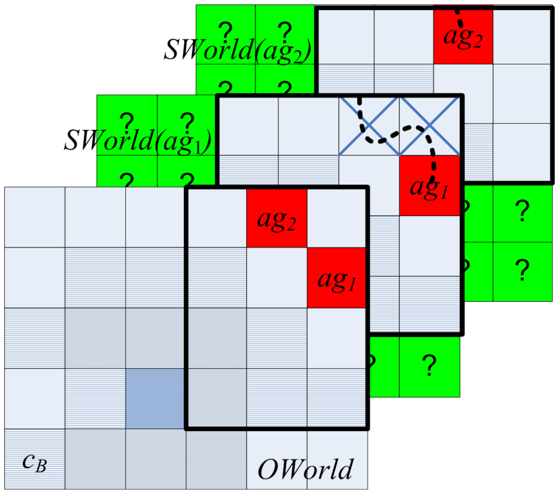


Fig. 1. The sample of the cellular automaton.

2.1. *Objective reality*

Let the objective reality layer *OWorld* consists of cells $(i, j) \in \mathbb{Z}^2$ with different impassability u_{ij}. The value of u_{ij} is the number of discrete time units which is required to pass the square ω_{ij} with coordinates (i, j). If ω_{ij} is completely impassable then put $u_{ij} = -1$. Also cells can include the information about an agent in a cell, the agent's destination square etc.

2.2. *Subjective reality*

The subjective reality layer consists of cells (i, j) so that each its cell (i, j) corresponds the cell (i, j) of the objective reality layer. Cells of the subjective reality layer *ag* contain the information about the current position of the agent *ag*, about the history of the *ag* motion and about the impassability of known to the agent *ag* cells.

2.3. *The automaton's functioning*

Let us denote

$$\mathcal{D} = \{(-1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1)\}.$$

Definition 6. *Let us call agent's cellular route the sequence*

$$M = \{(i_1, j_1), (i_2, j_2), \dots, (i_s, j_s) | (i_k, j_k) \in \mathbb{Z}^2, k = \overline{1, s}, (i_{k+1} - i_k, j_{k+1} - j_k) \in \mathcal{D}, k = \overline{1, s-1}\},$$

such as the agent in the square ω_{i_1,j_1} will be sequentially move into squares $\omega_{i_2,j_2}, \dots, \omega_{i_s,j_s}$. Denote the set of all cellular routes starting in the cell with coordinates $c_A \in \mathbb{Z}^2$ and ending in the cell with coordinates $c_B \in \mathbb{Z}^2$ as $\mathscr{M}(c_A; c_B)$.

Let us define a function

$$\theta(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0. \end{cases}$$

Introduce the notation:

$$\psi_3(u, T_{max}) = \begin{cases} u, & u \geq 0, \\ T_{max}, & u < 0. \end{cases}$$

If cell impassability does not change over a time, then it is not necessary to consider the routes containing impassable cells. However, if the impassable cell can become passable, these routes should be taken into account. To do this, define the functional

$$\tilde{T}_h(M) = \sum_{(i,j)\in M} \|d_{ij}\| \psi_3(u_{ij}, T_{max}).$$

We call weight of the route M for the agent *ag* the following:

$$\Lambda(M; \alpha, \beta, \gamma, T_{max}) = \alpha \tilde{T}_h(M) + \beta \sum_{(i,j)\in M} \theta(f_{ij}) + \gamma \sum_{(i,j)\in M} vis_{ij}(ag),$$

where u_{ij} is the impassapility of the square ω_{ij}, vis_{ij} is the number of visits of the square ω_{ij} (it is contained in the subjective reality layer $SWorld(ag)$), α, β, γ are parameters.

The agent in the square ω_{ij} each discrete time tick tries to find locally optimal (in a neighborhood $V_o(i, j)$ with radius o) route M_o such that $\Lambda(M_o; \alpha, \beta, \gamma, T_{max}) \to \min$ and go through this route. Therefore the agent's route at whole constructs from locally optimal subroutes.

Agent *ag* can apply previously described approach for route searching in undiscovered by this agent areas, i.e. in consisting of cells ω_{ij} with $vis_{ij}(ag) = 0$. More standard approaches to optimum route search, for example, Dijkstra's algorithm, can be used in areas composed of cells already visited. However, the use of standard methods of searching for an optimal route is limited to a rate of landscape change over time. It is possible that information about the visited cells become outdated (parameter $time_{ij}(ag)$ is used for determining the actuality of information), or even impassibility of the cells would change directly during the process of passing the route selected as the globally optimal.

