An algorithm for rough alignment of point clouds in three-dimensional space

A. Makovetskii[1], S. Voronin[1], V. Kober[1], A. Voronin[1]

[1]Chelyabinsk State University, Bratiev Kashirinykh 129, Chelyabinsk, Russia, 454001

Abstract. Commonly registration of 3D point clouds consists of two steps: preliminary coarse registration and precise registration. A typical algorithm for the precise registration is the Iterative Closest Point (ICP) algorithm. To obtain an admissible result of the registration for point clouds with quite different initial positions, a coarse registration algorithm should be also be used. In this presentation, we propose a pure geometric coarse registration algorithm. For two given point clouds with arbitrary initial positions, approximation of the orthogonal transformation between the clouds is suggested. Computer simulation results are provided to illustrate the performance of the proposed algorithm.

1. Introduction

Point cloud registration is an important step in 3D reconstruction. Point cloud data can be obtained at different viewpoints with acquisition devices such as 3D laser scanner and Kinect camera [1]. Point cloud registration is widely used in various applications such as teleoperations, nondestructive testing, autonomous vehicle navigation, pose tracking, and SLAM [2,3]. Generally, two point clouds of an object or scene are captured at two time instances, possibly from different camera positions due to object occlusion, surface layout or measurement methods [4]. Consequently, multiple point clouds from various coordinates can be merged into a composite model with a common coordinate system.

In this paper, we propose a new fast and efficient coarse registration algorithm. The proposed algorithm consists of three steps:

1) computation of the mass centers of the template and target point clouds;

2) computation of orthogonal vector triple of the point clouds based on pure geometrical characteristics of the clouds;

3) finding the orthogonal transformation that aligns the two point clouds.

Note that proposed geometrical method is similar to the inertia ellipsoid method known in mechanics [13]. After the coarse registration, the precise alignment can be done by various ICP algorithms [14-21]. Note, that the proposed algorithm extremely improves the convergence of ICP algorithms. Computer simulation results are provided to illustrate the performance of the proposed method.

2. Orthogonal transformation for two point clouds

Let $P = \{p_1, \ldots, p_s\}$ be a template point cloud, and $Q = \{q_1, \ldots, q_s\}$ be a target point cloud in $\mathbb{R}^3$. Suppose that relationship between points in P and Q is given in such a manner that

$$Q = R_{true}P + t_{true}, \tag{1}$$

where R_{true} is the rotation matrix, t_{true} is the translation vector. The aim of the coarse alignment is an approximate computation of the true geometrical transformation.

2.1. The mass centers

Let c_P and c_Q be the mass centers of the clouds P and Q, respectively.

$$\begin{cases} c_P = \frac{1}{n}\sum_{i=1}^{s} p_i \\ c_Q = \frac{1}{n}\sum_{i=1}^{s} q_i \, , \end{cases} \tag{2}$$

where $c_P = ((c_P)_x, (c_P)_y, (c_P)_z)$ and $c_Q = ((c_Q)_x, (c_Q)_y, (c_Q)_z)$.

2.2. The main axis of the point cloud

Suppose that the main axis contains the mass center c_P of the cloud P. Denote by $v = (v_x, v_y, v_z)$ the directing vector of the axis. Denote the vector that connects points c_P and p_i by cp_i

$$cp_i = (p_{ix} - c_x, p_{iy} - c_y, p_{iz} - c_z), \tag{3}$$

and the L_2 distance from the point p_i to the axis by d_i

$$d_i = \frac{||[v, cp_i]||}{|v|}. \tag{4}$$

We find the main axis of the point cloud as a straight line that is the closest to all points of the cloud in sense of the Euclidean distance in $\mathbb{R}^3$.

Suppose that $|v| = 1$, then the following condition holds:

$$d_i = \left| \begin{vmatrix} e_1 & e_2 & e_3 \\ v_x & v_y & v_z \\ p_{ix} - c_x & p_{iy} - c_y & p_{iz} - c_z \end{vmatrix} \right| =$$

$$= |(v_y(p_{iz} - c_z) - v_z(p_{iy} - c_y), -v_x(p_{iz} - c_z) + v_z(p_{ix} - c_x), v_x(p_{iy} - c_y) - v_y(p_{ix} - c_x))|. \tag{5}$$

Consider the variational problem:

$$\sum_{i=1}^{n} \frac{||[v, cp_i]||^2}{|v|^2} \to min, \tag{6}$$

subject to $|v| = 1$.

