Analysis of algorithms for implementing Delaunay triangulation

V. Mikhailov[1], P. Pchenitchnyi[1], R. Tagirov[1], R. Khabibrakhmanov[1], R. Shaimukhametov[1]

[1]*Kazan Federal University, Kremlevskaya 18, Kazan, Russia, 420008*

Abstract
The results of the analysis of some algorithms for the implementation of Delaunay triangulation for constructing surfaces and geometric figures on a plane in terms of execution time, ease of implementation and degree of complexity are presented.

Keywords
Delaunay triangulation, implementation complexity, analysis, execution time

1. Introduction

Some problems of modeling surfaces and geometric shapes on a plane have a solution by the Delaunay triangulation method [1, 2]. We present the results of the analysis of several methods for implementing Delaunay triangulation.

2. Calculations

The Delaunay triangulation is constructed using the following triangulation algorithms: incremental, recursive divide - and - conquer, sweep – line (single pass sweep - line and double pass strip) [3].

Tables 1 and 2 in rows show: average computation time in seconds - t_{av}, number of processing points - N_p, ease of implementation (the simpler, the more *) - Simplicity, implementation complexity - Complexity [4].

The calculations were carried out in the Mathematics 12.2 system on a desktop PC. At the first stage, a set of randomly generated points was constructed, and the Delaunay condition [5] was checked. Then, for each algorithm, 40 iterations were performed for 1000, 10000 and 100000 points, the difference in the results of each iteration is no more than 3% for 1000 points and no more than 5% for 100000 points. The table summarized the average values of the parameters presented.

Table 1
Performance indicators of Delaunay triangulation algorithms

Algorithm	Incremental	Divide - and - Conquer	Sweep – Line	Sweep – Line (band-pass)
		On the plane		
t_{av} (N_p =1000)	0,34	0,27	0,29	0,3
t_{av} (N_p =10000)	3,87	3,14	3,21	3,29
t_{av} (N_p =100000)	40,12	35,17	36,25	37,11
Simplicity	*****	**	****	***
Complexity	$O(N^{3/2})$	$O(N \cdot logN)$	$O(N \cdot logN)$	$O(N)$
		In three dimensional space		
t_{av} (N_p =1000)	0,48	0,37	0,38	0,41
t_{av} (N_p =10000)	5,88	4,25	4,23	4,66
t_{av} (N_p =100000)	51,44	40,51	41,63	43,87
Simplicity	***	*	***	**
Complexity	$O(N^{3/2})$	$O(N \cdot logN)$	$O(N \cdot logN)$	$O(N)$

Calculations have shown that the divide - and - conquer algorithm has the shortest execution time and one of the best indicators of labor intensity, but it is quite difficult to implement. The Incremental

algorithm has the worst performance, besides the ease of implementation. The sweep – line algorithm is slightly inferior in terms of time and has the same labor intensity on average as the divide - and - conquer algorithm, but it is quite simple to implement, both on a plane and in space.
An analysis of the effectiveness of two different versions of the Sweep - Line (band-pass) algorithm showed that the two-pass algorithm has better timing indicators. On a relatively small set of points, this difference is imperceptible, but on a sufficiently large number - a difference of several seconds.

Table 2
Efficiency indicators of one-pass and two-pass variants of the band-pass Delaunay triangulation algorithm

Algorithm	Single	Two-pass
	On the plane	
t_{av} (N_p =1000)	0,3	0,3
t_{av} (N_p =10000)	3,29	3,12
t_{av} (N_p =100000)	37,11	34,95
Simplicity	***	**
Complexity	$O(N)$	$O(N)$
	In three dimensional space	
t_{av} (N_p =1000)	0,41	0,4
t_{av} (N_p =10000)	4,66	4,41
t_{av} (N_p =100000)	43,87	41,17
Simplicity	**	*
Complexity	$O(N)$	$O(N)$

3. Conclusions

According to our calculations, the most optimal of the considered methods for implementing the Delaunay triangulation is the divide - and - conquer algorithm.

4. References

[1] Delaunay, B.N. Sur la sph`ere vide. A la m´emoire de Georges Vorono¨I / B.N. Delaunay // Bulletin de l’Acad´emie des Sciences de l’URSS. Classe des sciences math´ematiques et na. – 1934. – Vol. 6. – P. 793-800.
[2] Skvortsov, A.V. Delaunay triangulation and its application / A.V. Skvortsov. – Tomsk: Publishing house of vol. University, 2002. - 128 p.
[3] Sinitsyn, S.I. A hybrid recursive-incremental algorithm for constructing a Delaunay triangulation / S.I. Sinitsyn [Electronic resource]. – Access mode: https://studylib.ru/doc/2336718/inkrementnyj-algoritm-postroeniya-triangulyacii-delone
[4] Sinclair, D.A. S-hull: a fast radial sweep-hull routine for Delaunay triangulation [Electronic resource]. – Access mode: https://www.researchgate.net/publication/301880550_S-ull_a_fast_radial_sweep-hull_routine_for_Delaunay_triangulation.
[5] Skvortsov, A.V. Efficient construction algorithms Delaunay triangulations / A.V. Skvortsov, Yu.L. Kostyuk // Geoinformatics. Theory and practice. – Tomsk: University press, 1998. – Vol. 1. – P. 22-47.