Thus, depending on the speed of the landscape changes, it is necessary to find a compromise between the approach "Reacting", which evaluates the current situation in the immediate vicinity of the agent and the approach "Planning", in which searched globally optimal trajectory. For example, it is pointless to set the radius o of the neighborhood $V_o(i, j)$, in which locally optimal route is searched, more than the number of ticks during which the landscape has remained unchanged.

The example of the described cellular automaton is depicted on the fig. 1. Increasing of the impassability at the mentioned figure is indicated with a darker tone, crosses "×" in the layer $SWorld$ mark already visited cells, marks "?" correspond to cells whose status is unknown.

3. Function of obstacles

Turn to the continuous formulation of obstacle avoidance problem for the agent moving in the domain Ω with changing over time obstacles from the point A to the point B with route $r(t)$, $t \in [0, T]$ in the shortest time T to construct transfer function for the our CA. This problem has the form

$$\|\dot{r}(t)\| = v(t, r(t)), \tag{1}$$

$$r(0) = A, \quad r(T) = B, \tag{2}$$

$$T \to \min. \tag{3}$$

We will call further the function $v : [0, T] \times \Omega \to \mathbb{R}$ as "function of obstacles". Divide the segment $[0, T]$ k onto the k subsegments with length $\tau > 0$, domain Ω onto squares ω_{ij} with numbers $(i, j) \in \mathbb{Z}^2$ and the length of a side h. Approximate at the each moment of time $k\tau \in [0, T]$ on the square ω_{ij} function $v(k\tau, \cdot)$ with the constant function $v^k_{ij}(h, \tau)$.

It is necessary to go to the discrete time for the model simplifying. Let

$$t_h = \frac{h}{\max_{(i,j)\in I_h, k\in T_\tau} v^k_{ij}(h, \tau)}.$$

If it holds that

$$u_{ij}(k) = \frac{h}{t_h v^k_{ij}(h, \tau)} = \frac{\max_{(i,j)\in I_h, k\in T_\tau} v^k_{ij}(h, \tau)}{v^k_{ij}(h, \tau)}$$

define that square ω_{ij} on the state in moment $k\tau \in [0, T]$ is crossable in non-diagonal direction in $u_{ij}(k)$ ticks.

Moreover, it is possible to go to the integer values of the $u_{ij}(k)$ by discarding the fractional part and taking $\tilde{u}_{ij}(k) = [u_{ij}(k)]$. We associate with the agent in the square $\omega_{i,j}$ value $errc_{ij}$ of the cumulative discrete time error. Also we associate with the square ω_{ij} error value $err_{ij} = \{u_{ij}(k)\}$. When an agent starts to cross the next square $\omega_{i'j'}$, the value $errc_{i'j'}$ increments on the $err_{i'j'}$, sets $errc_{ij} = 0$ and if $errc_{i'j'} > 1$ then agent passes one tick independently from the value of the function of obstacles in the square $\omega_{i'j'}$ and sets $errc_{i'j'} = errc_{i'j'} - 1$.

Let

$$T : \mathscr{M}(c_A; c_B) \to \mathbb{R}$$

the functional of time which is required to going through cellular route.

If values of the u_{ij} do not change in a time of the movement from the point A to the point B, then the problem (1)–(3) can be represented as discrete problem

$$T(M) = t_h \sum_{(i,j)\in M} \|d_{ij}\| u_{ij} \to \min.$$

It is clear that possible to minimize functional

$$T_h(M) = \sum_{(i,j)\in M} \|d_{ij}\| u_{ij} \to \min$$

instead functional T.

It is possibly (but not very easy) to proof that the aforementioned CA actually finds the approximation of the solution of the problem (1)–(3) in some subdomain of the Ω. The sequence of such approximations r_h converges to the optimal solution r and the following estimate holds:

$$|r(l(t)) - r_h(l_h(t))| \le (h\sqrt{2} + \tau)(e^{\|\nabla_{(t,x,y)} v\|_{C([0,T]\times\Omega)} K} - 1) + h\sqrt{2}, \tag{4}$$

where l, l_h are parameterizations of routes, h is the length of the square ω_{ij} side, τ is the length of the time tick, $K > 0$ is the constant depending on a class of routes considered.