Denote $J(v, \lambda)$ the following functional:

$$J(v, \lambda) = \sum_{i=1}^{n} d_i^2 + \lambda(v_x^2 + v_y^2 + v_z^2 - 1), \tag{7}$$

where $\lambda > 0$ and consider the variational problem:

$$J(v, \lambda) \to min_{v, \lambda} \quad . \tag{8}$$

Note that constrained variational problem (6) is equivalent to the problem (8). Write the functional J(v, λ) in the form:

$$J(v, \lambda) = \sum_{i=1}^{n} \left(\left(v_y(p_{iz} - c_z) - v_z(p_{iy} - c_y) \right)^2 + \left(v_x(p_{iz} - c_z) - v_z(p_{ix} - c_x) \right)^2 + \right.$$
$$\left. + (v_x(p_{iy} - c_y) - v_y(p_{ix} - c_x))^2 + \lambda(v_x^2 + v_y^2 + v_z^2 - 1). \tag{9}$$

Denote by r_i the following vector:

$$r_i = (p_{ix} - c_x, p_{iy} - c_y, p_{iz} - c_z). \tag{10}$$

Rewrite the formula (9) in the form:

$$J(v, \lambda) = \sum_{i=1}^{n}((v_y r_{iz} - v_z r_{iy})^2 + (v_z r_{ix} - v_x r_{iz})^2 + (v_x r_{iy} - v_y r_{ix})^2 +$$
$$+ \lambda(v_x^2 + v_y^2 + v_z^2 - 1). \tag{11}$$

Compute the components of J(v, λ) gradient

$$\frac{\partial J}{\partial v_x} = 2v_x(\lambda + \sum_{i=1}^{n}(r_{iz}^2 + r_{iy}^2)) - 2v_y \sum_{i=1}^{n}(r_{ix}r_{iy}) - 2v_z \sum_{i=1}^{n}(r_{ix}r_{iz}), \tag{12}$$

$$\frac{\partial J}{\partial v_y} = 2v_y(\lambda + \sum_{i=1}^{n}(r_{iz}^2 + r_{ix}^2)) - 2v_x \sum_{i=1}^{n}(r_{ix}r_{iy}) - 2v_z \sum_{i=1}^{n}(r_{iy}r_{iz}), \tag{13}$$

$$\frac{\partial J}{\partial v_z} = 2v_z(\lambda + \sum_{i=1}^{n}(r_{ix}^2 + r_{iy}^2)) - 2v_x \sum_{i=1}^{n}(r_{ix}r_{iz}) - 2v_y \sum_{i=1}^{n}(r_{iy}r_{iz}). \tag{14}$$

We obtain the following system of linear equations:

$$\begin{pmatrix} \lambda + \sum_{i=1}^{n}(r_{iz}^2 + r_{iy}^2) & -\sum_{i=1}^{n}(r_{ix}r_{iy}) & -\sum_{i=1}^{n}(r_{ix}r_{iz}) \\ -\sum_{i=1}^{n}(r_{ix}r_{iy}) & \lambda + \sum_{i=1}^{n}(r_{iz}^2 + r_{ix}^2) & -\sum_{i=1}^{n}(r_{iy}r_{iz}) \\ -\sum_{i=1}^{n}(r_{ix}r_{iz}) & -\sum_{i=1}^{n}(r_{iy}r_{iz}) & \lambda + \sum_{i=1}^{n}(r_{ix}^2 + r_{iy}^2) \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{15}$$

Rewrite the formula (14) by the expressions:

$$\begin{pmatrix} \lambda + a & d & e \\ d & \lambda + b & f \\ e & f & \lambda + c \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{16}$$

$$\begin{pmatrix} -a - \lambda & -d & -e \\ -d & -b - \lambda & -f \\ -e & -f & -c - \lambda \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{17}$$

$$\left(\begin{pmatrix} -a & -d & -e \\ -d & -b & -f \\ -e & -f & -c \end{pmatrix} - \lambda E \right) \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{18}$$

$$\begin{pmatrix} -a & -d & -e \\ -d & -b & -f \\ -e & -f & -c \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \lambda \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \tag{19}$$

note that eigenvalues in the formula (19) are negative.

$$(-1) \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = (-1)\lambda \begin{pmatrix} av_x + dv_y + ev_z \\ dv_x + bv_y + fv_z \\ ev_x + fv_y + cv_z \end{pmatrix}, \tag{20}$$

Denote by A the matrix

$$A = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}. \tag{21}$$

Then we have

$$< Av, v >= \sum_{i=1}^{n} ((v_y r_{iz} - v_z r_{iy})^2 + (v_z r_{ix} - v_x r_{iz})^2 + (v_x r_{iy} - v_y r_{ix})^2, \tag{22}$$

$$Av = -\lambda v, \tag{23}$$

$$< -\lambda v, v >= -\lambda < v, v >= -\lambda ||v||^2 = -\lambda =$$
$$= \sum_{i=1}^{n} ((v_y r_{iz} - v_z r_{iy})^2 + (v_z r_{ix} - v_x r_{iz})^2 + (v_x r_{iy} - v_y r_{ix})^2. \tag{24}$$

2.3. The geometrical transformation

We compute the mass center c_P and orthogonal vector basis v_1, v_2, v_3 of the cloud P, mass center c_Q and orthogonal vector basis u_1, u_2, u_3 of the cloud Q. Suppose that matrix V consists of the vector-columns v_1, v_2, v_3, matrix U consists of the vector-columns u_1, u_2, u_3. Then coarse point cloud alignment takes the following form:

$$t = c_Q - c_P, \tag{25}$$
$$R = UV^{-1}. \tag{26}$$

3. Conclusion

3D point cloud registration consists of two steps: preliminary coarse registration and precise registration. We proposed a pure geometric coarse registration algorithm. For two given point clouds with arbitrary initial positions, an approximation of the orthogonal transformation between the clouds is suggested.

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5. References

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