Application of renyi mutual information in stochastic referencing of multispectral and multi-temporal images

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***Abstract*—The efficiency of relay stochastic algorithms for referencing multispectral and multi-temporal images is considered when using Renyi mutual information as a measure of similarity of the referenced images. A comparison is made with the situation of using correlation similarity measures.**

Keywords— image, referencing, image combination, mutual information, Renyi, stochastic algorithm, gradient, objective function.

1. INTRODUCTION

The necessity for image referencing arises in various applications: in the processing of medical and satellite images, the operation of autopilot systems, trajectory construction and many others [1,2]. One of the approaches to solve this problem is reduced to the problem of estimating the parameters of mutual geometric deformations of these images. The solution of the latter problem, in turn, is usually implemented iteratively as a search for the extremum of some multidimensional target function of evaluation quality in the space of referenced parameters [3]. The problem becomes more complicated when referenced multi-temporal and multispectral images, since in addition to geometric deformations caused by changes in the scene and the position of photodetectors, there are nonlinear brightness distortions in the images, the dependencies of which are not a priori known. This imposes additional strict requirements on the similarity measures of images, on the basis of which the target functions are constructed. Recently, information-theoretic [4, 5] has been increasingly used as such similarity measures. The paper investigates the effectiveness of using one of such measures – Renyi mutual information [6] – with stochastic non-identification image referencing.

2. DESCRIPTION OF ESTIMATION ALGORITHM

To synthesize an algorithm for estimating mutual geometric deformations of images $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ let us choose a recurrent relay stochastic procedure in the form [7]:

$$\hat{\boldsymbol{\alpha}}_t = \hat{\boldsymbol{\alpha}}_{t-1} \pm \boldsymbol{\Lambda}_t \operatorname{sign}\left(\boldsymbol{\beta}\left(\mathrm{J}\left(\hat{\boldsymbol{\alpha}}_{t-1}, \mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}\right)\right)\right), \qquad (1)$$

where $\hat{\boldsymbol{\alpha}}$ is a vector of estimates of deformation parameters $\boldsymbol{\alpha}$; $\boldsymbol{\beta}(\mathrm{J}(\bullet))$ is the stochastic gradient of target function $\mathrm{J}(\bullet)$ of estimation quality; $\boldsymbol{\Lambda}$ is a learning matrix that determines the rate of change in grades during estimation on iterations; $t = \overline{1, T}$ is the number of iterations. A two-dimensional local sample Z_t of pixels $z^{(1)}_{\mathbf{j}t} \in \mathbf{Z}^{(1)}$, $z^{(2)}_{\mathbf{j}t} \in \mathbf{Z}^{(2)}$ with small size μ is used at each iteration. On the base of it sample the stochastic gradient of target function is calculated (in this problem it is the gradient estimate), $\mathbf{j}_t \in \Omega_t$ is the coordinate vector of the element Z_t.

To find Renyi mutual information at the next iteration of the algorithm, it is necessary to estimate the probability density (PD) of images $\tilde{\mathbf{Z}}^{(1)}_t$ and $\mathbf{Z}^{(2)}$ on the base of a local sample Z_t, where $\tilde{\mathbf{Z}}^{(1)}_t$ is the oversampled image $\mathbf{Z}^{(1)}$ according to estimates $\hat{\boldsymbol{\alpha}}_{t-1}$. At the same time, in order to reduce computational costs, the Parsen window method was used [8]. The PD estimate is found as a superposition of approximating functions $f\left(z - z_i\right)$ (elementary PD) of the same shape, centered on the brightness of all pixels z_i, $i = \overline{1, \mu}$ that are in the local sample:

$$p(z) = \mu^{-1} \sum_{z_i \in Z_t} f\left(z - z_i\right). \qquad (2)$$

In the paper, a Gaussian function was used as an approximation function.

Renyi mutual information is found from Renyi entropy:

$$S = \left(H\left(\tilde{\mathbf{Z}}^{(1)}_t\right) + H\left(\mathbf{Z}^{(2)}\right)\right) \Big/ H\left(\tilde{\mathbf{Z}}^{(1)}_t, \mathbf{Z}^{(2)}\right) \qquad (3)$$

where $H(\mathbf{Z}) = (1-\alpha)^{-1} \lg \sum_i p^{\alpha}_{zi}$ is the image entropy, $H\left(\tilde{\mathbf{Z}}^{(1)}_t, \mathbf{Z}^{(2)}\right) = (1-\alpha)^{-1} \sum_i \sum_k p^{\alpha}_{zi,zk}$ is the joint image entropy, p_z and $p_{z1,z2}$ are estimates of PD and joint brightness PD, $i, k = \overline{1, \mu}$ [9].

When usage of Renyi mutual information as the target function to calculate the stochastic gradient of MI, it is necessary to find its partial derivatives with respect to the estimated parameters. In this paper, analytical expressions for derivatives were found. So for the derivatives of the single and joint Shannon entropy according to the estimated parameters, we obtain expressions (4) and (5), respectively

$$\frac{\partial \hat{H}\left(\mathbf{Z}^{(k)}\right)}{\partial \overline{\alpha}} = \sum_{z^{(k)}_i \in Z_b} \frac{\left(\sum\limits_{z^{(k)}_j \in Z_a} \nabla^k_{ij}\right)^{(\alpha-1)} \left(\sum\limits_{z^{(k)}_j \in Z_a} \nabla^k_{ij} \Delta^k_{ij} \dfrac{\partial \Delta^k_{ij}}{\partial \overline{\alpha}}\right)}{(1-\alpha)\sigma^2 \sum\limits_{z^{(k)}_i \in Z_b} \left(\sum\limits_{z^{(k)}_j \in Z_a} \nabla^k_{ij}\right)^{\alpha}} \qquad (4)$$