Definition 7. *Define the obstacle which is exists in the moment $t \in [0, T]$ as simply connected set $Obst \subset \Omega$ such that any $r_{obst} \in Obst$ is the point of a local minimum of the function of obstacles $v(t, \cdot)$ and exists $r_0 \in \Omega$ such that $v(t, r_0) > v(t, r_{obst})$.*

4. The model of a communication system and conflict

Definition 8. *Let us define communication graph as follows*

$$\Gamma(t) = (Ag, Comm, \varphi(t), M(t)),$$

where Comm is the set of channels, $\varphi(t) : Ag \times Comm \to \{0,1\}$ is the incidence function, $M(t) : Comm \to \mathbb{R}^n$ is the markup function in the moment of time $t \in [0, N_T]$. The M gives the features vector of the channel $comm \in Comm$. This features can be channel bandwidth, radio frequency etc.

We should study communication graph which is connected with the motion model. This means that should be given the function $pag : [0, N_T] \times Ag \to \mathbb{Z}^2$ which maps agent's coordinates to an agent in each moment $t \in [0, T]$. Also it means that $M(t)$ and $\varphi(t)$ depend on properties of cells in which incident agents are currently placed and on properties of cells between of them. Therefore connections between agents in the Γ can break and establish depending on the agents speed and landscape type.

Let us define that each agent $ag \in Ag$ has an own signal exchange timetable. It is possibly also to define specific signals like "enemy detecting", "grouping" etc. We can study various traffic models depending on the agents' timetables, motion speed and the landscape type.

Finally, we can define "requirements graph" Π and state that communication graph $\Gamma(t)$ should be similar with Π in some metric each moment of time. Such graph Π can be viewed as a fuzzy set of communication graphs, as an abstract container or as a generator of the stream of communication graphs.

Also, we developed the conflict model combined with the motion and communication model similar to described in the work [4] and its computer simulation "Bokohod". Agents emerge different kinds of tactics and exchange signals without any external control.

5. Computational experiment

Previously the author had developed the algorithm of the landscape generation with the given configuration enthropy. This algorithm constructs vector of the quantities of cells of the each class $V = (N_1, \ldots, N_l)$ by the given entropy S as follows:

S t e p 1. Solve the equation

$$S = -\frac{\beta(1-\beta^l) - (1-\beta) l \beta^l}{(1-\beta)(1-\beta^l)} \ln\beta + \ln\frac{1-\beta^l}{1-\beta}, \tag{5}$$

S t e p 2. Use the found solution $0 \le \beta \le 1$ and equation

$$N_1 = N\frac{1-\beta}{1-\beta^l}$$

to find N_1,

S t e p 3. Compose the vector $V_0 = (N_1, \beta N_1, \ldots, \beta^{l-1} N_1)$,

S t e p 4. Round the components of the V_0 up to integers and obtain the vector V_1 in this way. It is necessary to make rounding such that the sum of all components of V_1 would be equal to N.

The landscape was generated such as the discrete function of the obstacles $u : \mathbb{Z}^2 \to \mathbb{Z}$ would have local maxima strictly in $N_{obst} = V_l = \beta^{l-1} N_1$ cells. The author thinks that this method gives more natural-like landscapes as it makes "generally passable" area with some hardly passable subareas. As it known from [3] we can make very different landscapes with the same configuration entropy. By this reason, we will use the special, CA-based way of the filling landscape with cells of different classes. This method guarantees slow, near linear increasing of the TED at the increasing of the entropy.

Examples of obtained landscapes are shown on the fig. 2. Sample dependencies of the configuration entropy, TED and N_{obst} are shown on the fig.

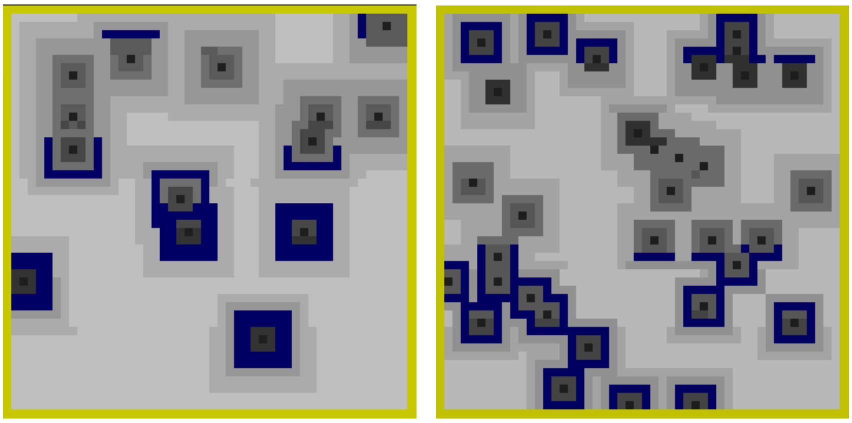
We set $l = 9$, $n = m = 48$, $o = 6$, choose $N_{obst} \in \{5\} \cup \{10i | i = \overline{1,25}\} \cup \{255\}$. For the each N_{obst} value generate landscape and perform experiment in moving agent from the cell ω_{11} to the cell ω_{nn} 100 times according to the algorithm, which was described earlier. Next we compute the time which is required for the experiment complexion T^i_{bok} and the time of the moving from the ω_{11} to the ω_{nn} by linear straight route T^i_{tup}. Then we calculate the mean value and standard deviation of the win of time for all of this series:

$$\overline{win} = \frac{1}{50}\sum_{i=1}^{50}\frac{T^i_{bok}}{T^i_{tup}}.$$

$$\sigma = \left(\sum_{i=1}^{50}(\overline{win} - \frac{T^i_{bok}}{T^i_{tup}})^2\right)^{1/2}.$$

It was found that the mean value of the win in transit time $\overline{win}$ and the configuration entropy of the landscape S are correlated with a correlation coefficient 0.959556 (see fig. 4.). The $\overline{win}$ and the TED of the landscape are correlated with a correlation coefficient 0.964763. The orange line in the figure corresponds to the curve $y = (S(N_{obst}) + 1)\ln 9$, the brown line corresponds to the curve

$$y = 0.922178 + 0.539383 TED(N_{obst}),$$



(a) $S = 1.6, N_{obst} = 14$ (b) $S = 1.8, N_{obst} = 33$

Fig. 2. Examples of Landscapes. A darker cell is more impassable.

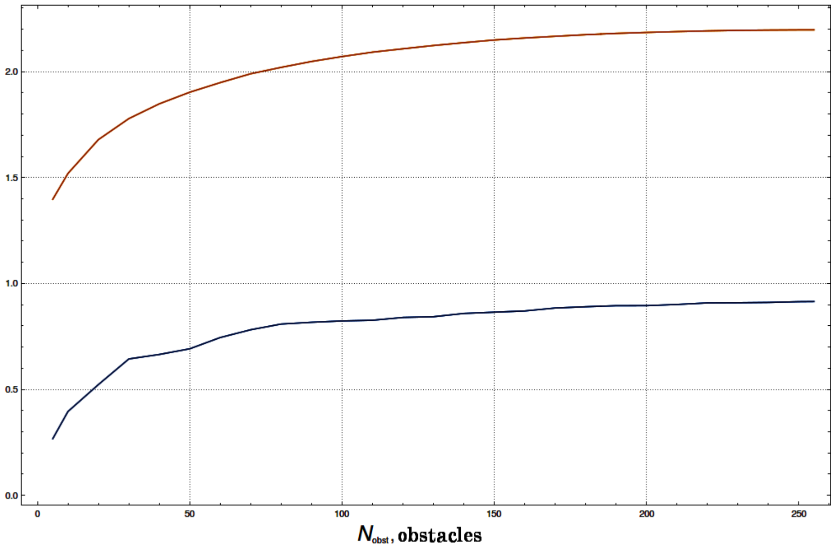


Fig. 3. Sample dependencies of the configuration entropy, TED and N_{obst}.

where $S(N_{obst})$ and $TED(N_{obst})$ are the entropy and the total edge density's mean value of the landscape with N_{obst} obstacles. Vertical bars correspond to the standard deviation of the win of time.

A computational experiment on finding velocity of the group formed from 48 agents have been also performed. This agents moved from the one side of the squared landscape to the opposite one. 50 random landscapes with the given entropy was generated by the aforementioned algorithm and dependence of the average number of agents N_{ag} have reached the target on the discrete time t was computed. It was found that this dependence is determined, basically, not by the particular kind of landscape, but by the landscape configuration entropy S (see fig. 5). Curves pictured below correspond to the function

$$N_{ag}(t, S) = \frac{48}{2}(1 + \operatorname{erf}(\phi_1(S)t - \phi_2(S))),$$

where parameters $\phi_1(S)$, $\phi_2(S)$ can be defined, for example, as follows

$$\phi_1(S) = -(-0.851367S^5 + 7.57352S^4 - 26.7493S^3 + 46.8414S^2 - 40.6265S + 13.9769),$$

$$\phi_2(S) = -(-48.2853S^5 + 442.611S^4 - 1603.33S^3 + 2865.57S^2 - 2521.87S - \frac{2.1176 \times 10^{-6}}{S - \ln 9} + 876.074)$$

and the values of S are 1.52949, 1.67909, 1.9899, 2.10915, 2.18557 from the left to the right.

6. Conclusion

There have been obtained dependence of the win of movement in accordance with the proposed algorithm on the landscape configuration entropy for certain types of landscapes. The immediately following result may be a comparison of a model of the conflict based on the proposed cellular automaton with the result of solution of the corresponding Osipov-Lanchester equations. Another possible result would be a comparison of models of the "diffusion" of agents into a given sub-area based on cellular automaton with the solution of the corresponding diffusion equation. Finally, we can simulate the sharing of the subjective reality layers between agents. In this case, one agent will use the information about the area, received from other agents and will transmit

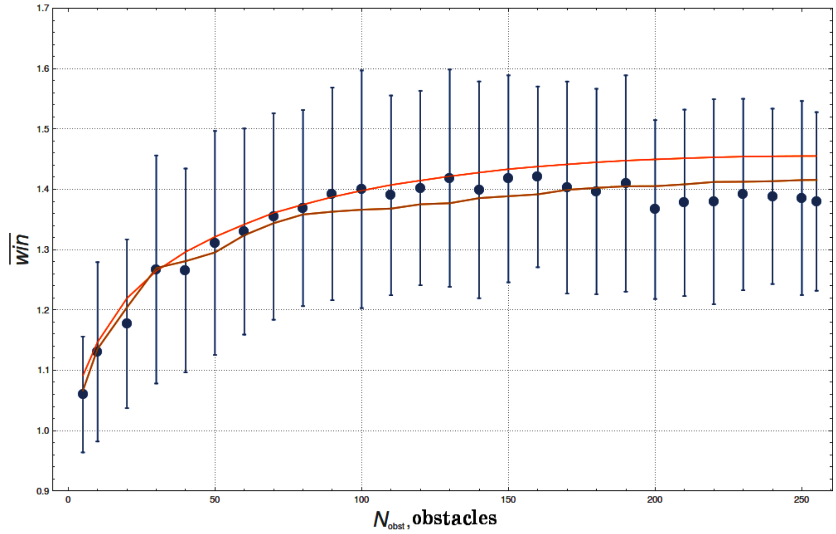


Fig. 4. Win of the time.

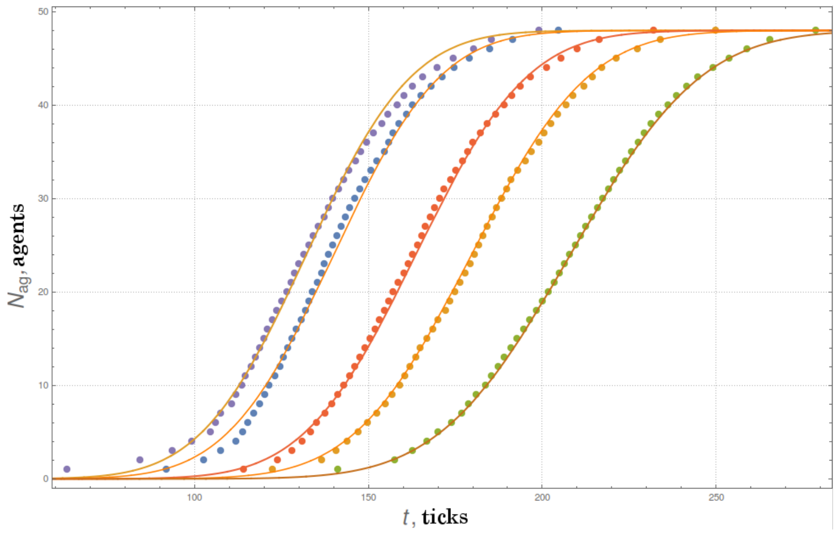


Fig. 5. The dependence of the average number of agents N_{ag} have reached the target on the discrete time t and entropy S.

such information to other agents itself. The algorithm described in the article can be applied to the mobile robot equipped with a transport base, navigation equipment (compass, GPS receiver, etc.), a sensor allowing to determine the impassibility of the terrain and the deciding unit, including memory.

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