$$\frac{\partial \hat{H}\left(\mathbf{Z}^1,\mathbf{Z}^2\right)}{\partial\bar{\alpha}} = \sum_{z_i^{(k)}\in Z_b} \frac{\left(\sum_{z_j^{(k)}\in Z_a}\nabla_{ij}^1\nabla_{ij}^2\right)^{(\alpha-1)}\left(\sum_{z_j^{(k)}\in Z_a}\nabla_{ij}^1\nabla_{ij}^2\Delta_{ij}^1\frac{\partial\Delta_{ij}^1}{\partial\bar{\alpha}}\right)}{(1-\alpha)\sigma^2\sum_{z_i^{(k)}\in Z_b}\left(\sum_{z_j^{(k)}\in Z_a}\nabla_{ij}^1\nabla_{ij}^2\right)^{\alpha}} \qquad (5)$$

where $\nabla_{ij}^{(1,2)} = f\left(z_i^{(1,2)} - z_k^{(1,2)}\right)$; $\Delta_{ij}^{(1,2)} = z_i^{(1,2)} - z_k^{(1,2)}$; σ^2 is variance of $f(\bullet)$; $Z_{a,b}\in Z_t$ (in the experiments the local sample is divided equally); $k=1,2$.

In this case, the gradient of Renya MI is

$$\boldsymbol{\beta} = \left(H^{-2}\left(\tilde{\mathbf{Z}}_t^{(1)},\mathbf{Z}^{(2)}\right)H\left(\tilde{\mathbf{Z}}_t^{(1)},\mathbf{Z}^{(2)}\right)\partial H\left(\tilde{\mathbf{Z}}_t^{(1)}\right)\Big/\partial\bar{\alpha} - \right.$$
$$\left. -\left(H\left(\tilde{\mathbf{Z}}_t^{(1)}\right)+H\left(\mathbf{Z}^{(2)}\right)\right)\partial H\left(\tilde{\mathbf{Z}}_t^{(1)},\mathbf{Z}^{(2)}\right)\Big/\partial\bar{\alpha}\right). \qquad (6)$$

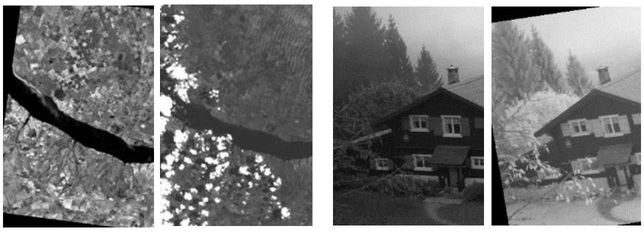
3. THE RESULTS OF THE ALGORITHM

The approbation of the developed algorithm was carried out on multi-temporal and multi-spectral images. The results were compared with the results of the algorithm based on the meringue cross-correlation similarity measure [10] (hereinafter the correlation algorithm). The parameters of the similarity model, including shift, rotation angle and scale factor, were used as estimated. So, in Fig. 1a, for example, fragments of two different-time satellite images of the same domain are shown, and in Fig. 1b of two images of the same object in the visible and infrared ranges of the spectrum, having mutual geometric deformations, in particular a rotation of 9^0. The results of estimating this parameter, averaged over 30 implementations, are presented in Fig. 2, where, for example, the dependences on the local sample size (LSS) of the variance of the estimated parameter (Fig. 2a) and the number of iterations required to achieve a steady-state evaluation mode are given. The steady-state mode is characterized by the fact that the change in the average value of estimates in a sliding window of some size does not exceed a specified threshold. In the same window, the variance of estimates of deformation parameters is also calculated.

4. CONCLUSION

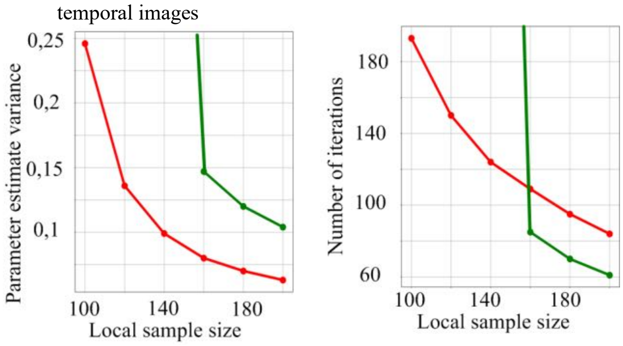
It can be seen from the graphs that with a small μ the correlation algorithm shows the estimation failure and only, starting from μ=160 the convergence of parameter estimates is observed. The proposed algorithm has stable convergence in the entire reduced range of LSS. In addition, it provides approximately 1.4 times less error of estimate parameters referencing, in the domain where the convergence of the correlation algorithm is achieved. It should also be noted that in the convergence domain, the correlation algorithm provides a slightly higher rate of convergence of deformation parameter estimates, which is explained by the fact that finding the numerical value of MI itself requires a larger sample size.

Thus, the usage of Renyi MI as the target function is advisable when image referencing of general morphology under conditions of uncertainty of large nonlinear brightness distortions.



(a) (b)

Fig. 1. Example of multispectral (visible and infrared spectrum) and multi-temporal images



(a) (b)

Fig. 2. The dependence of the number of iterations for convergence and variance on LSS

ACKNOWLEDGMENT

The work was supported by the Russian Science Foundation according to research projects No. 22-21-00513, https://rscf.ru/en/project/22-21-00513/.

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